the average shear stress or traction should reach a plateau. This is clearly not the case in Fig. 9 at 100°C.

The rheological model presented here was developed from the results of several high shear stress rheometers of two different types—translating concentric cylinder devices at the higher pressures and a rotating concentric cylinder at the higher shear rates. These devices have yielded consistent and complimentary results in agreement with viscosity measured in a falling body viscometer, shear modulus measured with ultrasonics by another laboratory, and with limiting shear stress measured with an impact shear plate by yet another laboratory.

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DISCUSSION

M. P. F. Sutcliffe[1] and K. L. Johnson[1]

For a number of years the authors have devoted their skill and energy into investigating the rheological properties of lubricants under the extreme conditions found in concentrated contact by techniques and experiments which are independent of the rolling-sliding contact situation. They are to be congratulated on this excellent work. Most other investigators, including ourselves, have taken the less demanding path of attempting to deduce fundamental fluid properties from rolling contact experiments. Ideally these two approaches should converge upon a common constitutive law for the fluid. The present situation seems to be that in some respects they do, but in others apparent discrepancies remain.

The main points of agreement are twofold:

(1) The behavior can be broadly characterized by a nonlinear Maxwell model which, in its simplest one-dimensional form, may be written (in the authors' notation)

$$\dot{\gamma}=\frac{d(\tau/G)}{dt}+\frac{\tau_r}{\mu}F(\tau/\tau_r) \tag{1}$$

where τ_r is a representative stress.

(2) At any given pressure and temperature a "limiting shear stress" τ_L is reached at which the fluid shears at a constant stress, independent of shear rate, in the manner of a plastic solid. τ_L is found to be linearly related to pressure by an equation of the form

$$\tau_L=A+Bp \tag{2}$$

where A and B are mild functions of temperature and the first term is generally small compared with the second.

The main point of discrepancy concerns the form and phys-

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ical interpretation of the nonlinear viscous function $F(\tau/\tau_r)$ in Eq. (1). When care is taken to make due allowance for viscous heating, disk machine isothermal traction tests in appropriate conditions give a linear variation of mean shear stress with log shear rate (sliding speed ÷ film thickness). This behavior strongly suggests thermally activated flow and has been interpreted by us and others (e.g., Hirst and Moore) in terms of the Eyring theory which predicts a hyperbolic sine for the function $F(\tau/\tau_r)$. We have called the representative stress the "Eyring stress," denoted by τ_o. It is measured from the gradient of the stress-log shear rate plot and indicates the stress above which significant nonlinearity occurs (authors' Eq. (2)).

It should perhaps be noted that the concept of thermally activated flow and that of a limiting stress are not mutually exclusive. When the mechanical work term τv_τ in the Eyring equation reaches the magnitude of the activation energy $(E+pv_p)$, thermal activation ceases to be a significant mechanism of flow compared with solid-like shearing of van der Waals' bonds.[2] This transition would be expected at some roughly constant fraction of the shear modulus about G/30, as found in the authors' experiments.

In their viscometric experiments (e.g., Fig. 6) the authors have not found clear evidence of thermally activated flow and have instead attributed nonlinearity in the stress-strain rate response to the transition from Newtonian flow to rate independent flow at the limiting shear stress. This transition is expressed in terms of the limiting stress τ_L by their equation (15), in which the quantity p^* is related to the independently measured glass transition pressure p_g.

In order to make an unequivocal choice between these two interpretations of the data, it is necessary to compare results obtained under similar conditions of pressure, temperature and strain rate. This has not proved easy through experimental limitations of both techniques.

One of the few truly comparable set of conditions is that shown by the lowest set of data in Fig. 9 (5P4E at a pressure of 0.47 GPa and temperature 100°C). In this case it is a matter of opinion whether the data should be interpreted in terms of the authors' Eq. (15) or the Eyring Eq. (2).

For traction experiments to show an extended range of Eyring-like response, the conditions should be such that the limiting stress τ_L and the onset of nonlinearity τ_o should be widely separated. Such conditions are found with lower viscosity fluids at higher temperatures. To our knowledge these are conditions which have not been explored by the authors' techniques. We hope they will be.

For example, we show in the attached figure traction test data for HVI 650 at a mean pressure of 0.94 GPa and at 100°C where the ratio of τ_L to τ_o is about 10. In order to apply the authors' Eqs. (14) and (15) to this example, we have taken the viscosity of HVI 650 at 100°C to vary with pressure according to:

$$\mu \simeq 0.027\exp(1.6\times10^{-8}p-4.1\times10^{-18}p^2) \quad (3)$$

where p is the pressure in Pa. No direct measurements of limiting stress at 100°C are available, but on the basis of experiments at lower temperatures and the data of Bair and Winer (1982) we take

$$\tau_L \simeq 0.060p \quad (4)$$

The glass transition pressure is 0.8 GPa at 30°C (from Fig. 14). Using the authors' data for the mineral oil N1 suggests a value of $p_g \simeq 1.5$ GPa at 100°C. In this way we have computed curves *A* and *B* in our figure. The correspondence of these predictions with the experimental traction curve is poor. Changing the viscosity μ in Eq. (15) would displace curve *B* along the $\dot\gamma$ axis without change of shape. To reduce the gradient of the theoretical curve to a value approaching that of the experimental results would require the glass transition pressure to be about 0.5 GPa as illustrated in curve *C*. This would seem to be an unrealistically low value, since the viscosity of HVI 650 at this pressure and temperature is less than 50 Pas. We have carried out a similar exercise for a low viscosity kerosene metal-rolling lubricant (Sutcliffe, 1991)[3] with the same conclusion: to fit Eq. (15) to the traction data necessitates a value for the glass transition pressure at which the viscosity is unreasonably low (~10 Pas).

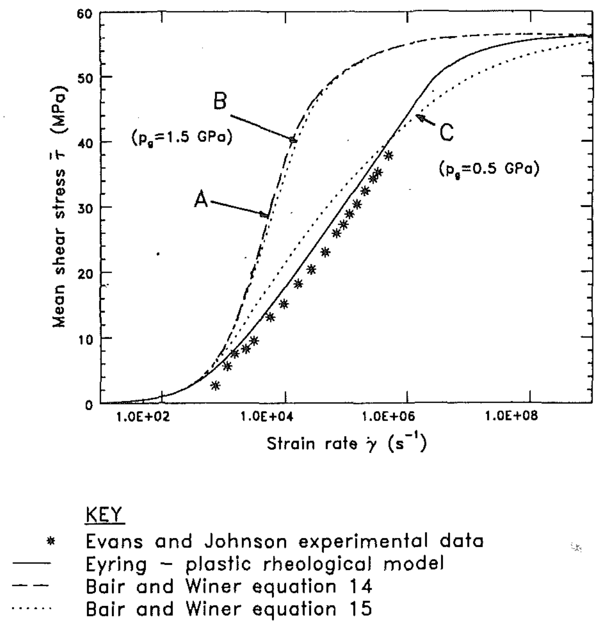

Fig. 14 Traction curve for HVI 650, $p = 0.94$ GPa, at 100°C

The authors have commented that traction test data, plotted in terms of the mean shear stress, may give a distorted picture of the true stress-strain rate relationship due to the variation in pressure through the contact. We accept that this criticism may have some validity in those tests where the mean shear stress is close to the limiting stress. To examine this point in a more general way we consider the example of HVI 650 at 100°C shown in the figure. On the assumption of Eyring response up to the limiting stress, we have computed the variation of shear stress through the contact, taking into account the effect of pressure on μ, τ_o, and τ_L. The mean shear stress is found by integration and is also plotted in the figure. In the range of the experimental data it is clear that, in this case, the theoretical curve still follows the Eyring shape, uninfluenced by the limiting stress and that the errors in the fitted values of μ and τ_o are small.

In conclusion we are inclined to speculate that, as in earlier controversies in this subject, both views will turn out to be broadly correct—in different regimes of operational variables!

K. T. Ramesh[4]

The authors are eminently qualified to attempt to bring together the existing information into one constitutive model, having already contributed the larger part of our current understanding of the high shear stress behavior of lubricants. The range of this work certainly reflects that background. This discusser would like to further explore some particular areas of interest.

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[2] E = activation energy at atmospheric pressure, v_τ and v_p are activation volumes for shear and pressure, respectively.

[3] Sutcliffe, M. P. F., "Measurements of the Rheological Properties of a Kerosene Metal-Rolling Lubricant," *Proc. Inst. Mech. Engrs.*, Pt. C, Vol. 205, 1991c, p. 215.

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