

AUTHOR'S CLOSURE

The author wishes to thank Dr. Prager for his discussion and is in agreement with the general views on plasticity which are expressed. The thought of the paper however, was not to obtain all inclusive stress-strain laws, but to write a simple law which describes for small strains, essential features of both elastic and plastic properties of a material. In view of Dr. Prager's discussion it would be interesting to write down the flow law which is implied by Equations [5] if it is assumed that the material, after undergoing permanent strain, still obeys Equations [5] for small new strains  $d\epsilon_{ij}$ . These and other considerations are included in a paper now being written by the author.

### Forced Torsional Vibrations With Damping: An Extension of Holzer's Method<sup>1</sup>

H. PORITSKY.<sup>2</sup> It will be noted that the authors have followed the use of the "shorthand notation," which is of the complex-amplitude method, for studying discrete masses, thus transforming Equation [1] into Equation [3] at a considerable saving of complexity over the equations which would have resulted from direct substitution of the real solutions [2] into [1]. However, in solving the problem of distributed inertia with damping, namely, Equation [4], they have followed scrupulously the complicated procedure consisting in utilizing Equation [5], resulting in two simultaneous differential Equations [6]; while pointing out that Equation [4] can be written in a form similar to Equation [3], they dismiss it with the remark that "it is full of imaginaries," and feel that a point has been scored in Equations [5] to [13], because all the quantities are real. The logic of the foregoing procedure is obscure to this reviewer, who feels that, properly introduced, complex amplitudes have their place in distributed linear problems with damping where they result even in more spectacular saving of complexity and in greater elegance.

The connection between the "shorthand notation" and sinusoidal quantities

$$\varphi = a \cos \omega t + b \sin \omega t \dots\dots\dots [1]$$

lies in the fact that Equation [1] of this discussion is the real part of the complex solution

$$\varphi = c e^{j\omega t}, \quad c = a - jb \dots\dots\dots [2]$$

By assuming a time factor  $e^{j\omega t}$  in the various quantities  $\varphi_n(t)$ ,  $\varphi(x,t)$

$$\varphi_n(t) = \varphi_n e^{j\omega t} \dots\dots\dots [3]$$

$$\varphi(x,t) = \varphi(x) e^{j\omega t} \dots\dots\dots [4]$$

and substituting in the equations one can eliminate the factor  $e^{j\omega t}$ , thus reducing the calculations to the complex amplitudes  $\varphi_n$ ,  $\varphi(x)$  whose real and imaginary parts yield the amplitude of 0-deg-phase and the 90-deg-phase vibration components. The expression [2] of this discussion is equivalent to the "vector-crank" diagram, where  $c$  corresponds to the initial position of the vector,  $e^{j\omega t}$  represents the uniform angular rotation of the crank,

<sup>1</sup> By J. P. Den Hartog and J. P. Li, published in the December, 1946, issue of the JOURNAL OF APPLIED MECHANICS, Trans. A.S.M.E., vol. 68, p. A-276.

<sup>2</sup> Engineering General Division, General Electric Company, Schenectady, N. Y. Mem. A.S.M.E.

and taking the real part to the projection on the axis of reals.<sup>3</sup> Proceeding thus, Equation [4] of this discussion furnishes

$$\varphi'' = -\lambda^2 \varphi, \quad \lambda^2 = \frac{I_1 \omega^2 - c_{10} j \omega}{k_1 + c_{11} j \omega} \dots\dots\dots [5]$$

whose solution, subject to the conditions  $\varphi' = 0$ ,  $\varphi = 1$  for  $x = 0$ , is

$$\varphi = \cos \lambda x \dots\dots\dots [6]$$

yielding

$$M = (k_1 + j \omega c_{11}) \lambda \sin \lambda x \dots\dots\dots [7]$$

The expression of the  $\sin \lambda x$ ,  $\cos \lambda x$  in terms of trigonometric and hyperbolic functions of  $ax$ ,  $bx$ , where  $\lambda = a + jb$ , is readily carried out yielding of course results agreeing with Fig. 5 and Table 2 of the paper.<sup>4</sup>

In conclusion, the writer wishes to emphasize that he is not taking any issue with the correctness of the authors' solution and that the point brought out refers merely to the relative merits of the complex versus the real method of arriving at the same result.

### An Improved Fuse Escapement for the Mark 18 and Other U. S. Navy Mechanical Time Fuses<sup>1</sup>

R. H. WHITEHEAD.<sup>2</sup> Fig. 2 of the paper illustrates the British mechanical time fuse driven solely by a coiled mainspring previously wound and using the form of escapement shown in Fig. 3. The beat of this escapement was also reduced by adding weights to the lever  $B$  of Fig. 3, as in the case of the improved escapement of the authors.

The escapement shown in Fig. 3 is a so-called "mixed escapement" and is the invention of the late Dr. Junghans of Schramberg, Germany, and used also in the German mechanical time fuses for anti-aircraft and other artillery applications.

The mixed escapement is basically an inferior timekeeper to the escapements used in fine watches but has the merit of a high rate of energy absorption, coupled with high-frequency operation or beat, and its ability to withstand setback and centrifugal forces during service firing. It is subject, however, to greater changes in rate with variations of applied power from the gear train, and in this respect has some of the poor timing qualities of a straight verge escapement with balance, but without the added hairspring.

The basic elements of a timing mechanism are as follows:

$a$  Source of power consisting of centrifugal or gravity weights, coiled mainspring, helical starting springs, etc.

$b$  The gear train delivering power to the escapement, and carrying hands or timing disk which starts and stops with each beat of the escapement.

<sup>1</sup> An alternative treatment utilizing the imaginary part of

$$\varphi = C e^{j\omega t}, \quad C = A + jB$$

and yielding

$$A \sin \omega t + B \cos \omega t$$

is possible, and would correspond more to the authors' Equation [2].

<sup>4</sup> The criticism has been voiced that mechanical engineers are behind times compared with electrical engineers as regards facility in handling vibration problems. While this discussor has heartily disagreed with such statements, he must admit that equal diffidence toward the use of complex amplitudes in treating the analogous electrical problem (a transmission line with resistance) could hardly be expected nowadays, though it may have occurred early in the century when complex amplitudes were first introduced.

<sup>1</sup> By F. G. Kelly and J. I. Zar, published in the December, 1946, issue of the JOURNAL OF APPLIED MECHANICS, Trans. A.S.M.E., vol. 68, p. A-285.

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