zero, which explains why Mr. Whitehead was able to run a fuse on only the starting spring.

Mr. Whitehead's remarks on the greater accuracy obtainable with laminated clock plates, pierced and shaved holes, and punched gears, are valuable additions to this discussion. While, as he says, millions of fuses were made during the war without hand-matching, ". . . there was a departure in the shape of the tooth . . . which would make the escapement more in line with calculated tooth form such as is . . . shown by the authors."

Mr. Whitehead's discussion has been stimulating, gratifying, and informative as to the history of the Mark 18 fuse and certain phases of the clock-manufacturing technique.

# Supersonic Nozzle Design[1]

A. R. Eaton, Jr.[2] An important assumption in the theory, as presented in the paper, is the one of linear velocity distribution in the throat of the nozzle. It is not easy to obtain such a distribution; and, if zero static pressure gradient in the test section is to be approached, it may be essential in the application of the method to compute the nozzle shape from an experimental velocity distribution in the throat.

Also with regard to the length of nozzles, there is one consideration that has not been mentioned. As the author has stated, it is usually easier to obtain a good static pressure distribution in the test section if the nozzle is long. On the other hand, if supersonic tests are to be entirely valid, they must be free from effects of condensation of water vapor in the test section. And when a nozzle is very long, there is more time for condensation to occur between the throat and the test section; consequently, a long nozzle must be operated with lower intake humidity than a short nozzle.

The difference may be important; for in the case of two nozzles already designed by the method discussed, for the same Mach number, the shorter nozzle can be safely operated with an intake relative humidity of about 10 per cent, while the longer nozzle must apparently be operated with an intake relative humidity about 2 per cent. Thus the design of drying equipment must be considered along with nozzle design; the nozzle must not be made so long that the drying job is prohibitively difficult.

K. J. DeJuhasz.[3] This paper is a valuable addition to the growing literature on supersonic flow which assumes today an ever-increasing importance in view of the recent developments in gas turbines and jet engines. The paper serves to clarify this subject to a considerable extent, and it is particularly to be commended that the author makes use of graphical methods, which, for most engineers are clearer and more readily visualized than purely mathematical derivations. It is felt, however, that the value of the paper would have been enhanced by the inclusion of some examples and their solutions, showing the actual construction of the nozzle, and the device in which it is to be used.

In this connection reference is made to a recent article[4] by P. de Haller, on dynamic problems in gases, which is probably but little known. This article makes even more extensive use of the graphical method, but treats the problem more generally and less specifically than does the author's paper.

Supersonic flow in nozzles is a branch of the broader group of phenomena of fluid flow under conditions such that the variation of the velocity of propagation and of density must be taken into consideration. Examples of engineering importance are explosions in space and in a cylindrical pipe, and intake, exhaust, and scavenging phenomena in engines and compressors. The theoretical groundwork for these problems has been laid by great mathematical minds, Rankine, Raleigh, Riemann, Hugoniot, and others. But there seems to exist no comprehensive work which would unify these theories and make them readily usable by the practical engineer.

The writer found the clearest and most consistent treatment by J. Ackeret.[5] Until now, the writer has not had opportunity to obtain and study references (2, 3) of the author's paper, and hopes that these fill the great need in this respect.

The writer has given considerable study to the basic theories in this field, in connection with engine intake and exhaust phenomena. Among these the work of Hugoniot[6] is by far the most basic and thorough; it is also rather difficult to follow, because its author was a man of immense mathematical imagination, but reluctant to use graphical methods of explanation and treatment. Unfortunately, he died just after the completion of the paper mentioned,[6] which was the beginning of what would have been his "magnum opus."

A comprehensive treatment of gas dynamics from the point of view of engineering, as distinct from aerodynamic applications is a challenge to some exceptional expositor (who should be preferably more an engineer than a mathematician), who would predigest the available literature in this field and would present it in an understandable and readily usable form for the practical designer.

A. H. Shapiro.[7] Scant attention has been paid in the technical literature to the question of how an incident wave, either expansion or compression, is reflected from a solid surface which is shielded by a boundary layer.

In the absence of boundary layer, it is easy to demonstrate that a wave incident on a straight wall must be reflected in like sense (e.g., an expansion wave is reflected as an expansion wave), as indicated in Fig. 6 of the paper. Here we make use of the fact that the stream leaving the reflected wave must have the same direction as the stream entering the incident wave.

When a boundary layer is present, however, the situation is considerably more complex, as the Mach waves do not then penetrate to the wall. Since the streamlines within the subsonic boundary layer may have curvature, there exists in general no unique relationship between the curvature of the solid wall on the one hand and the nature and strength of the reflected wave and the net change in direction of a supersonic streamline on the other hand.

If the effect of boundary layer on the mode of wave reflection is of major consequence, the method of nozzle design first proposed by Prandtl and Busemann may have to be modified considerably.

It is the writer's impression that the nozzles used in supersonic wind tunnels in Germany were not all satisfactory with respect to uniformity of velocity at the exit. Deviations from the theoretical condition of a uniform velocity profile seem in general to be attributed to (*a*) uncertainties as to the extent of boundary-layer displacement thickness, and (*b*) uncertainties as to the velocity profile near the throat of the nozzle. The writer offers for discussion the question of whether or not the effect of the boundary

[1] By A. E. Puckett, published in the December, 1946, issue of the Journal of Applied Mechanics, Trans. A.S.M.E., vol. 68, p. A-265.

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[4] "The Application of a Graphic Method to Some Dynamic Problems in Gases," by P. de Haller, *Sulzer Technical Review*, 1945, no. 1, pp. 6–24.

[5] "Handbuch der Physik," vol. 7, Julius Springer, Berlin, Germany, 1927; "Gasdynamik," by J. Ackeret, chapt. 5.

[6] "Sur un Théoreme Relatif au Mouvement Permanent et a'l'écoulement des Fluides," by H. Hugoniot, Comptes Rendus, vol. 103, 1886, pp. 1178–1181.

[7] Assistant Professor of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, Mass. Jun. A.S.M.E.

layer on the nature and strength of the reflected waves may not be of major significance.

The author suggests that the channel be widened to accommodate the displacement thickness of the boundary layer. This procedure of course is not connected with the point brought up by the writer, but serves better to approximate the design Mach number at the nozzle exit. In view of the uncertainties as to the effect of boundary layer on wave reflections, the writer asks, in a spirit of inquiry, whether an equally good result might not be obtained by designing on the basis of frictionless flow with a Mach number slightly higher than that ultimately desired, using previous experience as a guide for determining the relation between the "frictionless" Mach number and the actual Mach number to be expected.

Perhaps the only conclusion which can be drawn from this discussion is that there exists a need for a great deal more experimental work on the interaction between waves and boundary layers along the general lines followed by H. Liepmann.

## Author's Closure

The problem of avoiding condensation of water vapor in the nozzle of a supersonic wind tunnel is, as Mr. Eaton has mentioned, a serious one. Such condensation may influence test results not only through its effect on wind-tunnel Mach number, but perhaps also in details of the flow around a model, such as conditions in the wake of a blunt body. For this reason it appears that accurate wind tunnel testing may require in any case very low absolute humidities, probably less than 0.1 per cent, corresponding to intake relative humidity less than 2 per cent for air at 70 F. If this is accomplished, as Mr. Eaton points out, the length of the nozzle is probably not such a critical choice.

Professor DeJuhasz calls attention to a problem which is confronting the theoretical aerodynamicist very forcibly today—that of making new and refined developments in gas dynamics available in useful form to the engineer. It is well to have this problem clearly emphasized, and it is hoped that the increasing interest of engineers in the field will stimulate efforts in this direction.

The question of manner of reflection of a finite shock wave from a boundary layer, raised by Mr. Shapiro, certainly requires much further study. However this problem probably does not enter critically in the design of a supersonic nozzle. The family of finite waves used in the graphical construction of the nozzle shape is, as stated in the paper, only an approximation to the infinite number of infinitesimal waves which must represent the real, continuous flow in the nozzle. The streamlines at the edge of the boundary layer, therefore, do not experience any sudden or discontinuous changes in direction. The "curvature" of the streamlines in the boundary layer must then, within the scope of the usual Prandtl boundary-layer theory, remain small. The essential problem is then only one of computing the behavior of a compressible fluid boundary layer in the presence of a continuous pressure gradient. If this can be done, it can be shown that the additional curvature imposed on the streamlines is represented approximately by the increase of the displacement thickness of the boundary layer. It therefore appears that a reasonable correction for the effects of boundary later is the addition to a "perfect fluid" nozzle shape of the displacement thickness of the boundary layer, to arrive at an equivalent real boundary for the flow.

It is clear that the value of this correction depends entirely on the accuracy of the computation of boundary-layer growth. This can be accomplished approximately in several ways, with a fair amount of labor. However for a quicker answer without this refinement, or perhaps in view of the uncertainty of the computation, an alternate correction is certainly the one mentioned by Mr. Shapiro—to design the nozzle for a Mach number slightly higher than that desired. This, of course, still leaves unsettled the question of final slope of the nozzle wall at its exit, which must be somewhat greater than zero to account for the continuing rate of increase of boundary layer at that point. An approximate computation of boundary layer growth at least gives an approximate answer to this question.