similar performance might occur in cylindrically shaped surfaces, i.e., standard circular bearings.

Dr. S. Kyropoulos indicates the possibility that the experiments were carried out under conditions of mixed nonfluid (boundary) and fluid lubrication. His explanation of the low starting friction of silver bearings is interesting.

The authors believe that their work will give an impetus for similar correlation of friction and surface-finish data. Developed surface-finish apparatus provide complete facilities for such accurate measurements. Friction versus finish data will undoubtedly constitute a field for further development in laboratories and in industry. In this connection, the collaboration of physicists and engineers will prove very beneficial in elucidating the complex phenomena of friction which some authors consider close to the "terra incognita."

# A Photoelastic Study of Stresses in Rotating Disks[1]

C. W. MacGregor.[2] The author has presented an interesting study of the stresses in rotating disks containing holes by applying the newer three-dimensional stress-freezing technique to what is essentially a two-dimensional problem.

A question which occurred to the writer, however, concerns the ratio of the diameter of the holes used to the thickness of the plate. In model tests $A$ and $B$ the diameter-thickness ratios are 0.5 and 0.75, respectively, according to the data given in the paper. This means that the problem falls somewhere between the cases of plane strain and plane stress.

It was shown some time ago by R. W. Vose[3] that a considerable reduction in the stress-concentration factor determined photoelastically was found for the case of a hole in a plate subjected to tension in one direction, as compared to the theoretical value when the ratio of hole diameter to the thickness of the plate was less than 2 to 1. This would seem to indicate that the stresses found photoelastically in the case of the rotating disks described in the present paper are lower than the actual values. Although a direct numerical correction based upon Vose's values, as applied to the case under discussion, may not give the true picture, it is indicative of the order of magnitude of the effect. The factors by which the photoelastic-stress value should be multiplied to obtain the theoretical value in the case of the plate with a single hole subjected to tension in one direction only would be 1.305 and 1.167, respectively, for the diameter-to-thickness ratios used in model tests $A$ and $B$ of the paper.

The writer would like to ask the author if any experiments were made to determine whether or not such effects were of appreciable magnitude in the present tests.

A. Yorgiadis.[4] Referring to Equation [4$a$] of the paper and the preceding paragraph, the ratio of the cross-sectional area, along the circle of centers before drilling the holes, to the area after drilling is not

$$\frac{2\pi a}{2\pi a - 6d} \quad \ldots\ldots [5] \quad \text{but} \quad \frac{2\pi}{2\pi - 24 \arcsin (d/4a)} \quad \ldots\ldots [6]$$

Even this corrected expression would be unsuitable for this purpose because the maximum reduction in area does not occur in the line of centers, but in the circle, the radius of which is $\sqrt{a^2 - 0.25\, d^2}$ and at that section the reduction in area due to drilling is

$$\frac{2\pi}{2\pi - 12 \arcsin \dfrac{d}{2a}} \quad \ldots\ldots [7]$$

When $d$ is small relative to $a$, the author's equation introduces only a slight error. However, this error becomes appreciable as this ratio increases. For example when $a = 1$ in. and $d = {}^3/_8$ in., Equation [7] of this discussion gives a value 0.3 per cent more than Equation [5]. When $a = 1$ in. and $d = {}^7/_8$ in., Equation [7] gives a value 23 per cent more than Equation [5]. At the limiting value, $d = a$, when the drilled holes are tangent to each other, the author's equation yields $\frac{\pi}{\pi - 3}$, whereas this value should be $\frac{\pi}{\pi - \pi}$, or $\infty$ as calculated from Equation [7].

### Author's Closure

Referring to the corrections suggested by Dr. MacGregor, the author wishes to state that no such corrections were made because none was believed necessary. It seems to the author that Vose's experimental results have been seriously misinterpreted by a number of photoelasticians. Although it is not defined in Vose's article, the quantity which he has called the "stress-concentration ratio" seems to be the ratio of the change of thickness at the point of highest stress to the change of thickness at a point in the uniform stress field. Customary usage would define the stress-concentration factor in the problem of the hole in a tension field as the ratio of the maximum principal stress to the tension in the uniform field. The two definitions will yield identical values as long as a condition of plane stress exists in the plate. However, when the ratio of the diameter of the hole to the thickness of the plate is small the stress system in the plate is three-dimensional and the strain gage does not measure the sum of the two principal stresses in the plane of the plate. Theory of elasticity shows that the stress-concentration factor is equal to 3 for the limiting cases of plane stress and plane strain and there is no apparent reason to anticipate an appreciable change in the intermediate range. Some unpublished experimental results (obtained optically, by use of a polariscope) of M. M. Frocht confirm this opinion.

In the problem of the rotating disk the variation of the body forces causes a difference in the theoretical solutions under the assumptions of plane stress and plane strain. It is well known that the drilling of a small hole in the center of a solid rotating disk of uniform thickness doubles the maximum principal stress in the disk. Although this stress-concentration factor is usually derived under the assumption of plane stress, it is equally correct for the case of plane strain, despite the difference in the magnitudes of the stresses in the two cases. There is no available theoretical solution for the problem dealt with experimentally in the author's paper, but, in the light of the foregoing considerations, it seems reasonable to assume that the correction suggested by Dr. MacGregor is unwarranted.

The author readily concedes that Mr. Yorgiadis' mathematics is correct. At the time of writing of the paper the approximate nature of the correction factor, Equation [5], was recognized, but this form was used for the sake of simplicity. The 0.3 per cent error involved is considerably smaller than the probable experimental error. In the extreme hypothetical cases cited by Mr. Yorgiadis not even his "corrected" correction factors would be suitable, because the assumption stated in the paper, upon which the use of the correction factor is based, is grossly violated.

[1] By R. E. Newton. Published in the June, 1940, issue of the Journal of Applied Mechanics, Trans. A.S.M.E., vol. 62, p. A-57.

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[3] "An Application of the Interferometer Strain Gage in Photoelasticity," by R. W. Vose, Journal of Applied Mechanics, Trans A.S.M.E., vol. 57, 1935, p. A-99.

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