Network- and Differential-Analyzer Solution of Torsional Oscillation Problems Involving Nonlinear Springs[1]

C. S. L. ROBINSON.[2] Some complete solutions are given here for systems with nonlinear springs. These have not been adequately studied before because the mathematics becomes awkward. It is true that other methods of solution have been used.[3] However, in view of the facts that there are about as many nonlinear springs as there are linear ones and that damping is usually nonlinear, this paper is much needed.

Admittedly most materials follow Hookes law, but many mechanical arrangements are made so that the resultant load-deflection relations are nonlinear, especially at the loads which occur near vibration peaks. The paper shows that whenever manufacturers refer to such a construction as "nonresonant" per se, they are entirely wrong. (This has actually been done!)

It is true that resonant frequencies can be located by the conventional mathematics using linear spring constants which approximate the lower part of the deflection curves. Hence, a general statement may be made that nonlinearity in spring constants affects only resonant amplitudes and the range over which they occur.

In amplitude calculations, the great drawback to accuracy is damping. The author did not give any solutions for nonlinear damping, but it can be imagined that this would affect only the magnitudes of the peak responses. Damping is apt to increase at high amplitudes for two reasons:

1 Material hysteresis increases with stress.
2 Static friction between rubbing parts is overcome.

As mechanical engineers, we usually excuse our inability to solve perplexing nonlinear vibration problems by saying that the parameters in the usual case are not accurately known. We hope this will not always be so, and we thank the electrical engineers for supplying the tools for analysis.

[1] By C. Concordia, published in the March, 1945, issue of the JOURNAL OF APPLIED MECHANICS, Trans. A.S.M.E., vol. 67, 1945, p. A-43.

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[3] Reference 3 of the paper; also, "Forced Vibrations of Systems With Nonlinear Restoring Force," by K. O. Friedrichs and J. J. Stoker, Quarterly of Applied Mathematics, vol. 1, July, 1943, pp. 97–115.

AUTHOR'S CLOSURE

We wish to thank Mr. Robinson for his comments, and to state that we are in entire agreement with them. Although they were not shown in the paper, we have included the effects of nonlinear damping and hysteresis, and of coulomb friction in some studies. Such damping effects become especially important in the study of control systems as well as of other types (one example is the problem of aerodynamic flutter) wherein it is of interest to determine the degree of stability of the system and the possibility of continuous self-excited oscillations.