Discussion

Tension Tests at Constant True Strain Rates[1]

LAWFORD H. FRY.[2] The authors discuss the effect of temperature and strain velocity on the stress-strain properties of various materials. In describing testing methods, the authors say, "The value of the true strain rate was controlled by adjusting the head motion of the testing machine." This reads as though the assumption were made that the travel of the crosshead of the testing machine is the same as the strain of the specimen. This is certainly not the case.

The writer has shown:[3] "Only a small fraction of the crosshead travel appears as strain in the specimen. The proportion of strain to crosshead travel varies during a test as the load is increased, but is unaffected by the rate of crosshead travel." Methods are given in the paper[3] just quoted for comparing the travel of the crosshead with the rate of strain. It would be interesting to have information as to this relationship in the case of the machine used by the authors and the various specimens tested.

Incidentally as a minor point, attention is called to the authors' statement, "The standard 0.505-in-diam tension specimen was employed throughout the testing program." Presumably the authors refer to the standard 2-in-gage-length specimen of the American Society for Testing Materials. Reference to A.S.T.M. Specification E 8-42, Fig. 3, will show that the diameter of the 2-in. specimen is $^1/_2$ in. plus or minus 0.01 in.

S. TIMOSHENKO.[4] The writer would like to bring to the attention of American engineers some recent investigations made in Russia by N. N. Davidenkov. This investigation deals with calculation of true stresses and true strains at the minimum diameter of tension test bars of Armco iron stretched to various degrees of reduction of the cross-sectional area. Eight 1-in-diam bars were stretched up to reduction of area from 41 per cent to 69.2 per cent, after which they were cut at the minimum diameter. The minimum cross sections were polished and microscopically investigated. Numerous measurements of average changes of dimensions of grains showed that radial and tangential strains ϵ_r and ϵ_t are equal and are constant over the entire cross section. Their values obtained from these measurements were in very good agreement with the average values obtained from measurements of the minimum diameter of the test pieces. This indicates that the strains calculated from the measurements of the reduction of area represent true strains.

To obtain the true stress distribution, Davidenkov makes the usually accepted assumption

$$\frac{\sigma_l - \sigma_t}{\epsilon_l - \epsilon_t} = \frac{\sigma_l - \sigma_r}{\epsilon_l - \epsilon_r} \quad [1]$$

from which he concludes that for $\epsilon_t = \epsilon_r$ we must also have $\sigma_r = \sigma_t$. To calculate these stresses and also the stresses σ_l in the axial direction, the curvature $1/R$ of the longitudinal fibers of the stretched bars at the surface was measured and it was assumed that for any inside fiber at the distance x from the axis of the bar the curvature is

$$\frac{1}{\rho} = \frac{1}{R}\frac{x}{a} \quad [2]$$

where a is the minimum radius of the stretched bar. This expression for curvature was substituted into the equation of equilibrium of an infinitesimal element and in this way was obtained

$$d\sigma_r = \frac{1}{\rho}(\sigma_l - \sigma_r)\,dx = \frac{x}{Ra}(\sigma_l - \sigma_r)\,dx$$

By integration, we find

$$\sigma_r = (\sigma_l - \sigma_r)\frac{a^2 - x^2}{2Ra} \quad [3]$$

It is seen that the radial stress σ_r increases as we proceed from the surface of the specimen to its axis. It vanishes at the surface and becomes a maximum at the axis.

From Equation [1] of this discussion we conclude that for $\epsilon_r = \epsilon_t$ the difference $\sigma_l - \sigma_r$ must be constant for the entire cross section. Denoting that constant value by σ_0, we obtain

$$\sigma_l = \sigma_0 + \sigma_r \quad [4]$$

The summation of stresses σ_l must give us the tensile force S stretching the bar. Hence, using Equations [3], and [4], we find

$$\int_A \sigma_l\,dA = \sigma_0 \int_0^a \left(1 + \frac{a^2 - x^2}{2Ra}\right) 2\pi x dx = \sigma_0 \pi a^2 \left(1 + \frac{a}{4R}\right) = S$$

$$\sigma_0 = (\sigma_l)_{\min} = \frac{S}{\pi a^2 \left(1 + \frac{a}{4R}\right)} \quad [5]$$

$$(\sigma_l)_{\max} = \sigma_0 \left(1 + \frac{a}{2R}\right) \quad [6]$$

It is seen that the longitudinal stress is not uniformly distributed over the minimum cross section of the stretched bar and that its maximum value at the axis of the bar depends upon the magnitude of the ratio $a/2R$.

This represents the principal result of Davidenkov's work. We see that the simple tensile test brings us to a complicated stress distribution at the reduced cross section. Thus it seems desirable, in studying the effect of speed on the stress-strain relation, to use the experiments in which stress distribution is not affected by plastic deformation. Such conditions can be approached in torsion of tubular specimens. Several experiments of this kind with tubes of lead were made some time ago by James Jamieson at the University of Michigan.

[1] By C. W. MacGregor and J. C. Fisher, published in the Dec., 1945, issue of the JOURNAL OF APPLIED MECHANICS, Trans. A.S.M.E., vol. 67, p. A-217.

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[3] "Speed in Tension Testing and Its Influence on Yield Point Values," by L. H. Fry, Proceedings of the A.S.T.M., vol. 40, 1940, p. 625.

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Authors' Closure

The question of the control of the strain rates was brought up by Mr. Fry.

No assumption was either made or used in our experiments to the effect that the travel of the crosshead of the testing machine is the same as the strain of the specimen as mentioned by Mr. Fry. In fact the paper emphasizes that one of the reasons for developing the true-strain-rate approach was just to avoid such an inequality between rate of head motion and the rate of straining of the specimen. Mr. Fry is referred to the first paragraph under the heading "Testing Procedure" which brings this out clearly. The last sentence of this paragraph states "If the rate of head motion is constant it is not true that the strain rate is also constant."

In the authors' tests, the *true strain rate of the specimen* was maintained *constant* by *varying* the rate of head motion of the testing machine throughout each test so that the proper minimum diameter was attained at the appropriate time. Since the true strain $\epsilon = 2 \log_e \frac{d_0}{d}$, the absolute value of the true strain rate is

$$\dot{\epsilon} = 2\frac{\dot{d}}{d} \qquad [1]$$

where $\dot{\epsilon}$, d, and $\dot{d}$ are the true strain rate, the instantaneous diameter, and the time rate of change of the instantaneous diameter, respectively. To maintain a constant true strain rate it is only necessary then to force the diameter of the specimen to follow a certain predetermined schedule of values with respect to time by varying the rate of head motion of the testing machine through adjustments of the control valve. Hence it is the true strain rate of the specimen which is maintained constant and *not* the rate of head motion.

In addition to the difference between the rate of head motion and the strain rate of the specimen to which Mr. Fry refers, it is also true that due to the increase in gage length of a specimen under tension, the true strain rate is not obtained by dividing the change in gage length per unit of time by the original gage length. While this procedure does not make any appreciable error at the yield point, the error increases with the amount of strain. In Fig. 2 of Mr. Fry's paper,[5] for example, the effect of strain rate on the tensile strength is shown. Unless a correction was made for increase of gage length these values do not represent true strain rates.

Likewise, during necking, even if a correction is made for increased gage length which originally was, say, 2.0 in., the definition of strain rate customarily used has little physical significance due to the great variation in strain rate along the test bar. These various disadvantages are overcome through the use of the true-strain-rate definition employed in the paper and given by Equation [1] of this closure.

Professor Timoshenko has very kindly called the authors' attention to a recent interesting paper by N. N. Davidenkov and N. I. Spiridonova in which a discussion is given of the true stress and true strain conditions in a necked tension specimen. Previous contributions on this same subject have been made by E. Siebel[6] and P. W. Bridgman.[7] The Davidenkov formula is the same as the one given earlier by Siebel.

[5] "Speed in Tension Testing and Its Influence on Yield Point Values," by L. H. Fry, Proceedings of the A.S.T.M., vol. 40, 1940, p. 625.

[6] "Effect of Lateral Contraction in Tensile Test on Strain Hardness of Metals," by E. Siebel, *Berichte der Fachausschusse des Vereines deutscher Eisenhuttenleute*, vol. 71, no. 5, 1925, pp. 1 and 2.

[7] "Stress Distribution at Neck of Tension Specimen," by P. W. Bridgman, Transactions, American Society for Metals, vol. 32, 1944, p. 553.

The formula developed in the paper referred to by Professor Timoshenko is based in part on the assumed validity of Equation [1] of his discussion, namely, the equality of the stress difference and strain difference ratios. Davidenkov points out that this has not yet been proved experimentally as fully accurate, but that it is used as a first approximation.

Davidenkov attempts to prove experimentally that the axial strain is constant across the necked specimen by measuring grain sizes after the specimen has necked down. The scatter of his observations and the absence of control measurements on undeformed specimens reduces the significance of his results.

Tests conducted by the authors two years ago on large 2-in-diam. round steel specimens showed that at the minimum section the average true axial strain $\epsilon = \log_e \frac{A_0}{A}$ determined from diameter measurements was substantially greater than the true axial strain at the surface of the specimen $\epsilon_s = \log_e \frac{l}{l_0}$ as determined by the separation of closely spaced scribed circumferential lines at the minimum section. The possible effect of the curvature of the surfaces at the neck was investigated, and found to be negligible for scribed lines spaced as closely as in these tests. For two such tests carried to strains of about 0.6 to 0.65 the value of $\epsilon = \log_e \frac{A_0}{A}$ was in fact greater than the value of $\epsilon_s = \log_e \left(\frac{l}{l_0}\right)_{\text{at surface}}$ by 8 per cent in the first test and 10 per cent in the second at the minimum section. The second test was carried to a slightly greater true strain than the first. This variation is difficult to reconcile with a constant axial strain value across the minimum section.

Since there is some question as to the validity of the assumptions used by Davidenkov and Spiridonova in their analysis, as well as some question as to the validity of their experimental results as indicated in particular by the existence of experimental contradiction to their observations, the authors have felt that any "correction" of the average true stress would not be justified at this time, as the determination of the correction factor depends on uncertain and controversial premises. Further, if a "correction" of the average true stress is made, then the results of the authors' tests on round bars indicate that a similar correction of the average true strain should be made. Until the question of the strain and stress distribution in the neck of a tension specimen is more satisfactorily determined and conflicting observations have been reconciled, the authors prefer to deal directly with the average true stress (P/A at the minimum section) and the average true strain ($\epsilon = \log_e \frac{A_0}{A}$ at the minimum section).

Professor Timoshenko mentions the use of hollow torsion bars as a more suitable form of test. Our experience has been that if sufficiently thin-walled torsion-test specimens are used to provide a uniform state of stress, considerable difficulty is encountered due to plastic buckling. If the uniform gage length is made small to overcome this, the effect of an unknown restraint is introduced. On the other hand, if the wall is thick enough to prevent buckling, an appreciable error is introduced in the calculation of the stress. The use of solid torsion bars is thus preferable for these reasons and also because of the greater amount of machining necessary for hollow specimens.

The authors would like to take this opportunity to thank Mr. Fry and Professor Timoshenko, and also those who contributed oral discussion to this paper, for bringing to their attention several problems of interest in connection with the constant-true-strain-rate tension test.