

performs differently in the horizontal than in the vertical direction, in spite of the fact that he neglected gravity is: "What a clever disk!" Closer examination, however, shows that the disk has no choice in the matter, since Mr. Hiersch treats a case without damping, in which there is a certain amount of vibration (represented by the constants  $M_1$  and  $M_2$ ) at the natural frequency of the system going on *ad infinitum*. His example is chosen in such a way that the natural vibration in the horizontal and in the vertical directions have different phase angles with the forced vibration and, since the forced vibration is assumed to have a frequency 4 times the natural frequency, the disk describes the Lissajous figure shown in Fig. 2 of the discussion. This motion gives the "pulsating torque" about which Mr. Hiersch is concerned.

The case of no damping is, however, only of academic interest. Actually, as is well known, the presence of damping means that any initial amount of natural vibration must die out ( $M_1$  and  $M_2$  become zero) so that only the forced vibration remains, on which the balancer can go to work.

Mr. Fleischmann proves in mathematical fashion that a stiff rotor can be balanced by two weights and finds the correct amount of weight to be added for the balancing process shown by the author in Figs. 17(a) and 17(b).

Mr. Brown reports good results with "compensated weights" applied in such a manner that for a "stiff" rotor they would result in a zero reaction at one of the bearings.

This practice of inserting two trial weights at the time should prove successful, especially when the rotor is operating at a speed well below the first natural frequency of the system as in the usual shop-balancing operation.

However, where the speed is high and the elastic system complex, as on large turbine generators, the advantage of compensated weights is lost, and it becomes more convenient to work only with single trial weights. This also leads to better relations with the service man who has to install the weights sometimes under adverse conditions.

## Investigation of Cross-Spring Pivot<sup>1</sup>

P. G. EXLINE.<sup>2</sup> The author of this paper is to be congratulated upon his work which represents a valuable contribution to the literature on elastic hinges. Use of such elastic members has extended to many applications, and the design data presented in the paper will assist in further extension.

It is felt that the term "cross-spring pivot" describes the subject member somewhat ambiguously, and it is suggested that the term "flexure pivot" be used for a system of crossed, flat springs, and that "fulcrum plate" be used for a single, flat spring acting as a hinge.

The author does not discuss one very important aspect of this problem, namely, that in which the springs are in tension due to heavy loads. This does not detract from his paper, but it does permit questioning note (d) under "Design Notes" wherein it is stated: "When used as a pivot for a heavily loaded lever system, the springs should be placed so that one of the springs is in direct tension."

The writer is constructing a device which employs a set of crossed tapes as a pivot, subjected to a load of 30,000 lb. If one tape were in line with the load, would not its stretch cause a dish-ing of the other tape which would introduce substantial instability at the zero position?

<sup>1</sup> By W. E. Young, published in the June, 1944, issue of the JOURNAL OF APPLIED MECHANICS, TRANS. A.S.M.E., vol. 66, p. A-113.

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D. E. WILLIAMSON.<sup>3</sup> The cross-spring pivot has been used extensively in the design of profilometer tracers used for the measurement of surface roughness. In these applications, pivots of very small dimensions are used in order to reduce the weight of the moving parts and to provide a very "soft" spring action. In use the pivots are given a static deflection of about 0.03 deg. Upon this static deflection is superimposed an oscillatory deflection of about 0.003 deg double amplitude.

Substituting in the formula for pivot-point travel, the static deflection produces a shift of about 0.000012 in. for a 90-deg spring 0.1 in. long. The oscillatory motion will produce only  $1/10$  of this small value or 0.000012 in. Inasmuch as the lever arm which the spring supports extends 1 in. on either side of the pivot, this value is happily inconsequential.

It is, however, important that the pivot system be rigid with respect to transverse forces applied normal to the axis of rotation. Since we are dealing primarily with the oscillatory motion, it cannot be assumed that optimum design would be that which would best carry the static load.

It has been found in some applications that transverse flexibility gives rise to unwanted motion great enough to obscure completely the motion which it is desired to measure.

The present paper does not attempt to discuss transverse stiffness at small amplitudes of deflection. It is felt, however, that the success of the pivot may frequently depend upon the ratio of transverse to angular stiffness, and it is suggested that this phase of the work be considered. The content of the present paper is excellently chosen and presented, and should provide valuable information to designers of small instruments.

### AUTHOR'S CLOSURE

The term, "cross-spring pivot" is perhaps as Mr. Exline points out, a bit ambiguous particularly since in many applications, the springs do not actually cross. "Flexure pivot" describes this type of hinge more accurately.

By heavy loads, it was intended to refer to loads which might cause a column effect in the spring members. Under optimum loading conditions seven pounds would be about the limit for the three-inch springs. In tension this load would result in a total strain of about 0.000155 in. which is small when referred to a total length of three inches. Although Mr. Exline does not mention the physical dimensions of his springs, the magnitude of the load would suggest that these springs might be highly stressed, perhaps close to the yield point, in which case the elongation might be a considerable factor in the reaction between the spring members. This effect has not been investigated but presents interesting possibilities.

In general a slight transverse flexibility has not been considered serious but in the application mentioned by Mr. Williamson, in which such small motions are being recorded, transverse motion might produce a considerable error in measurement. At present, the best that can be suggested is that it may sometimes be necessary to sacrifice light weight and sensitivity for stiffness. An attempt should also be made to keep the resonant frequency of the system as far as possible from the range of frequencies likely to be encountered in the operation of the instrument.

It should be pointed out that the design formulas were developed primarily for larger deflection angles than those used by Mr. Williamson, and for this reason the results he obtains may be only approximations. However, it may still be concluded that the shift of this pivot is very small relative to a 1-in. lever arm.

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