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The components of the vorticity vector are

$$\left(-G'\frac{\partial\psi}{\partial y},\ G'\frac{\partial\psi}{\partial x},\ \frac{\partial v_x}{\partial y}-\frac{\partial v_y}{\partial x}\right)=\left(\rho G'v_x,\ \rho G'v_y,\ \frac{\partial v_x}{\partial y}-\frac{\partial v_y}{\partial x}\right)$$

and the vortexes are defined by

$$\frac{dx}{\rho G'v_x}=\frac{dy}{\rho G'v_y}=\frac{dz}{\dfrac{\partial v_x}{\partial y}-\dfrac{\partial v_y}{\partial x}}$$

$\dfrac{dx}{v_x}=\dfrac{dy}{v_y}$ implies that the vortexes lie on cylinders whose projections onto planes parallel to the x-y plane are the old streamlines. If the original flow F were irrotational, the vortices of F' would coincide with the old streamlines.

Author's Closure

The author wishes to thank Mr. Giese for his interesting generalization which provides many fields of flow derivable from the same two-dimensional flow, and retaining the same pressure and density distribution. For steady two-dimensional flow the extension is obtained by adding a normal velocity which is independent of the axial distance, is constant along each streamline, but varies from one streamline to the next one. A direct physical proof of this extension follows from the consideration that the addition of the same normal velocity component along each stream line does not effect the acceleration, and hence the pressure distribution, even though the added velocity component along different stream lines is different; similarly, consideration of flow tubes shows that with density unaffected, the condition of continuity is satisfied.

As pointed out by Mr. Giese, if the added normal velocity varies with the stream lines, the flow is not irrotational. Where the fluid starts from rest, it follows from Kelvin's circulation theorem that the vorticity vanishes and the flow is consequently irrotational. Thus only when the added normal velocity component is constant does an initially irrotational two-dimensional flow lead to an irrotational one.

Calculation of the Multiple-Span Critical Speeds of Flexible Shafts by Means of Punched-Card Machines[1]

M. A. Prohl.[2] In Appendix 1 of his paper the author derives the fundamental equations of the calculation procedure by dealing directly with the mechanical system, but states, "the logic used in the Appendix is itself based upon the electric-circuit methods by which it was preceded." The writer would like to present a derivation which leads to essentially the same results but which is in no way restricted by the demands of the electric circuit; in other words, a straightforward mechanical approach.

Assume that the rotor has been divided into a series of massless shaft sections of constant diameter with the rotor weight concentrated at the division points, as indicated in Fig. 1 of this discussion. Let the shearing force S and the bending moment M be positive as shown, the slope be positive when the inclination is upward to the right, and the deflection y be positive upward. These conventions will then apply for all sections of the rotor. The symbol W denotes the total weight concentrated at each division point.

[1] By A. W. Rankin, published in the June, 1946, issue of the Journal of Applied Mechanics, Trans. A.S.M.E., vol. 68, p. A-117.

[2] Turbine Engineering Division, General Electric Company, West Lynn, Mass. Jun. A.S.M.E.

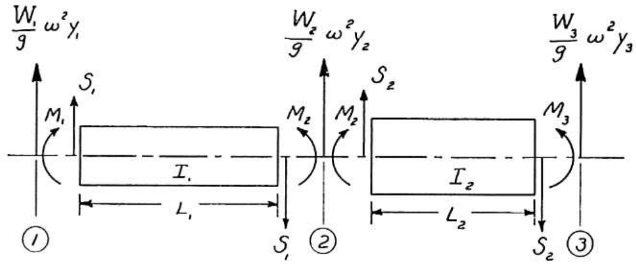

Fig. 1

It may be shown that a straightforward application of the elementary beam theory yields the following equations for bending moment, slope, and deflection

$$M_2 = M_1 + L_1S_1 \qquad [1]$$

$$\theta_2 = \theta_1 + \frac{L_1}{EI_1}M_1 + \frac{L_1^2}{2\,EI_1}S_1 \qquad [2]$$

$$y_2 = y_1 + L_1\theta_1 + \frac{L_1^2}{2\,EI_1}M_1 + \frac{L_1^3}{6\,EI_1}S_1 \qquad [3]$$

Since the shaft section is massless, the shear is constant along its length. A finite change in shear occurs at the division point due to the inertia force of the concentrated weight

$$S_2 = S_1 + \frac{W_2}{g}\omega^2 y_2 \qquad [4]$$

Equations [1], [2], [3], and [4] are similar in form to Equations [16d], [16b], [16a], and [16c], respectively, of the author's paper with the exception that Equation [3] contains one more product than Equation [16a]. In other words, the equations derived directly from the beam theory without the introduction of any auxiliary variables involve seven products as compared with six for the equations obtained from the electric-circuit analysis.

The L_1, θ_1 term in Equation [3] can be eliminated by the following substitution

$$B_1 = M_1 + \frac{2EI_1}{L_1}\theta_1 \qquad [5]$$

Making this substitution in Equations [1], [2], and [3] yields the following results

$$B_2 = B_1 + \frac{2\,EI_2}{L_2}\theta_2 + L_1S_1 - \frac{2\,EI_1}{L_1}\theta_1 \qquad [6]$$

$$\theta_2 = -\theta_1 + \frac{L_1}{EI_1}B_1 + \frac{L_1^2}{2\,EI_1}S_1 \qquad [7]$$

$$y_2 = y_1 + \frac{L_1^2}{2\,EI_1}B_1 + \frac{L_1^3}{6\,EI_1}S_1 \qquad [8]$$

Note

$$B_2 = M_2 + \frac{2\,EI_2}{L_2}\theta_2$$

Equation [6] may be simplified by eliminating S_1 and θ_1. Multiply both sides of Equation [7] by $\dfrac{2\,EI_1}{L_1}$ and rearrange

$$\frac{2\,EI_1}{L_1}\theta_2 - 2\,B_1 = L_1S_1 - \frac{2\,EI_1}{L_1}\theta_1$$

The foregoing result is introduced in Equation [6]. The four

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basic equations are listed in the order in which they appear in the author's paper

$$y_2 = y_1 + \frac{L_1^2}{2\,EI_1} B_1 + \frac{L_1^3}{6\,EI_1} S_1 \qquad [9]$$

$$\theta_2 = -\theta_1 + \frac{L_1}{EI_1} B_1 + \frac{L_1^2}{2\,EI_1} S_1 \qquad [10]$$

$$S_2 = S_1 + \frac{W_2}{g} \omega^2 y_2 \qquad [11]$$

$$B_2 = -B_1 + \left(\frac{2\,EI_1}{L_1} + \frac{2\,EI_2}{L_2}\right) \theta_2 \qquad [12]$$

Equations [9], [10], [11], and [12] of this discussion are identical with Equations [16] of the author's paper, except for algebraic signs. These equations possess the following advantages over those of the electric-circuit analysis: (1) There is no alternation in the sign of any of the coefficients in going from one shaft section to the next; (2) the same conventions for the positive directions of S, M, θ, and y apply to each shaft section. Obviously, if the writer had used the same conventions as those required by the electric-circuit analysis, there would be no difference in the two sets of equations.

The writer readily admits that the introduction of the auxiliary variable herein denoted by B is not a step which one would intuitively take in attacking this problem solely from a mechanical point of view. This then is the specific contribution which the electric-circuit analysis has made in the field of the numerical calculation of critical speeds.

For a rotor with a large number of changes in shaft diameter it would be desirable to combine several pieces of shaft of different diameter into one equivalent massless section, since any reduction in the number of shaft sections greatly reduces the amount of labor involved in the trial calculations. This is particularly important when a manually operated calculating machine is used. It has been the writer's experience that good accuracy can be obtained with surprisingly few shaft sections per span provided that the elastic constants for these shaft sections are properly evaluated.

Such a reduction in the number of shaft sections can readily be made if Equations [1], [2], [3], and [4] of this discussion are used as the basis for the critical-speed calculation. If a massless shaft section having several diameters or having a continually varying diameter is to be treated, it is only necessary to modify the coefficients of the S_1 and M_1 terms in Equations [2] and [3], or the values of a, b, c, and d in the following generalized equations

$$\theta_2 = \theta_1 + a_1 M_1 + b_1 S_1 \qquad [13]$$

$$y_2 = y_1 + L_1\,\theta_1 + c_1 M_1 + d_1 S_1 \qquad [14]$$

These coefficients a, b, c, and d have the following significance: If the shaft section is temporarily considered as a cantilever fixed at its left-hand end and loaded by a unit bending moment, then the coefficient a is numerically equal to the slope of the free end, and the coefficient c is equal to the deflection of the free end. If the shaft section is then loaded so that the bending moment is zero and the shear is unity at the fixed end, then the coefficient b is numerically equal to the slope of the free end, and the coefficient d is numerically equal to the deflection at the free end. For a shaft section composed of several portions of different diameters these slopes and deflections may be computed by successively applying Equations [1], [2], and [3] of this discussion.

It appears to the writer that the calculation procedure derived from the electric-circuit analysis or the procedure represented by Equations [9], [10], [11], and [12] of this discussion, is limited to shaft sections of constant diameter. Any attempt to introduce the general coefficients for a nonuniform shaft section would destroy the simplicity of these equations.

Author's Closure

In reply to Mr. Prohl's discussion, the author wishes to point out that the important contribution of this paper seems to have been overlooked. The major contribution of this paper is in its introduction of business machines to handle the appalling number of calculations required. By means of these business machines, a calculation which heretofore was too involved to be applied to routine analysis of the four- and five-bearing sets in which the author is interested has been reduced from a somewhat academic method to a routine calculation which has already been applied to over a dozen machines and is at present being continuously extended.

The derivation of the analytical equations is not of primary importance in this paper. As the author points out in the sixth paragraph, any of the published analytical methods—Prohl (7),[3] Myklestad (6), Kron (8) etc., could have been used; Kron's method was used because the author personally preferred it to the others.

Concerning Mr. Prohl's discussion of the development of the equations in Appendix 1, the author's remark (Par. 2, Appendix 1) that the logic used in the mechanical development was based upon the electric-circuit methods applies only to the introduction of the pseudo-moments F_r and pseudo-shears F_y (designated by B in the subject discussion). The analytical work of this Appendix is taken directly from elementary beam equations as is stated in the third paragraph. This can be seen by noting, for example, that Prohl's Equations [1], [2], and [3] are identical with the author's Equations [7*b*], [9*b*], and [9*a*] except for sign and nomenclature changes.

The various sign alterations have not proved confusing to the slightest degree in the continued application of these punched-card machines. The original cards are punched with the correct signs of the Z and Y quantities during the preliminary manual calculations and all operations follow automatically from then on.

The author does not agree that the number of shaft sections can be easily reduced, particularly for the practical case of turbine shafts with sudden changes in shaft diameter. We have tried reducing the number of shaft sections and have not been satisfied with the results. Furthermore, since the punched-card machines are doing the major portion of the calculating, there is little to be gained by reducing the number of shaft sections, particularly if this reduction complicates the preliminary manual calculations.

In practical applications, the calculation procedure presented in this paper has been found by experience to give sufficiently accurate results even for shaft sections whose diameters are not constant. We have found that by keeping the shaft sections sufficiently small in length, the approximate effective inertias can be found easily.

In conclusion, the author wishes to point out one of the improvements which has been made in this calculating procedure since the publication of this paper. In the first study made, the shaft-property data for each section for each assumed speed were manually calculated and manually punched into the cards. This involved the hand punching of about two thousand cards. The system now in use uses the business machines themselves to calculate and punch the majority of the shaft-property data, and it is now necessary to hand-punch only about sixty cards rather than the two thousand that were needed in the first study.

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[3] See Bibliography of paper.

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