Since the rolling problem is indeed very complicated with such a large number of variables entering the problem, the method adopted by Dr. Nádai of inquiring into the effect of changing one variable at a time appears to be perhaps the best procedure to give us the fundamental information so needed to clarify this problem. It is hoped that he will continue the work along these lines.

Author's Closure

The surface friction between rolls and the piece to be rolled mentioned by Dr. MacGregor deeply influences the structure of the rolled piece and the distribution of the stresses in the contact areas. Conditions in this respect may be widely different in the case of hot rolling or when a strip is cold-rolled and at the same time one of the high-pressure lubricants is applied. Axial tension must also have a deep effect on the stress distribution and probably makes it more uniform along a line which is perpendicular to the plane of the sheet. It is known that a few cases have been treated, in which the nonuniform stress distributions were determined in a layer of a plastic material squeezed out under pressure between rigid plates. The case of parallel plates and that of two slightly inclined plates have been treated. It appears, therefore, that it may be possible perhaps to improve the assumptions which were made in the paper for describing the states of stress in the compressed portion of a sheet, so that account also of the tangential forces is taken retarding the flow of the plastic material when it is compressed between the rolls. The experiments by Dr. MacGregor may throw additional light on the influence of the surface friction and should be of assistance in the future, when such improved computations should be carried out.

Mathematical-Machine Determination of the Vibration of Accelerated Unbalanced Rotor[1]

F. M. Lewis.[2] A solution of the problem of the vibration of an accelerated unbalanced rotor can be obtained by the same procedure, using complex functions, which was utilized in the writer's paper[3] "Vibration During Acceleration Through a Critical Speed," which was referred to by the author.[1]

If the eccentricity of the mass M is e then the magnitude of the exciting force becomes

$$M \frac{\omega^2}{{\omega_n}^2} e{\omega_n}^2 = \frac{r^2}{q^2} ek \qquad [1]$$

and if the resonance factor R is now equal to x/e, then the dimensionless solution corresponding to Equation [15] of the 1932 paper[3] becomes

$$R = -\frac{i\pi e^{i\sigma}}{q^2\delta} e^{2\pi i u_1 r}\left[\int_0^r r^2 e^{i\pi[(r^2/q)-2ru_1]}\,dr - e^{2\pi i u_2 r}\int_0^r r^2 e^{i\pi[(r^2/q)-2ru_2]}\,dr\right] \qquad [2]$$

Here R is the same quantity as the x/e or u of Baker's paper,[1] but the other symbols are identical with those of the 1932 paper.[3]

We will indicate the method of solution without carrying out all the work in detail.

With the complex substitution

$$r = \sqrt{\frac{iz_1 q}{\pi}} + qu_1 \qquad [3]$$

the equation obtained from Equation [2] of this discussion contains three integrals

$$\int_0^r e^{-z_1}\,dz_1 \qquad [4]$$

$$\int_0^r \frac{1}{\sqrt{z_1}} e^{-z_1}\,dz_1 \qquad [5]$$

$$\int_0^r \sqrt{z_1}\, e^{-z_1}\,dz_1 \qquad [6]$$

The solution of Equation [4] is obvious, while Equation [5] appears in the solution for an exciting force of constant amplitude.

Integrating by parts, Equation [6] of this discussion can be written

$$\int_{z_1}^{\infty r} \sqrt{z}\, e^{-z}\,dz = \sqrt{\pi}\, K + \sqrt{z_1}\, e^{-z_1} + \frac{1}{2}\int_{z_1}^{\infty r} \frac{1}{\sqrt{z}} e^{-z}\,dz \qquad [7]$$

After these substitutions and reduction there is obtained the solution

$$R = -\frac{1+i}{\sqrt{2}} + \frac{\sqrt{\pi q}}{2\delta}\left[\frac{i}{2\pi q}\left\{\varphi_1 - \varphi_2 + 2\sqrt{\pi}\, e^{z_1} K\right\} + {u_1}^2\left(\varphi_1 + 2\sqrt{\pi}\, e^{z_1} K\right) - {u_2}^2\varphi_2\right] \qquad [8]$$

The φ_1 and φ_2 integrals were evaluated for the 1932 paper[3] so that results for the present case can be easily obtained, the only additional operations needed being multiplication and division, provided that the same $r^2\pi q$ values are used for which φ_1 and φ_2 were previously computed.

The writer has evaluated the solution for $q = 40.8$, $\gamma = 0.051$ accelerated motion, which is very close to Baker's curve for $q = 40.8$, $\gamma = 0.050$ and by interpolating between Baker's curves for $\gamma = 0.05$ and $\gamma = 0.1$ a close check can be obtained in this case.

A comparison between points on the curve $q = 40.8$, $\gamma = 0.051$ by the machine and complex integral calculation is given in Table 1 of this discussion.

TABLE 1

β	u_0 or R_e: Machine	u_0 or R_e: Lewis' Equation [8]
0.924	4.0	3.98
1.000	7.6	7.58
1.076	12.6	12.42
1.107	13.7	13.45
	Peak	
1.122	13.7	13.30
1.150	11.5	11.34
1.198	3.7	3.68

The curves are almost identical in shape, having their maximum at the same β value. The peak value of the differential-analyzer curve appears to be high by $2^1/_2$ or 3 per cent.

The peak value of the deceleration curve was computed and the machine curve appears to be high by $3^1/_2$ per cent.

In view of the difficult problem given the machine, virtually amounting to summing a considerable number of positive and

[1] By J. G. Baker. Published in the December, 1939, issue of the Journal of Applied Mechanics, Trans. A.S.M.E., vol. 61, 1939, p. A-145.

[2] Professor of Marine Engineering, Massachusetts Institute of Technology, Cambridge, Mass. Mem. A.S.M.E.

[3] "Vibration During Acceleration Through a Critical Speed," by F. M. Lewis, Trans. A.S.M.E., vol. 54, 1932, paper APM-54-24, p. 253.

negative loops, such an agreement can be considered remarkably close. It is to be expected that for lower q values and higher γ values a still closer check could be obtained.

Author's Closure

In the oral discussion attention was called to a paper, "The Transient Vibrations of Machines," by W. E. Johnson, published in the *General Electric Review.* The author was not aware of this paper, which should have been included in the references noted. The present paper differs mainly by including the effect of damping.

Professor Lewis' discussion is especially valuable in that it not only demonstrates an entirely mathematical method of obtaining the results of the paper, but also substantiates the accuracy of the mathematical machine used. The author is grateful for Professor Lewis' contribution.