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Statistical Determination of Decision-Making Regions for Branching Paths: An Algorithm With a Wheelchair Assistance Application

Typically, mobile vehicles follow the same paths repeatedly, resulting in a common path bounded with some variance. These paths are often punctuated by branches into other paths based on decision-making in the area around the branch. This work applies a statistical methodology to determine decision-making regions for branching paths. An average path is defined in the proposed algorithm, as well as boundaries representing variances along the path. The boundaries along each branching path intersect near the decision point; these intersections in path variances are used to determine path-branching locations. The resulting analysis provides decision points that are robust to typical path conditions, such as two paths that may not clearly diverge at a specific location. Additionally, the methodology defines decision region radii that encompass statistical memberships of a location relative to the branching paths. To validate the proposed technique, an off-line implementation of the decision-making region algorithm is applied to previously classified wheelchair path subsets. Results show robust detection of decision regions that intuitively agree with user decision-making in real-world path following. For the experimental situation of this study, approximately 70% of path locations were outside of decision regions and thus could be navigated with a significant reduction in user inputs. [DOI: 10.1115/1.4046578]

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1 Introduction

This work is motivated by human-assistive devices, particularly robotic or semi-automated wheelchairs, that seek to significantly enhance mobility. When using these vehicles, many users are challenged by a diminished dexterity and a decreased amplitude of movement [1]. In contrast, vehicle guidance applications tend to require continuous, rapid (high-bandwidth) steering and velocity inputs for path keeping. When employed in a wheelchair, path-keeping inputs usually come from a joystick. As wheelchair users age, or as motor functions decline, the dexterity needed to control a wheelchair's joystick may become cumbersome and demanding. This decreased dexterity may compromise the reliability of the input signals, creating error between the desired and actual wheelchair paths. The inability of the user to maintain path tracking can further compromise mobility, as poor tracking control often requires slower speeds. Ultimately, a need for high-bandwidth joystick inputs for path following may sacrifice the use of the wheelchair altogether.

The motivation of this work is that wheelchair guidance may be improved if users do not need to continuously supply directional inputs. Instead, the inputs could be analyzed to provide partial or complete automation for path selection, with the wheelchair performing corrections to follow the user's path choice. This capability assumes that the common paths of a wheelchair are known a priori and that the decision locations for choosing a particular path option are well defined.

However, it can be unclear how to define a common or average path, or more importantly, when the user or algorithm should be queried to determine which branch of a path should be followed. This work develops a statistical determination of decision-making regions for branching paths. The technique employed determines the locations in which users or operators are most likely to supply path guidance information. While the presented implementation is for wheelchair assistance, the algorithm can be applied to any robotic platform, including connected or autonomous road vehicles.

1.1 Prior Work Defining Path Subsets. Prior work by the authors developed a statistical method of determining a wheelchair user's common paths [2]. Three techniques were presented: (1) a methodology to calculate the average path from a group of similar traversals; (2) a calculation of a path's uncertainty due to variances between similar paths; and (3) a probabilistically determined goal location based on the number of times a particular path has been traversed. This work builds on that past effort and defines a means to determine the spatial locations in which users are most likely to input a path-specific decision as two or more paths begin to branch.

An important aspect of the prior work is the method in which paths are defined. While a plethora of studies have explored path planning (e.g., Refs. [3–7]), limited work has investigated robotic path or trajectory averaging. Though spline and other linear interpolation methods were able to be applied in Ref. [8], a line projection technique was ultimately chosen for Ref. [2] to combine multiple paths, particularly those that are non-unique (e.g., lack a one-to-one mapping between x and y positions).

Two additional techniques that are applied to path and trajectory averaging are stochastic approximation and path ensemble averaging. Stochastic approximation algorithms are recursively updated rules that can be used to solve optimization problems [9]; ensemble averaging considers trajectories of dynamic systems and the

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description of the resulting bundle. Ensemble averaging is applied to climate change, quantum systems, oscillators, neural networks, and even cardiography [10]. For example, a recursive stochastic optimization procedure was applied to trajectory averaging and yielded the highest possible asymptotic convergence rate for stochastic approximation algorithms [11].

1.2 Intersection-Based Decision-Making. A potential application of the current work is intersection-based decision-making for vehicles. With 30% of roadway fatalities related to intersections, their detection remains challenging for autonomous vehicles [12–14]. In a study by Wang et al., driving scenarios are divided into two categories: (1) normal road without intersections and (2) complex road intersections [15]. In the latter, accurate locations and orientations of all intersection branches are needed for decision-making and path planning. To recognize intersections, 3D-LiDAR scans were analyzed for admissible space in front of autonomous vehicles. A ν-support vector regression model is then used to estimate the pose of each intersection branch, and a fusion method is proposed to locate the position of the intersection.

Additionally, the techniques of this paper may be applicable to biomedical applications, such as intravenous injections. For example, in the study by Brewer and Salisbury, an algorithm is proposed that searches venous networks for bifurcations, regarded as the optimal intravenous insertion points [16]. Since the desired insertion point is not at the center of the bifurcation, the difficulty in their proposed algorithm is the detection of closely spaced bifurcations, which appear as a single, noisy bifurcation. Similar challenges arise in the datasets examined in this work with wheelchairs, namely bifurcations in paths can exist close to each other in space, resulting in a noisy bifurcation of many paths nearby each other, rather than distinct decision points. Thus, one can observe that path selection is not simply a challenge of traversing a connected graph, but also in analyzing the spatial location, and possible similarity, in the nodes of the graph to enable discrimination of distinct decisions in space and time.

1.3 Spatial Planning for Decision-making. Critical to the implementation of the proposed technique is a qualitative spatial understanding of the robot's current and future positions. Rodic and Katic present a robot that acquires information about different paths and builds a corresponding topology map [17]. The robot can then follow the same trajectories in later tasks. Additionally, the robot's current position can be compared with memorized discrete positions on the previously mapped paths to continue the robot's motion along a known trajectory. The techniques employed in Ref. [17] are very similar to those presented in Ref. [2] and the present work, the novelty of the current work being that a decision region is determined for path-branching locations. Additionally, in the current study, the goal is to use previously mapped paths and previously measured pose information to verify location prediction based on the robot's current position.

The aim of this paper is to apply a statistical approach to determine decision-making regions for branching paths. This technique of finding key decision-making regions, and only querying a user or algorithm at these regions, has the potential to significantly reduce user or operator inputs.

2 Decision-Making Region Methodology

This section presents a statistical approach to define the decision-making regions for branching paths. In prior work by the authors, an experimental study was performed in which user-driven robotic wheelchair traversals to predetermined locations were recorded [2]. Path averages and variances were then calculated for path subsets with the same starting and ending locations. The path averages and variances along each path are used to derive an average path in a spatial coordinate along the path called the s-coordinate. Analysis of these paths for real use applications showed clear branching points between paths. These points where paths branch to different goal locations are where users typically make decisions about their intended goal destination. Here, we want to define the regions at which users choose one path over another.

A naive approach would be to simply examine average paths to identify the exact point at which the paths deviate. In many cases, the average paths do not clearly separate from each other at a distinct point. For example, Fig. 1 shows the common paths in the laboratory testing region for the team's intelligent wheelchair. The point designated as "A" is a starting point, and the points numbered 1 to 4 represent common destination points within the lab. Upon repeated traversals from point A, one observes that paths commonly branch around obstacles in the lab—tables in this case. A branching location occurs as the paths deviate from point A at position ($X=0.25$, $Y=4.50$). These results, obtained from the instrumented wheelchair, illustrate that able-bodied persons and vehicles do not typically switch from one path to another at the same location in space. Rather, vehicles and people alike, including wheelchair users, may trend toward one path over a spatial range. Consequently, a decision point, defined as any point along a set of paths in which the paths are no longer statistically distinguishable, becomes a decision-making region. Now, the decision-making region, or decision space, can be defined as the area within which users must make a decision regarding their intended path.

The following procedure outlines a statistically determined approach to identify and refine decision points and their corresponding decision-making regions:

Step 1: Identify Path Variance Intersection Points. First, the two closest branching paths must be established. Additionally, the boundaries representing user-selected standard deviation levels n in the path variances are calculated. In this work, the standard deviation levels range from $n_{user}=0.25$–4-sigma standard deviations. Next, the intersection points between the overlap of the path variance boundaries are identified. These points are defined as the path variance intersection points or PVIPs.

Step 2: Fit a Linear Regression Line. The second step is to fit a least-squares linear regression line through the PVIPs.

Step 3: Identify Snap-fit PVIPs. Next, the closest point on the linear regression line to each PVIP is found. The closest points are identified by projecting a line from each PVIP onto the linear regression line. These newly established points on the regression line are called the "snap-fit" PVIPs (p_{snap}).

Step 4: Compute the Radius for Each Snap-fit PVIP. An arbitrary origin (x_o, y_o) and ending location (x_f, y_f) on the linear regression line is chosen. From this origin, one can then calculate the radii

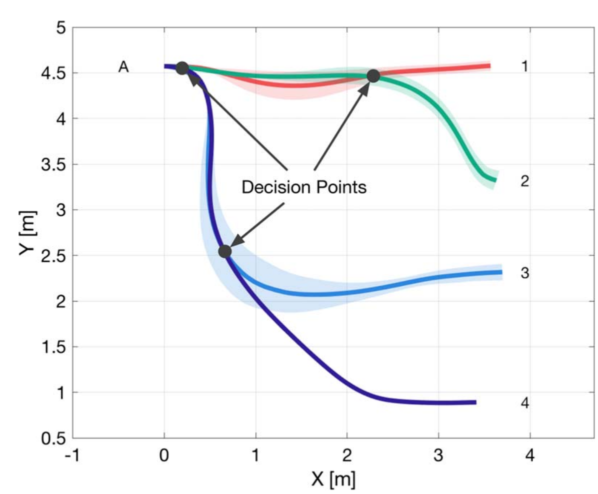

Fig. 1 Path 1-sigma variances for each average path, starting at location A to each final destination (1–4) with branching locations (decision points) occurring at positions ($X=0.25$, $Y=4.50$), ($X=2.50$, $Y=4.45$), and ($X=0.75$, $Y=2.55$)

to each snap-fit PVIP using:

$$r_\sigma = \sqrt{(p_{snap,x} - x_o)^2 + (p_{snap,y} - y_o)^2} \tag{1}$$

Step 5: Fit a Second Linear Regression Line to the Radii. A second least-squares linear regression line is fit through the radii identified in Step 4.

Step 6: Define Origin and Variance of the Decision-Making Distribution. Implementation of the linear regression line applied in Step 5 provides the origin of the PVIP distribution, as well as the variance of the PVIP distribution. Here, the actual locations of the snap-fit PVIPs are defined, such that p_{dist} is the origin of the distribution and σ_{dist} is the variance of the distribution.

Step 7: Identify the Center of the Decision Region. Next, the center of the decision-making region is identified using the origin of the distribution p_{dist}.

Step 8: Calculate the Decision Region Radii. Finally, the last step of the procedure is to calculate the decision region radii $r_{decision}$. This value is computed using the user-selected standard deviation level n_{user} and the PVIP distribution σ_{dist}:

$$r_{decision} = n_{user} \cdot \sigma_{dist} \tag{2}$$

An algorithmic summary for finding the path decision points and decision-making regions is presented in Algorithm 1. Additionally, pictorial illustrations of the decision region algorithm are shown in Fig. 2.

Algorithm 1 DecisionPointRegions

Data: σ, p_{sigma}, such that p_{sigma} are the path variance intersection points (PVIPs) between the average path variances for a variety of standard deviations, σ.

Result: $p_{decision}$, $r_{decision}$, such that $p_{decision}$ is decision point in x- and y-coordinates and $r_{decision}$ is the decision region radius for a given σ.

begin

$lin_{reg} \leftarrow$ fit linear regression line to PVIPs p_{sigma}
$p_{snap} \leftarrow$ locate closest point on lin_{reg} to p_{sigma}
$x_o \leftarrow$ arbitrary x-origin on linear regression line
$x_f \leftarrow$ arbitrary x-ending location on linear regression line
$r_\sigma = \sqrt{\left(p_{snap,x} - x_o\right)^2 + \left(p_{snap,y} - y_o\right)^2}$
$lin_{r_\sigma} \leftarrow$ fit linear regression line to r_σ
$lin_{r_\sigma} = [\sigma_{dist}\ r_{dist}]$
$\mathbf{v} = [x_f\ y_f] - [x_o\ y_o]$
$\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|}$
$p_{decision} = [x_o\ y_o] + \mathbf{u} r_{dist}$
$r_{decision} = n_{user} \cdot \sigma_{dist}$

3 Implementation for a Robotic Wheelchair

This section implements the application of the decision-making region identification technique. To demonstrate the proposed approach, measurements of a robotic wheelchair's motion are used as an application example. A modified robotic wheelchair

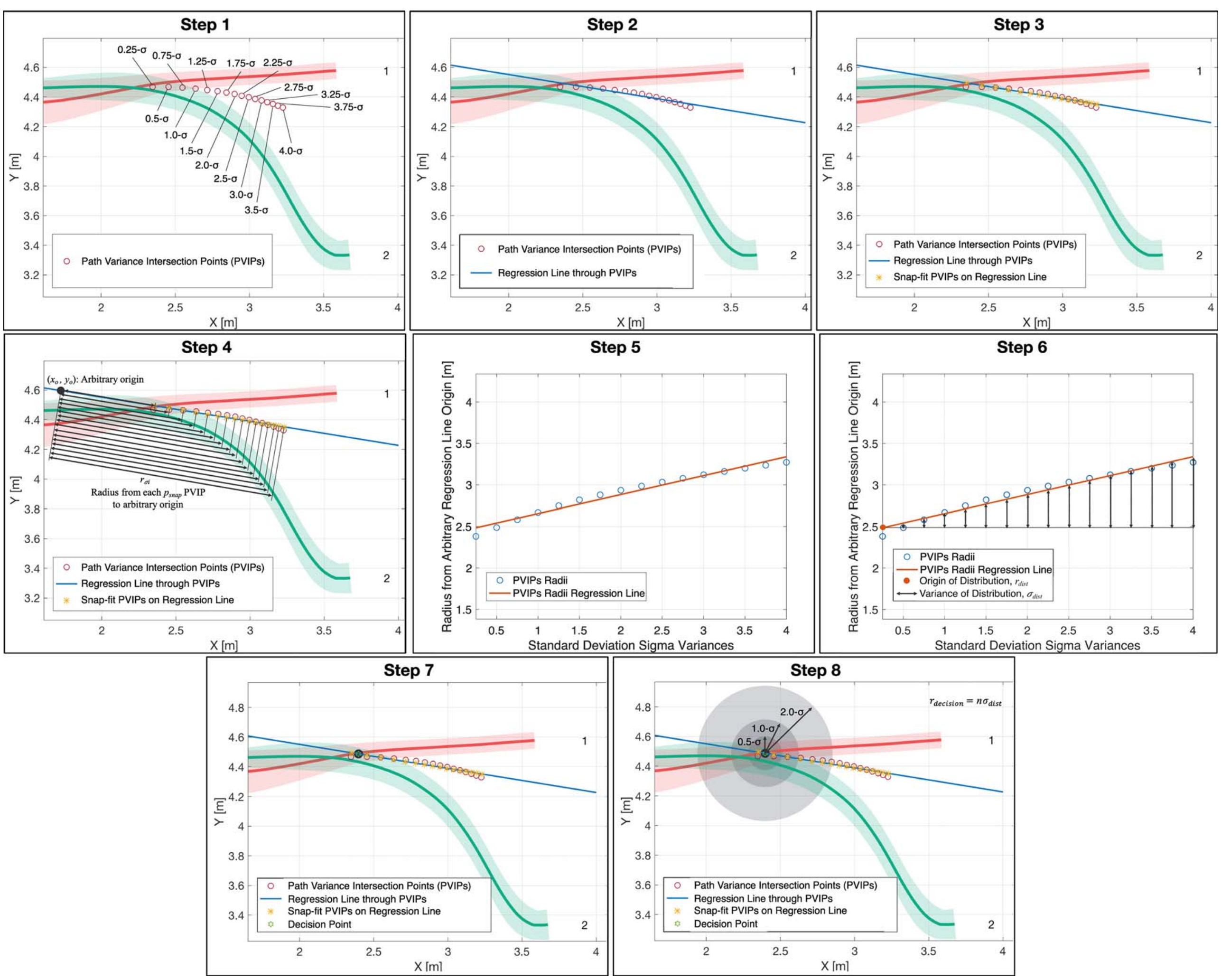

Fig. 2 Pictorial illustration of decision-making region identification technique showing Steps 1–8

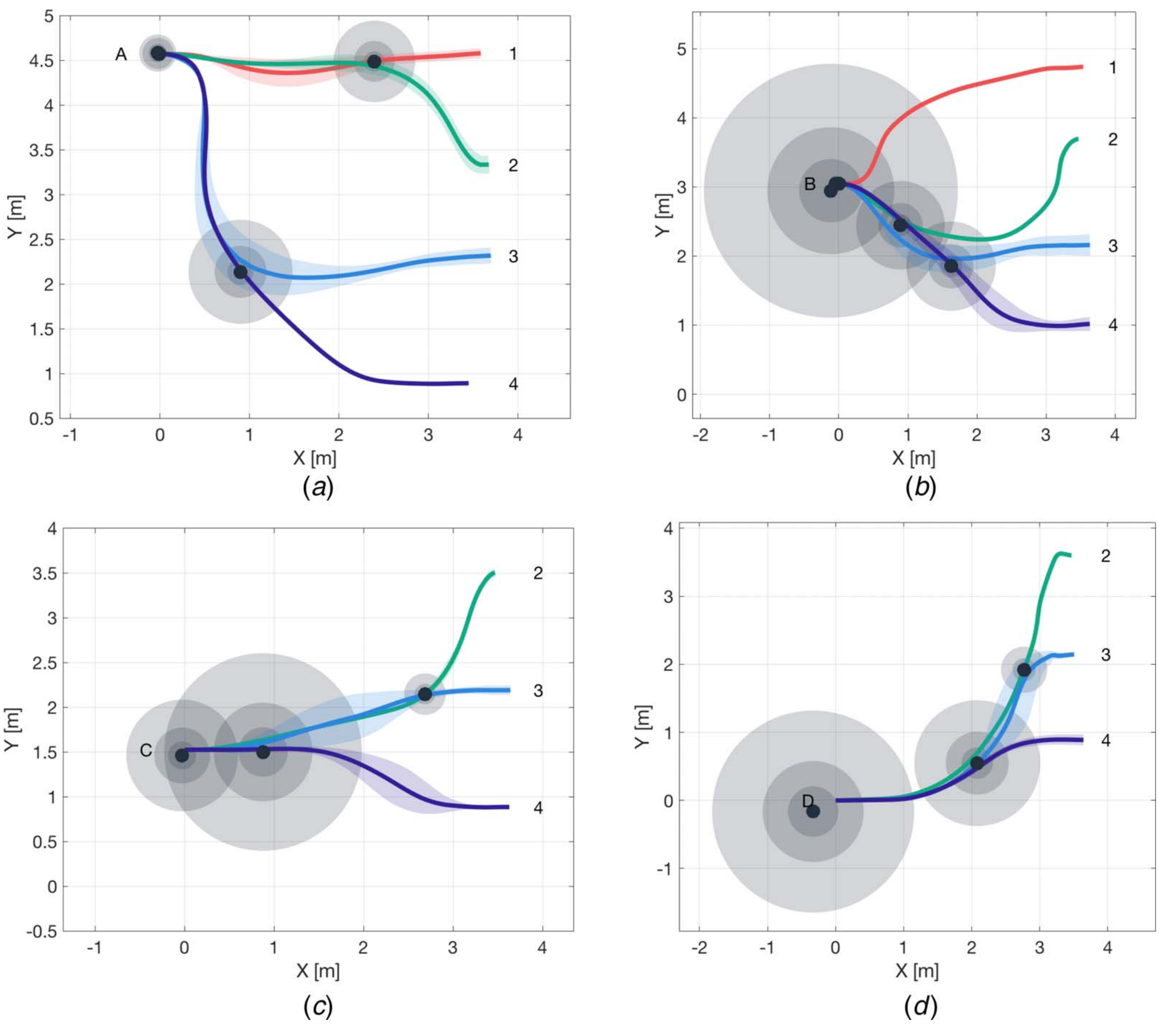

Fig. 3 Path decision points and spaces for each average path from four different start locations (A, B, C, and D) to each final destination (1, 2, 3, and 4 colored consistently by destination across subplots). The decision space circles radiate from the decision points at variances of 0.5, 1, and 2 sigma, with gray shading lightened respectively.

(Jazzy Pride Select 6, Pride Mobility Products, Corp., USA) was used to collect path traversals within an office setting. Pose measurements were collected via optical wheel encodes (HB6M Hollow Bore Optical Encoders, US Digital, USA) mounted directly to the direct-drive wheels of the wheelchair. Joystick steering commands were also collected from the wheelchair.

The same datasets are used for this implementation as those analyzed in Ref. [2]. The wheelchair was manually driven from one of four fixed starting locations (labeled A–D) to one of four target destinations (numbered 1–4), as illustrated in Fig. 1. To validate the repeatable accuracy of the measurements and the proposed algorithm, a total of 45 path traversals were driven.

The off-line implementation of the proposed decision-making region algorithm to each of the collected datasets is presented in Fig. 3. Here, the 0.5-, 1-, and 2-sigma decision regions for the previously classified path subsets are shown. It is important to note that by applying this approach, the result may yield an origin of the distribution that is asymmetrically located between the two closest paths (i.e., the origin may appear to lie closer to one path than another). This location is the result of both the user-selected percentile level and the number of times a particular path set has been traversed.

Of particular importance is the potential for a significant reduction in user input when implementing the decision-making region. Using the statistical description of a decision-making region, the user or operator need only input a navigation command while within this region. As a result, user control is alleviated from fine-control of following a path to coarse control of selecting a path; this result has the potential to reduce strain from the user, particularly in the case of wheelchair users suffering from a range of neurodegenerative diseases, including amyotrophic lateral sclerosis.

The minimum potential reductions in user input for each starting location to each final destination using the $n_{user} = 2$ standard deviation decision regions from Fig. 3 are presented in Table 1. These percent reductions were determined by dividing the diameter of the decision-making region by the total length of the corresponding path. Note that the decision-making regions in the collected dataset

Table 1 Minimum and overall reduction of user inputs for each starting and ending location

	Starting location			
Ending location	A	B	C	D
1	75%	97%	–	–
2	78%	59%	86%	74%
3	79%	55%	41%	59%
4	80%	70%	41%	52s%
Minimum average		**67.6%**		
Overall average		**81.4%**		

are intentionally close together, so that reduction in less confined settings would be expected to produce even better results.

$$I_{reduction} = \frac{2 \cdot r_{decision}}{L_{path}} = \frac{D_{decision}}{L_{path}} \quad (3)$$

where $I_{reduction}$ represents the input reduction, $r_{decision}$ and $D_{decision}$ represent the radius and diameter of the decision-making region, respectively, and L_{path} represents the length of the corresponding path. The resulting minimum average reduction of user inputs for all possible paths is 68%, which satisfies the primary objective of a significant reduction of user/operator inputs. Across all possible path permutations, an overall average of 81% user input reduction was achieved.

Moreover, while this implementation is applied to wheelchair assistance, it is also important to recognize that this work is applicable to any indoor robotic platform, such as janitorial or industrial robotics. Additionally, the decision-making region algorithm could be applied to autonomous or semi-autonomous vehicles, for example, aiding in intersection decisions.

4 Conclusions

The key contribution of this paper is the proposal of a statistically determined decision-making region identification technique for branching paths. The novelty of the proposed algorithm is the use of path variance intersections to determine path branching locations, rather than the naive assumption that key path decision information is held exclusively at the intersection of path averages. The primary objective of this technique was to reduce user or operator inputs; the implementation of the technique with previously collected wheelchair path sets yielded a minimum average reduction of 68% and an overall reduction of 81%.

Additionally, while the present paper applied the decision-making region identification technique to wheelchair assistance, the proposed approach can be readily applied to any indoor robotic platform and can be modified and applied to intelligent vehicles for decision-making at intersections. Along these lines, Nazario-Casey et al. make the case that utilizing autonomy in conjunction with online decision-making offers the potential for generalization, but also makes implementation more challenging [18]. The authors of the present paper are in agreement with this statement, but believe this generalization is highly beneficial to an array of robotics-based applications. Moreover, by creating modular algorithms and testing them on a variety of robotic platforms, the proposed approach can be expanded upon, modified, and evaluated for new systems and their associated challenges [19].

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