Toward A Mobile Robot for Vibration Control and Inspection of Power Lines

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As power demand across communities increases, focus has been given to the maintenance of power lines against harsh environments such as wind-induced vibration (WIV). Inspection robots and fixed vibration absorbers (FVAs) are the current solutions. However, both solutions are currently facing many challenges. Inspection robots are limited by their size and considerable power demand, while FVAs are narrowband and unable to adapt to changing wind characteristics and thus are unable to reposition themselves at the antinodes of the vibrating loop. In view of these shortcomings, we propose a mobile damping robot (MDR) that integrates inspection robots mobility and FVAs WIV vibration control to help maintain power lines. In this effort, we model the conductor and the MDR by using Hamilton's principle, and we consider the two-way nonlinear interaction between the MDR and the cable. The MDR is driven by a proportional-derivative (PD) controller to the optimal vibration location (antinodes) as the wind characteristics vary. The numerical simulations suggest that the MDR outperforms FVAs for vibration mitigation. Furthermore, the key parameters that influence the performance of the MDR are identified through a parametric study. The findings could setup a platform to design a prototype and experimentally evaluate the performance of the MDR. [DOI: 10.1115/1.4050957]

Keywords: control applications, dynamics and control, intelligent systems, vibration control, vibrations

1 Introduction

Wind-induced vibration (WIV) is a major concern for flexible engineering structures such as airplane wings, spacecraft, power transmission lines, and cable-stayed bridge (see Fig. 1). For power transmission lines, these oscillatory motions typically take the form of Aeolian vibrations, which are caused by vortex shedding and are relatively high-frequency, low-amplitude vibrations. Aeolian vibration frequencies vary between 3 and 150 Hz, and the peak-to-peak amplitude is usually smaller than the cable diameter [1–7].

When left uncontrolled, WIV may lead to power lines (PLs) failure, thereby undermining public safety and resulting in considerable economic losses. The Department of Energy (DOE) reported that weather-related annual outage costs were estimated to be between \$18 and \$33 billion [8]. These unfortunate events have also resulted in deaths. The conventional methodology for vibration mitigation employs fixed passive vibration absorbers (FPVAs) (Fig. 1(*a*)). The problem with the FPVAs is that their effectiveness is dependent on the wind characteristics, which change with time, thus frequency and optimum absorber location (i.e., antinode of the vibrating loop) are also time varying. These challenges can be overcome using a mobile damping robot (MDR) capable of adapting to the changing environment and move to an antinode.

Currently, the most common methods of power line inspection and maintenance are foot patrol and helicopter-assisted inspection [9]. Both techniques are expensive, laborious, and can be dangerous for electrical technicians and pilots. For these reasons, numerous grid owners, institutions, and researchers have developed inspection robots [10–17]. However, the implementation of these inspection robots is limited by the cost, the high power demand, the short operation time, and the considerable weight. For instance, the LineScout robot developed by Hydro Quebec weighs about 120 kg [18] (Fig. 1(*b*)), and the TI robot from EPRI is about 2 m long [14]. We plan to transform our recent patent on passive WIV control to a robot for not only smart vibration control but also for intelligent power line monitoring, inspection, and repair. Unlike conventional inspection robots, our novel robot will be lightweight, compact, and permanently mounted on the power lines.

To realize an MDR for intelligent WIV control and maintenance of power lines, we need to understand the linear and nonlinear dynamic interactions of a moving robot, cable, and wind forces. Few researchers have explored the possibility of a moving damper for improved vibration suppression in manufacturing [19] and for civil structures [20,21]. More recently Bukhari et al. [22] extended the moving vibration absorber idea to power lines. In that work, the mobile damper responded to a predefined profile of wind characteristics. However, it did not include an active controller to travel to the antinode as needed. In this study, we redefine the moving damper as an intelligent mobile damping robot and explore further its ability to suppress vibrations of PLs and carry out other tasks such as monitoring and inspection of PLs. To do so, we model the MDR with a proportional-derivative controller to travel across the conductor using Hamilton's principle [23]. Then, we provide a numerical analysis of the system carried out in MATLAB®. Finally, we perform a parametric study to determine the key parameters of the robot.

2 System Description

This section presents the mathematical derivation of the mobile damping robot attached to a conductor as portrayed in Fig. 2. This concept was developed by integrating our recently patented Aeolian vibration damper [24] with an inspection robot. The conductor, the in-span mass, and the vibration damper [24] were reduced to an equivalent tensile Euler–Bernoulli beam with an in-span mass–spring–mass and viscous damping system (see Fig. 3). The combination of the in-span mass and suspended mass constitutes the total mass of the MDR. Note that modeling a conductor as a beam instead of a string is more accurate since it helps capture the flexural rigidity of the conductor [25].

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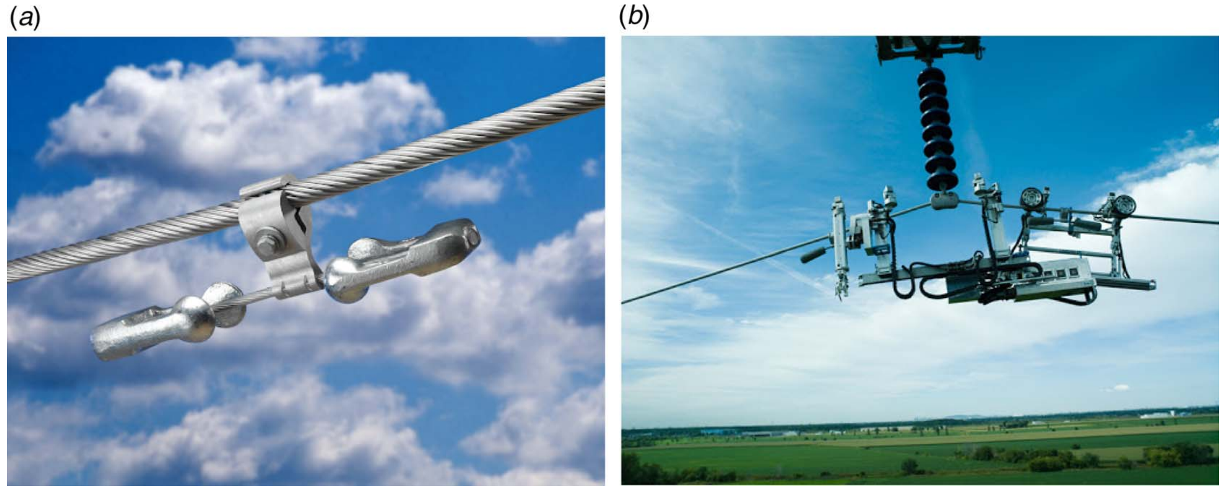

Fig. 1 Presentation of a FPVA and an inspection robot: (*a*) stockbridge damper and (*b*) Hydro-Quebec lineScout inspection robot

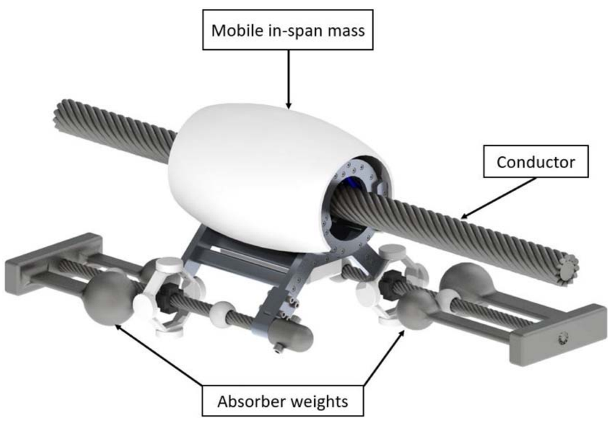

Fig. 2 Conceptual design of the mobile damping robot attached to a conductor

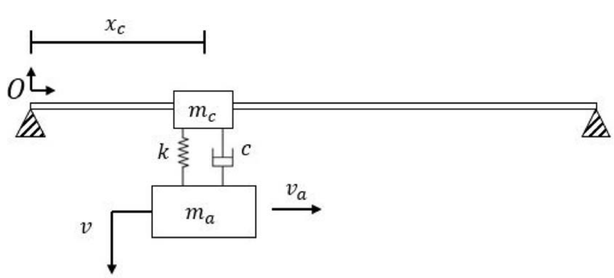

Fig. 3 Schematic of the mobile damping robot on the cable

Starting from the position vector of the beam, the in-span mass, and the suspended absorber and using Hamilton's principle, the governing equations of motion (EOMs) of the system can be expressed as follows:

$$EIy'''' + m\ddot{y} + Ty'' = F(x, t) - (F_1 + F_2)\delta(x - x_c) \tag{1}$$

$$[(m_a + m_c)\ddot{x}_c + m_c(\ddot{y} + 2\dot{y}'v_a + y''v_a^2 + y'\ddot{x}_c)y']\delta(x - x_c) = F_c \tag{2}$$

$$[m_a\ddot{v} - F_2]\delta(x - x_c) = 0 \tag{3}$$

The transverse displacement of the cable and the absorber are denoted by y and v, respectively. The longitudinal displacement and the velocity of the MDR are denoted by x_c and v_a, respectively. EI is the flexural rigidity, m is the mass of the beam, T is the tension of the beam, m_c is the mass of the in-span mass, and m_a is the mass of the absorber. $\delta(x - x_c)$ represents the Dirac delta function used to determine the instantaneous location of the MDR. $F(x, t)$ is the uniform wind input force expressed as follows:

$$F(t) = f_0 \sin(\omega_n t) \tag{4}$$

where f_0 is the drag force defined as follows [22]:

$$f_0 = 0.5\rho D C_d V_w^2 \tag{5}$$

where D is the diameter of conductor, ρ is the density of the fluid (wind), C_d is the drag coefficient, and V_w is the velocity of the wind.

F_1, F_2, and F_c are given as follows:

$$F_1 = m_c\left(\ddot{x}_c + (\ddot{y} + 2\dot{y}'\dot{x}_c + y''\dot{x}_c^2 + y'\ddot{x}_c)\right) \tag{6}$$

$$F_2 = k(y - v) + c(\dot{y} + y'\dot{x}_c - \dot{v}) \tag{7}$$

$$F_c = k_p(r - x_c) + k_d(\dot{r} - v_a) \tag{8}$$

In Eq. (7), k represents the spring constant of the MDR and c represents its damping coefficient. In Eq. (8), k_p and k_d represent the proportional and the derivative gain, respectively. r represents the position target, while $\dot{r}$ represents the velocity target. In our model, the position target can be determined based on the input wind force. For instance, if the wind input excites the first mode, the antinode will be located at the mid-span. It should be noted that this equation includes the nonlinear coupling forces, i.e., centrifugal and Coriolis terms.

The vertical displacement of the cable is the solution to the EOMs. It can be expressed using the Galerkin decomposition method as follows:

$$y(x, t) = \sum_{r=1}^{\infty} \Phi_r(x)A_r(t) \tag{9}$$

where $A_r(t)$ are time functions of the transverse displacement and $\Phi_r(x)$ are the normalized eigenfunctions (mode shapes). Following Barry [4], the eigenfunctions are chosen as the mode shapes of a simply supported beam with tension as follows:

$$\Phi_r(x) = \sqrt{\frac{2}{ml}} \sin\left(\left(\sqrt{\frac{-T}{2EI} + \sqrt{\frac{T^2}{4(EI)^2} + \frac{m\omega_r^2}{EI}}}\right)x\right) \tag{10}$$

where the natural frequencies of the bare beam are given by

$$\omega_r = \left(\frac{\pi}{L}\right)\sqrt{\frac{EI}{m}\left(r^4 + \frac{r^2 TL^2}{\pi^2 EI}\right)} \tag{11}$$

Following [23,26], Eqs. (1)–(3), which are a set of PDEs, can be transformed into ODEs for computation using the orthogonality condition. Assuming a constant MDR velocity, we obtain the following set of ODEs

$$\begin{aligned}
&\ddot{A}_p(t) + M_c\left[\sum_{r=1}^{\infty}\ddot{A}_r(t)\Phi_r(d) - 2\dot{A}_r(t)\Phi_r^{'}(d)v_a + A_r(t)\Phi_r^{''}(d)v_a^2\right] \\
&\quad \times D_p(t) + 2\zeta\omega_p\dot{A}_p(t) + \omega_p^2 A_p(t) \\
&\quad + \left\{k\left[\sum_{r=1}^{\infty}A_r(t)\Phi_r(d) - v(t)\right]\right. \\
&\quad \left. + c\left[\sum_{r=1}^{\infty}A_r(t)\Phi_r(d) + A_r(t)\Phi_r^{'}(d)v_a - \dot{v}(t)\right]\right\}D_p(t) = N_p(t)
\end{aligned} \tag{12}$$

$$\begin{aligned}
&(M_a + M_c)\dot{v}_a(t) + M_c \\
&\quad \times \left[\sum_{r=1}^{\infty}\ddot{A}_r(t)\Phi_r(d) - 2\dot{A}_r(t)\Phi_r^{'}(d)v_a + A_r(t)\Phi_r^{''}(d)v_a^2\right] \\
&\quad \times A_r(t)\Phi_r^{'}(d)D_p(t) = k_p(r - x_c(t)) + k_d(\dot{r} - \dot{x}_c(t))
\end{aligned} \tag{13}$$

$$\begin{aligned}
&M_a\ddot{v}(t) - k\left[\sum_{r=1}^{\infty}A_r(t)\Phi_r(d) - v(t)\right] \\
&\quad + c\left[\sum_{r=1}^{\infty}A_r(t)\Phi_r(d) + A_r(t)\Phi_r^{'}(d)v_a - \dot{v}(t)\right] = 0
\end{aligned} \tag{14}$$

where $N_p(t)$ and $D_p(t)$ can be defined as follows:

$$N_p(t) = \int_0^L \Phi_r(x)F(x, t)dx, \quad r = 1, 2\cdots \tag{15}$$

$$D_p(t) = \int_0^L \Phi_r(x)G(x, t)dx, \quad r = 1, 2\cdots \tag{16}$$

and d is the position of absorber, which is equal to $v_a t$.

It is important to note that unlike in Ref. [22] in which a step function was employed to sequentially move the robot, and here, we determine the location of the antinode and select values for k_p and k_d to effectively drive the absorber to the desired position.

3 Numerical Simulations

The numerical simulations were performed on a 200-m span length, using 795 Drake ACSR cable. The cable and the uniformly distributed load parameters are presented in Table 1. The cable natural frequencies were determined and used as wind excitation inputs. Table 2 lists five different cable modes. We note that the fundamental frequency falls outside of the Aeolian vibration frequency range (3 Hz–150 Hz). The other frequencies presented in the table correspond to frequencies within the Aeolian vibration frequency range (see Table 3).

Next, we verify that regardless of the simulation method, we can obtain the same results. Hence, we simulate the model of the MDR attached to the conductor using two MATLAB computation methods. The two methods considered are Bode and ODE45. Each computation method relies on distinct theories. The Bode function is used by defining the transfer function of the system and by analyzing the frequency response. The ODE45 method is founded on the acceleration analysis of the system using the EOMs. As shown in Fig. 4, both methods generate the same results. This particular experiment can be extended to determine the optimal placement of the robot as we will discuss in the subsequent sections.

Having validated the mathematical model, we provide a presentation of the PD controller used for the proposed MDR. The robot relies on a control scheme to move to the antinode and help mitigate the vibration of the cable. To determine the proportional and derivative gains for optimal control, the control requirements for the design need to be specified. The mobile robot is also required to reach the antinode in a reasonable amount of time, and hence, the rise time is also a key design parameter. Indeed, the faster the robot reaches the antinode, the quicker the vibration of the cable is reduced. In addition, we also desire to minimize the steady-state error to ensure that the robot reaches and stays at the antinode. With these specified requirements, we can iterate the values of k_p and k_d. Figures 5(*a*) and 5(*b*) show the horizontal displacement of the mobile robot as a function of time when the robot is subject to step inputs corresponding to the target antinode. For different combinations of k_p and k_d, we obtain a different response. Figure 5(*a*)

Table 1 Parameters of the cable and applied load

L (m)	m (kg/m)	T (N)	EI (N.m^2)	w_n (rad/s)	f_0 (N/m)
200	1.6286	27,840	1602	w_n	Eq. (5)

Table 2 Parameters of the mobile robot

m_c (m)	m_a (kg/m) T (N)	k (N/m)	c (Ns/m)	k_p	k_d
24.8	1.30	$m_a \times \omega_n^2$	5	1	6

Table 3 First five natural frequencies for a 200 m cable in Hz

w_1	w_{10}	w_{16}	w_{30}	w_{60}
0.32	3.27	5.23	9.87	20.11

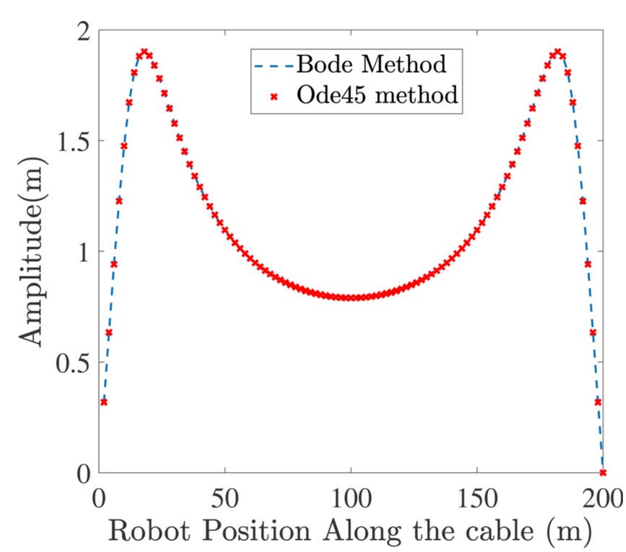

Fig. 4 Model validation by comparing the response of the cable-robot system using two methods: ODE45 and bode. The cable is excited by a uniformly distributed wind force.

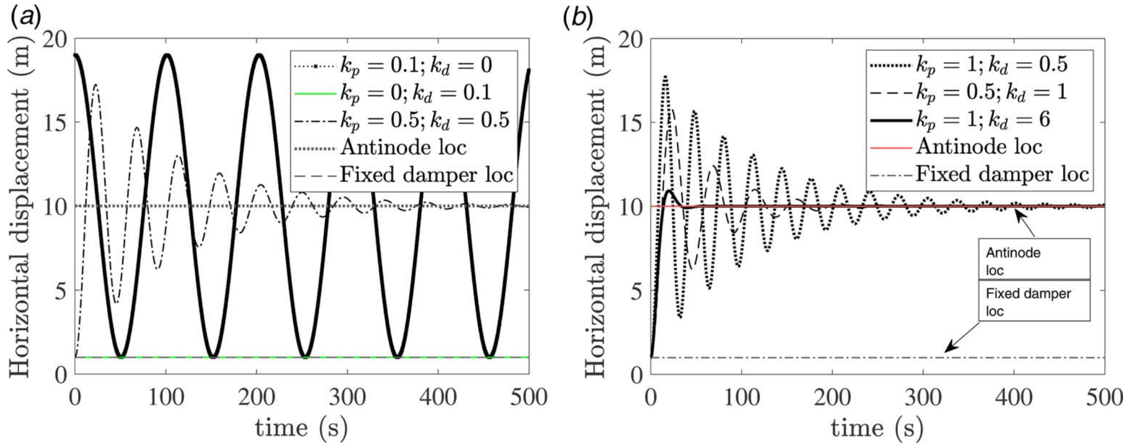

Fig. 5 Comparing the values of k_p and k_d for optimal control

shows that without a proportional gain, the robot cannot follow the target and stays at its original location. The results also show that without a derivative gain, the robot moves, but it showcases significant overshoot and steady-state error. Furthermore, when k_p matches k_d, the robot moves to the desired location but it still shows significant overshoot. However, over time, the device settles to the target antinode (minimal steady-state error).

Figure 5(*b*) is a further investigation of the combination of k_p and k_d. It is important to note that for controller gain, in general, it is the relative value between a gain and another that impacts the performance of the controller. The results in Fig. 5(*b*) show that when k_p is larger than k_d, we get significant overshoot, but there is no significant steady-state error. When k_d is larger, we can observe that the percent overshoot significantly decreases, and the steady error also remains small. This finding suggests that with k_d larger than k_p, we can meet the specified performance requirements of the proposed mobile robot for wind-induced vibration control of power lines. For the subsequent work, we will use $k_p = 1$ and $k_d = 6$.

We have shown how to select k_p and k_d, we now focus on the required target location for maximum vibration reduction. The optimal location for vibration suppression corresponds to the conductor's antinodes [27]. For Aeolian vibrations, the wind frequency input ranges from 3 Hz to 150 Hz. Considering this frequency range, a frequency analysis can be done to evaluate the effect of the robot position on the achieved vibration attenuation. Figure 6 shows the frequency responses of the bare cable (Fig. 6(*a*)) and the mobile vibration absorber (Fig. 6(*b*)) attached to the cable at different positions as the wind excitation frequency varies. The frequency in Fig. 6 is normalized by the cable fundamental frequency. The results show that the worst-case frequencies correspond to low harmonics. The vibration of the cable depends on the absorber position. The grayscale used shows the variation of vibration magnitude. Lighter regions show high vibration while darker regions show low vibration. If the absorber is tuned to the wind input frequency and it is properly placed at an antinode, it becomes fully effective by mitigating the vibration of the cable. On the other hand, if the absorber coincides with a node, it becomes ineffective. Consequently, we understand that fixed vibration absorbers (FVAs) will be limited because there is a chance that the FVA position may coincide with a node, especially at higher frequencies (see Fig. 6(*b*)). Moreover, the relevance of the mobile damping robot becomes apparent. By consistently self-adjusting its position to an antinode, the mobile damping robot can potentially increase its effectiveness.

Having established that the position of the damper is a key for vibration reduction, we now compare the performance of a fixed damper to the moving damper. From the literature on vibration design [27], if the damper is well tuned and does not fall on a node, it should provide significant vibration reduction. In addition, the closer it is to the antinode, the better the vibration reduction should be. In the following, we test this theory by placing the fixed damper and the moving damper at the same initial location and we determine the performance of each device. Figure 7 shows the response of the cable when attached to a fixed damper and a moving damper when the cable is subject to an excitation matching its 10th mode (3.27 Hz). The fixed damper and the moving damper are placed at different locations close to the antinode or node, and we evaluate their performance. The results show that when both devices are placed close to the node, the fixed damper is ineffective. In this case, the cable vibrates at resonance. The moving damper is able to readjust itself to the vibrating loop antinode. This results in significant vibration reduction. The results also show that when the dampers are placed within the vibrating loop, their performance is comparable. In addition,

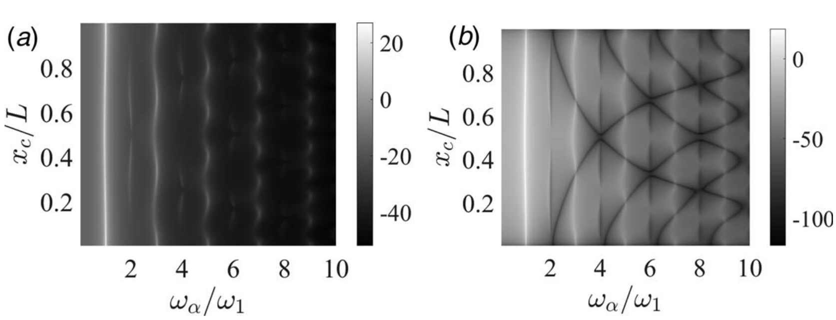

Fig. 6 Frequency response of the cable and the robot along the span: (*a*) normalized cable frequency response along the span and (*b*) normalized mobile damping robot frequency response along the span

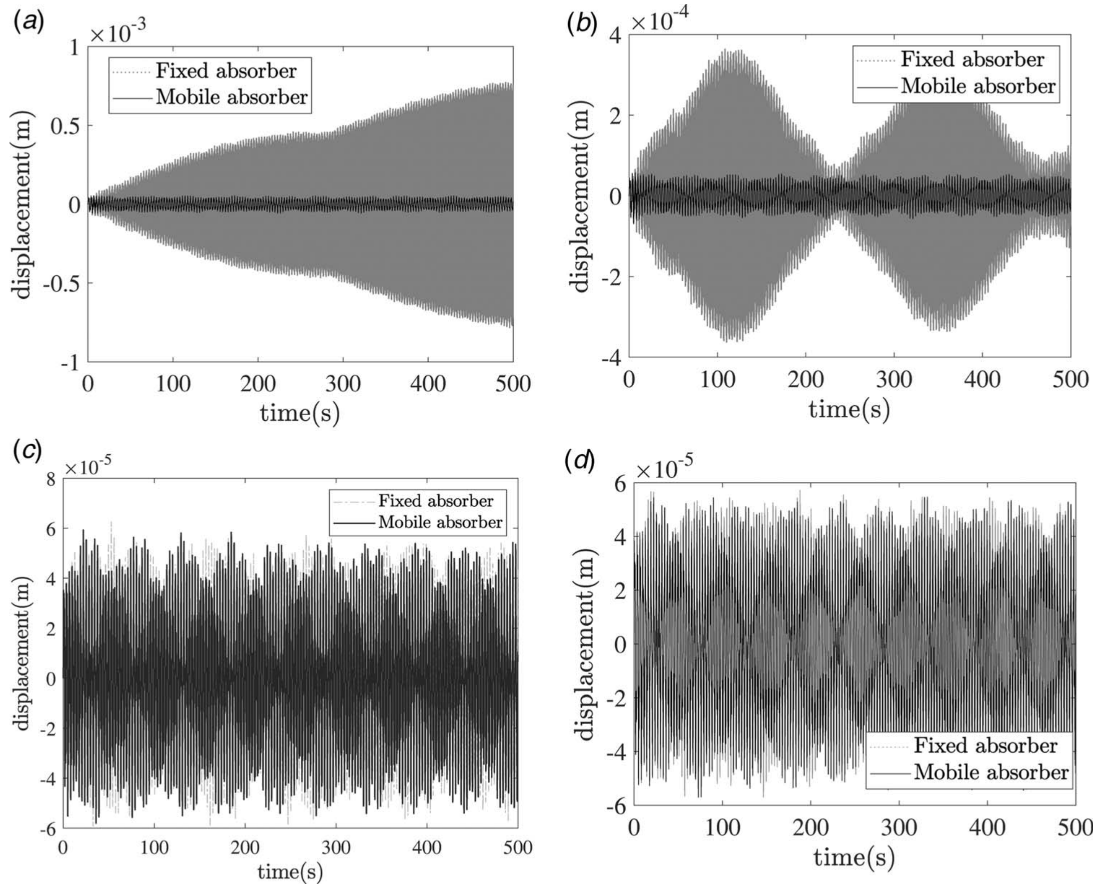

Fig. 7 Comparison of a fixed damper and the mobile damping robot for $w_{10} = 3$ Hz: (*a*) at node, (*b*) 10% away from node, (*c*) 30% away from antinode, and (*d*) at antinode

the results also show that the vibration mitigation is maximized at the antinode. In summary, we note that as long as the damper is within the vibration loop and is tuned to the resonance frequency, it is effective to reduce vibration. However, if the damper gets close or coincides with a node, it loses its efficiency. In these cases, the proposed moving robot becomes the solution of choice as it can move toward the antinode and maximize vibration reduction.

Having established that the moving damper ensures vibration reduction by being able to avoid nodes, we now focus on a parametric study. In particular, we attempt to evaluate the impact of the relative mass of the in-span mass to the suspended damper mass. For this, we consider the total mass of the device, which represents 8% of the total mass of the cable. We then vary the relative mass of the in-span to the suspended mass by introducing the parameter α. α ranges from 10% to 90% of the total mass m_T of the moving device. The in-span mass is then obtained as $m_1 = \alpha m_T$, and the suspended mass is obtained as $m_2 = (1-\alpha)m_T$. Figure 8(*a*) shows the displacement of the cable as a function of time for each moving damper assembly. Figure 8(*b*) shows the maximum displacement of the cable as a function of α. The results show that increasing the in-span mass degrades the performance of the mobile device.

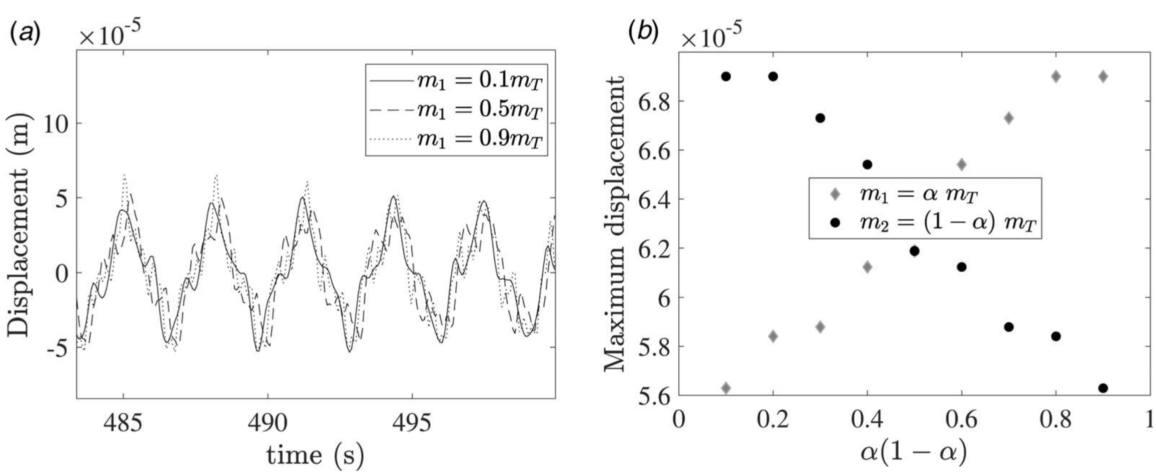

Fig. 8 Comparing the effect of mass ratio of the mobile robot

Indeed, as the in-span mass increases, the maximum displacement of the cable increases at a steady state. It can also be said that the vibration reduction is maximized for a larger suspended mass relative to the in-span mass. Thus, for future design consideration, the in-span mass needs to be minimized.

4 Conclusion

In this article, a mobile damping robot was modeled and analyzed to determine its performance in vibration mitigation of power lines. This study contributes to the ongoing research of continuous systems vibration mitigation. The proposed solution tracks the cable antinodes as the wind characteristic changes. The findings showed that when both the fixed damper and moving damper fall within a vibration loop, vibration reduction performance is comparable if both devices are properly tuned. However, the moving damper outperforms the fixed damper when the latter falls close to a node, Moreover, the numerical simulations showed that decreasing the in-span mass relative to the suspended mass maximizes vibration reduction. The current study sets a platform to design a prototype robot to gather data and understand the interaction between the power line and the MDR. For future work, the focus will be given to the control framework that helps the MDR to detect and actively track the antinode. This control framework will be compared to the simple PD controller used in this study. Moreover, more parametric studies could be done to tackle questions that involve the robot power management and overall efficiency.

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Conflict of Interest

The authors declare that they have no conflict of interest.

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