

Magnetohydrodynamics in liquid metal interfacial flows

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ABSTRACT

Liquid metal interfacial flows occur in the fields of nuclear fusion and electromagnetic metallurgy. Due to the electrically conductive characteristics of the liquid metal, the presence of magnetic fields in these application scenarios has significant impacts on the interfacial flow behaviors. Then typical interfacial flows under the influence of magnetic fields, such as the free surface liquid metal flow, the liquid metal droplet impacting problems, and the bubble motion in liquid metal, are discussed in the present review. We comprehensively illustrate the flow characteristics of free surface liquid metal flow, the spreading of liquid metal droplets impacting onto solid or liquid surfaces, outcomes of collisions between metal droplets, and bubble dynamics in liquid metal, under the influence of magnetic fields along different

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34 *directions. Meanwhile, we briefly review the current concepts of liquid metal free surface flow Plasma*
35 *Facing Components (PFCs) in fusion reactors and finally make a summary for the open questions related to*
36 *the fundamental research and industrial applications of interfacial flow magnetohydrodynamics in the*
37 *future.*

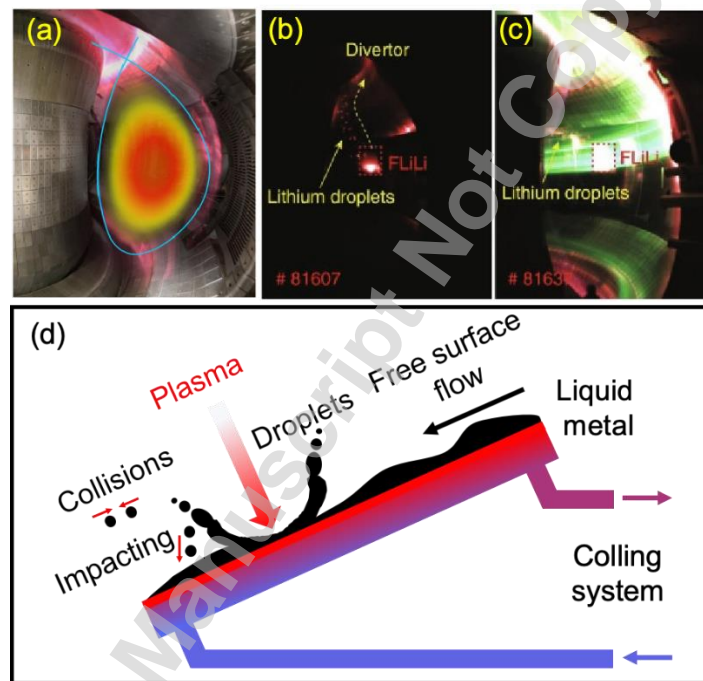
38 **1 INTRODUCTION**

39

40 Interfacial flow refers to the motion of at least two kinds of fluid with differing
41 physical properties, such as the boundary between immiscible fluids (e.g., oil and
42 water), the surface between a liquid and a gas, or the interface between a liquid and a
43 solid substrate. These flows exist diversely in nature and industry, including free surface
44 film flows, droplet impact dynamics, bubble motion in liquids, and particle motion in
45 fluid environments [1,2]. In this review, we only focus on the liquid metal interfacial
46 flow with gas serving as a secondary fluid phase, a scenario commonly encountered in
47 nuclear fusion reactors and electrometallurgy. Specifically, we consider the effects of
48 magnetic fields on these interactions, namely, the magnetohydrodynamic (MHD) effects.

49 In the Tokamak system, the plasma-facing components (PFCs) are important
50 parts to contain and control the plasma, manage extreme thermal loads, protect against
51 erosion, handle impurities, and assist in neutron shielding. Despite extensive testing of
52 various solid materials for PFCs, an ideal solution has yet to be discovered. Given the
53 benefits of flow liquid metal in engineering applications, scientists introduced the
54 innovative concept of covering the solid PFCs with a layer of free surface liquid metal
55 flow, known as liquid metal PFCs, in fusion reactors to address these challenges [3,4].
56 Nevertheless, there are still lots of issues that need to be solved due to the existence of

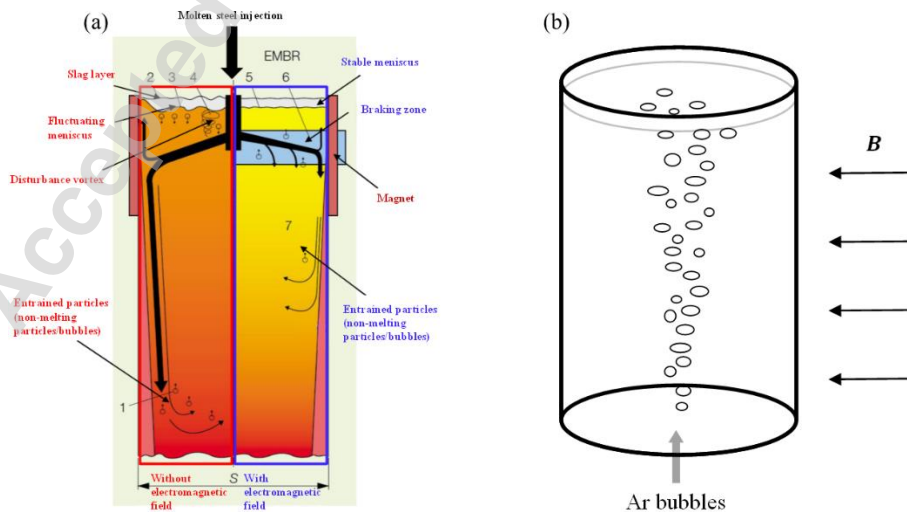
57 the strong magnetic field in the Tokamak system. Achieving a continuously flowing
58 liquid lithium film characterized by stable surfaces and uniform thickness presents a
59 significant challenge, primarily due to the influence of the MHD effect. Specifically,
60 when the liquid metal film flow intersects a magnetic field (B_0) the resultant Lorentz
61 force alters the force balance, leading to a flow behavior that is markedly different from
62 that observed without a magnetic field.



63
64 Fig.1. (a) Photo of plasma in the EAST device. Droplets are generated when the
65 plasma is without (b) and with (c) NBI heating [5]. (CC BY 4.0) (d) Schematic diagram of
66 the possible liquid metal flow when the PFCs work in the Tokamak device, i.e., free
67 surface flow, droplets impact.

68 Hu et al. [6] carried out plenty of experiments on a continuously flowing liquid
69 lithium (FLiLi) limiter in the Experimental Advanced Superconducting Tokamak (EAST).
70 Their results indicated that achieving complete coverage of solid PFCs with lithium-free

71 surface flow was challenging, and the interaction with plasma led to the formation of
72 numerous droplets on the lithium surface (Fig.1(b) and (c)). The formation of these
73 droplets was closely associated with the magnetic field, the stability of the film flow, and
74 the plasma characteristics. On the one hand, the instability of the film flow surface
75 facilitates the generation of droplets; on the other hand, these droplets, upon impacting
76 the liquid film surface, introduce further perturbations to the flow, complicating the
77 stability of the film. At the same time, the sputtered droplet motion will generate some
78 possible phenomena, such as mutual collisions and impacts on unwetted solid surfaces
79 of PFCs. Figure 1(d) shows the typical interfacial flows that may happen during the liquid
80 metal PFCs working in the Tokamak environment, i.e., the liquid metal free surface flow,
81 and the droplets impacting. To ensure a stable free surface liquid metal flow in the
82 design of PFCs, a comprehensive understanding of liquid metal interfacial flow
83 phenomena is essential. Up to now, numerous researchers have carried out relevant
84 work and contributed to the discovery and understanding of these phenomena, which is
85 the first main objective of this review.



86

87 Fig. 2. Schematic view for bubble plume applied in metallurgical engineering and
88 fusion reactors, where magnetic fields are at present. (a) Bubble plume in metallurgic
89 processing. (b) Configuration of the bubble column used in experiments and numerical
90 simulations [7]. Copyright by the Cambridge University Press (2007). Reproduced with
91 permission.

92 Bubble-driven flows are also common phenomena in metallurgical processes and
93 nuclear fusion devices, where bubbles are introduced into molten metals to enhance
94 mixing and heat transfer [7-15]. In metallurgical engineering, magnetic fields are
95 employed to control bubble motion in a contactless manner, thereby improving the
96 uniformity of liquid metal properties and the overall quality of final products. Figure 2(a)
97 gives a schematic view of the bubbly flows in metallurgical processing, where the gas
98 bubbles are injected to enhance mixing in the melt and to support chemical reactions. In
99 fusion reactors, bubbles are particularly useful in mitigating turbulence suppression
100 caused by MHD effects, thereby improving energy transfer efficiency [3]. While the
101 integration of bubbles and magnetic fields has significantly advanced these applications
102 (see Figure 2(b) for a sketch of the experimental setup [7]), the precise effects of MHD
103 on bubble interactions and distributions remain inadequately understood, necessitating
104 further in-depth research. The rising behavior of single bubbles in transparent fluids,
105 such as in water, has been extensively studied, leading to well-characterized motion
106 patterns like zigzagging, spiral, and even chaos trajectories. However, liquid metals pose
107 additional complexities due to their opacity, high density, and electrical conductivity,
108 which make direct observation and experimentation challenging [16, 9]. In the absence

109 of a magnetic field, high Reynolds number (Re , defined as the ratio of the inertial force
110 of the fluid to the viscous force) conditions in liquid metal lead to path instabilities of a
111 rising bubble, with the trajectory exhibiting a zigzag or spiral path due to wake
112 instabilities and vortex shedding [17-20]. At lower Re , bubbles tend to follow more
113 stable, straight paths. Besides, the wake structures are observed to be more complex in
114 liquid metal than in water, owing to the much higher rising Re and the much stronger
115 shape deformation in liquid metal. In the presence of magnetic fields, the magnetic
116 fields introduce Lorentz forces that can modulate the surrounding liquid flow, thereby
117 affecting the bubble's trajectory, velocity, shape evolution, and void distribution. Recent
118 studies have provided a certain level of understanding of these phenomena, which is
119 another main topic of this review.

120 **2 GOVERNING EQUATIONS AND NUMERICAL SCHEMES**

121 2.1 Flow computations

122 The incompressible free surface flows exposed to external magnetic fields (MFs)
123 are governed by some physical principles, which are, respectively, the conservation of
124 mass equation, the momentum equation, and the Maxwell equation. The governing
125 mass and momentum equations are given by

$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \nabla \cdot [\mu(\nabla \mathbf{u} + \nabla \mathbf{u}^T)] + \mathbf{F}_s + \mathbf{F}_l + \mathbf{S}, \quad (1)$$

$$\nabla \cdot \mathbf{u} = 0, \quad (2)$$

126 where \mathbf{u} is the fluid's velocity, ρ and μ denote the density and dynamic viscosity
127 of the immiscible fluids. The two sources in terms of \mathbf{F}_l and \mathbf{F}_s are the Lorentz force and
128 the surface tension force, respectively. The former is calculated as $\mathbf{F}_l = \mathbf{J} \times \mathbf{B}$, where \mathbf{J}

129 denotes the induced current density and \mathbf{B} is the MF. Note that for the free surface
130 flows concerned in the present review, the magnetic Reynolds number $Re_m =$
131 $\mu_m \sigma_e L U \ll 1$ in liquid metal is very low ($\sim 10^{-3}$), with μ_m denoting the magnetic
132 permeability, σ_e the electrical conductivity, and U the characteristic fluid velocity.
133 Hence, the induced MFs by the fluid motion are much smaller than the applied MFs, and
134 therefore a quasi-static approximation can be used [21], leading to $B = B_0$, where B_0 is
135 the external magnetic field. In this regard, the electric potential method can then be
136 used to compute the current densities, expressed by Ohm's law as below:

$$J = \sigma_e (-\nabla \varphi + \mathbf{u} \times \mathbf{B}), \quad (3)$$

137 where φ is the induced electric potential. Then due to the conservative property
138 of the charge, the divergence-free condition of $\nabla \cdot \mathbf{J} = 0$ should be maintained during
139 computations that an additional electric potential Poisson equation requires to be
140 solved, given as

$$\nabla \cdot (\sigma_e \nabla \varphi) = \nabla \cdot (\sigma_e \mathbf{u} \times \mathbf{B}). \quad (4)$$

141 Consequently, the combination of Eqs. (3) and (4) leads to the solution of the
142 current densities \mathbf{J} , and then the Lorentz forces in Eq. (1) are obtained. On the other
143 hand, the surface tension force, in the absence of a tangential Marangoni stress, has the
144 expression of $\mathbf{F}_s = \sigma \kappa \delta_s \mathbf{n}$, where σ and κ are the surface tension coefficient and the
145 local curvature of the interface, respectively, while \mathbf{n} is the normal direction of the
146 interface and δ_s the Dirac distribution function. At last, \mathbf{S} appeared at the right-hand
147 side of Eq. (1) is the remaining source term, which always stands for the gravity $\rho \mathbf{g}$ in
148 interfacial flows.

149 To solve the equations governing the fluid flow, i.e. Eq. (1), the essential step is
 150 to ensure the divergence-free condition of the velocity in terms of Eq. (2). Upon on this,
 151 some numerical methods have been proposed to decouple the solution of the pressure
 152 field from the momentum equation, and the most popular approach is the fractional-
 153 step method [22,23]. Then this type of methodology has been widely applied to solve
 154 the incompressible magnetohydrodynamics [19,20,24-29]. We take the numerical
 155 method proposed by Zhang et al. [19] for instance, the discretized forms of Eq. (1) are
 156 given as

$$\rho_c^{n+\frac{1}{2}} \left(\frac{\mathbf{u}^* - \mathbf{u}^n}{\Delta t} + \mathbf{u}^{n+\frac{1}{2}} \cdot \nabla \mathbf{u}^{n+\frac{1}{2}} \right)_c \quad (5)$$

$$= \nabla_c \cdot \left[\mu_f^{n+\frac{1}{2}} (\nabla \mathbf{u} + \nabla \mathbf{u}^T)^* \right] + (\mathbf{J} \times \mathbf{B})_c^{n+\frac{1}{2}} \\ + \left[(\sigma \kappa \delta_s \mathbf{n})^{n-\frac{1}{2}} - \nabla p^n \right]_{f \rightarrow c},$$

$$\mathbf{u}_c^{**} = \mathbf{u}_c^* - \frac{\Delta t}{\rho_c^{n+\frac{1}{2}}} \left[(\sigma \kappa \delta_s \mathbf{n})^{n-\frac{1}{2}} - \nabla p^n \right]_{f \rightarrow c}, \quad (6)$$

$$\mathbf{u}_f^{n+1} = \mathbf{u}_{c \rightarrow f}^{**} + \frac{\Delta t}{\rho_f^{n+\frac{1}{2}}} \left[(\sigma \kappa \delta_s \mathbf{n})^{n+\frac{1}{2}} - \nabla p^{n+1} \right]_f \quad (7)$$

157 where the velocity and the pressure fields are solved at time step $(n + 1)$. We see that
 158 in this discretization process, based on the principle of the balanced-force model [30],
 159 the discretization of the surface tension force \mathbf{F}_s should always stay with the pressure
 160 gradient ∇p , to minimize the spurious flow in the vicinity of the interface. Besides, the
 161 subscripts c and f indicate where the variables are stored, corresponding to the
 162 "center" and the "face" of the discretized grids, respectively, while the symbol $c \rightarrow$

163 $f(f \rightarrow c)$ means a second-order interpolation scheme from cell center (face) to cell face
 164 (center) by applying a simply arithmetic average. In the methodology proposed by
 165 Zhang et al. [19], all the variables, i.e. the velocity, the pressure, and other scalar fields,
 166 are located at the center of the grids, while the pressure gradient (∇p) and the surface
 167 tension force ($\sigma\kappa\delta_s\mathbf{n}$) are calculated on the grid faces. Then a projection method is
 168 applied on Eq. (8) to enhance the divergence-free condition of the flow, that is:

$$\nabla_c \cdot \left[\frac{\Delta t}{\rho^{n+\frac{1}{2}}} \left((\sigma\kappa\delta_s\mathbf{n})^{n+\frac{1}{2}} - \nabla p^{n+1} \right) \right]_f = \nabla_c \cdot \mathbf{u}_f^{n+1} - \nabla_c \cdot \mathbf{u}_{c \rightarrow f}^{**} \quad (8)$$

169 where $\nabla_c \cdot \mathbf{u}_f^{n+1}$ is zero. Because $(\sigma\kappa\delta_s\mathbf{n})^{n+\frac{1}{2}}$ and $\mathbf{u}_{c \rightarrow f}^{**}$ are already obtained in solving
 170 Eq. (7) and (8), therefore (p^{n+1}) could be computed through applying an iterative
 171 approach on Eq. (8). After that, the grid face velocity of \mathbf{u}_f^{n+1} is updated by substituting
 172 (p^{n+1}) into Eq. (8), and the grid center velocity of \mathbf{u}_c^{n+1} is interpolated through

$$\mathbf{u}_c^{n+1} = \mathbf{u}_c^{**} + \frac{\Delta t}{\rho_c^{n+\frac{1}{2}}} \left[(\sigma\kappa\delta_s\mathbf{n})^{n+\frac{1}{2}} - \nabla p^{n+1} \right]_{f \rightarrow c} \quad (9)$$

173 Additionally, in the presence of external magnetic fields, a consistent and
 174 conservative scheme [24,26] is now usually employed to compute the electric currents
 175 and Lorentz forces, in terms of Eqs. (3) and (4). Because of this, the progression of the
 176 numerical solution on the electromagnetic part from time level n to level $n + 1$ is given
 177 below:

$$\nabla_c \cdot (\sigma_e \nabla \varphi)_f^{n+\frac{1}{2}} = \nabla_c \cdot (\sigma_e (\mathbf{u} \times \mathbf{B}))_f^{n+\frac{1}{2}} \quad (10)$$

$$\mathbf{J}_f^{n+\frac{1}{2}} = (\sigma_e(-\nabla\varphi + \mathbf{u} \times \mathbf{B}))_f^{n+\frac{1}{2}} \quad (11)$$

$$\mathbf{F}_l^{n+\frac{1}{2}} = (\mathbf{J} \times \mathbf{B})_{f \rightarrow c}^{n+\frac{1}{2}} \quad (12)$$

178 Then, the remaining technique that requires clarification is the discretization of
179 the surface tension term $\mathbf{F}_s = \sigma\kappa\delta_s\mathbf{n}$. As will be demonstrated later, accurately
180 estimating \mathbf{F}_s proves to be a non-trivial task in numerical simulations.

181

182 2.2 Interface tracking and surface tension computation

183 The most intuitive way to update the position of the free surface is to describe
184 the shape of the interface by distributing a finite number of material particles on the
185 interface. The positions of these particles (and thus the shape of the interface) could
186 then be updated using

$$\frac{d\mathbf{x}_I^k}{dt} = \mathbf{u}, \quad (13)$$

187 where \mathbf{x}_I^k denotes the position of the interface in the k -th grid, and this equation
188 complies with the Lagrangian system. This class of methods is often called front-tracking
189 or marker methods [31], while they can be implemented and solved with high accuracy.
190 However, to ensure an appropriate description of the interface, marker particles need
191 to be redistributed periodically along the interface, which complicates practical
192 implementations. Upon on this methodology, Schwarz and Fröhlich [32] developed an
193 immersed boundary method to model the single bubble rising under the influence of a
194 vertical magnetic field, while the surface of the bubble is described using a set of

195 Lagrangian marker points interconnected by a triangular mesh. Then the motion of a
196 single bubble is obtained by solving its linear and angular momentum equation.

197 Alternatively, the other way to track the interface is to define the interface
198 position implicitly, i.e., through an arbitrary function such that $\mathcal{F}(\mathbf{x})_I = 0$, and rather
199 than discretizing the interface itself, one can then choose to discretize the implicit
200 function \mathcal{F} . With this regard, \mathcal{F} must also evolve in time to reflect the evolution of the
201 interface, yielding that

$$\frac{\partial \mathcal{F}}{\partial t} + \mathbf{u} \cdot \nabla \mathcal{F} = 0, \quad (14)$$

202 and this means \mathcal{F} is simply advected by the flow as an ordinary tracer. This approach
203 always falls into an Euler framework, and the most popular methods include the level-
204 set (LS) [33,34] and the volume of fluid (VOF) [35,36], the former defines \mathcal{F} as the signed
205 distance to the interface while the later defines \mathcal{F} as the volume fraction the continuous
206 or disperse phase. LS does degrade the accuracy with which the position of the interface
207 \mathbf{x}_I is estimated because it is thus necessary to periodically re-initialize \mathcal{F} , and this in turn
208 can cause large errors in mass conservation. The LS method has been implemented in
209 the in-house code of HIMAG [37], which was developed at UCLA for the modeling of the
210 free surface MHD flows. In contrast, the VOF method can conserve the mass of the fluid
211 better, and the implementation of the numerical algorithm, for the discretization of Eq.
212 (14), is also simple. In particular, a split-direction advection scheme, i.e. using the one-
213 dimensional scheme for alternative propagation, is always applied to update the volume
214 fraction \mathcal{F} to next time step $n + \frac{1}{2}$. During this split-direction process, the interface can
215 be reconstructed either using a geometrical scheme [36] or an algebraic scheme [38], to

216 maintain the sharpness of the immiscible interface. The application of the VOF method
217 can be also found in the MHD research community [20,39,40]. After obtaining $\mathcal{F}^{n+\frac{1}{2}}$, a
218 physical property ϕ (i. e., ρ , μ and σ_e) at a given position of the fluid domain is evaluated
219 by linear interpolation as $\phi^{n+\frac{1}{2}} = \mathcal{F}^{n+\frac{1}{2}}\phi_1 + (1 - \mathcal{F})^{n+\frac{1}{2}}\phi_2$, where 1 and 2 are the
220 properties of the two phases.

221 Now we turn to the discretization of the surface tension force $\mathbf{F}_s = \sigma\kappa\delta_s\mathbf{n}$,
222 which is closely related to the interface position defined by \mathcal{F} . There are two different
223 methodologies to compute this term. On the one hand, volumetric formulations are
224 based on a numerical approximation of the surface Dirac function, which allows a direct
225 evaluation of the volumetric force, while most methods belong to this category
226 independently of the way the interface is represented. For instance, for the widely used
227 Continuum Surface Force (CSF) method developed by Brackbill et al. [41], the interface
228 is represented through the volume fraction field \mathcal{F} and one simply sets $\mathbf{F}_s = \sigma\kappa\delta_s\mathbf{n} =$
229 $\sigma\kappa\nabla\mathcal{F}$, which makes the characteristic interface thickness to be non-zero but spread
230 over several grids. In contrast, the other kind of method maintains the sharpness of the
231 interface when computing the surface tension, and it is usually associated with LS
232 interface representations known as the Ghost Fluid Method [42,43]. Such a method
233 treats the surface tension as the corresponding jump condition in the discretization of
234 the pressure gradient operator, i.e., $p_{i-\frac{1}{2}}^- = p_i \pm (\sigma\kappa)_{i-\frac{1}{2}}$ and thus leads to a sharp
235 interface representation, in contrast with the other methods. Both of these two types of
236 numerical methods have been applied in modeling the surface tension force of the

237 interfacial MHD flows, respectively, by Zhang et al. [39,40] and Jin et al. [20], and they
238 both show good agreements compared to the experimental studies.

239 **3 FREE SURFACE LIQUID METAL FLOW**

240

241 Film flow and open channel flow are two typical free surface flows. Open
242 channel flow is common in rivers and canals where the liquid is deep, while film flow is
243 common in sheets of rainwater on glass. This review focuses mainly on the liquid metal
244 free surface flow used in liquid metal diverters in nuclear fusion reactors, where the
245 magnetic field is present. A summary of the effects of magnetic field magnitude,
246 direction, flow rate, and wall conductivity on the characteristics of film flow is provided.

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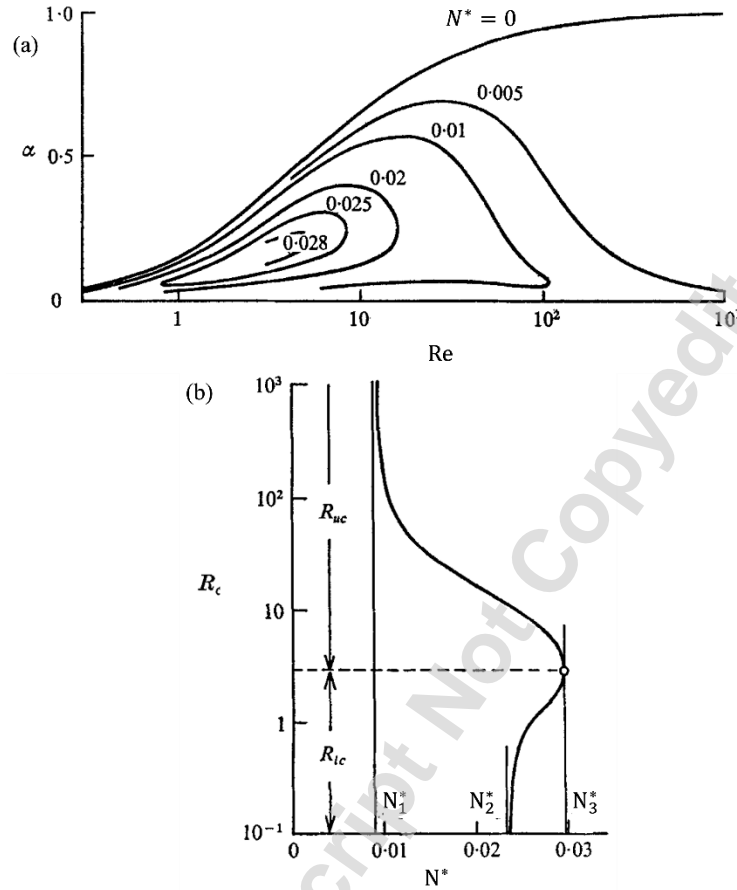
248 **3.1 Stability of liquid metal free surface flow**

249 Stability is a crucial aspect in the study of free surface flow. It can help us
250 understand and predict flow behavior, essential for optimizing various industrial
251 applications. Kapitza et al. [44] are pioneers in investigating the wave dynamics on thin
252 liquid layers. By adopting the long-wave expansion, Benjamin [45] studied the origin of
253 film flow instability. The long-wave hydrodynamic instability mainly stems from the flow
254 direction component of gravity, inertia, and hydrostatic pressure [46].

255 The introduction of MFs gives rise to the generation of an additional
256 electromagnetic force acting within the free surface flow, which in turn exerts an
257 influence on the stability of the free surface flow. Hesieh [47] firstly investigated the
258 stability of liquid films influenced by magnetic fields using the long-wave approximation.

259 Results indicated that magnetic fields have a stabilizing effect on long-wave

260 disturbances. Similarly, Ladikov [48] and Shen et al. [49] arrived at the same conclusion.
261 Rai [50] and Mukhopadhyay et al. [51] adopted the asymptotic approximation of the
262 linear stability equation based on the long wave assumption and the weakly nonlinear
263 analysis to study the stability of free surface flow within the influence of magnetic field,
264 respectively. Subsequently, the Galerkin method [52] or the Chebyshev collocation
265 method [53] was employed to solve the Orr-Sommerfeld (OS) eigenvalue problem.
266 Generally, a neutral stability curve can be obtained in the (K, Re) plane, where K is the
267 wave number in the flow direction. The region inside the curve indicates instability,
268 while the area outside the curve signifies stability, thereby allowing for the
269 determination of parameters such as the critical Reynolds number for instability. Gotoh
270 [54] adopted the neutral stability curve (Fig. 3) to determine the parameter range of Re
271 and N^* , corresponding to the stability and instability of free shear flow under the
272 influence of a vertical magnetic field, here N^* is defined as Ha^2/Re^2 .



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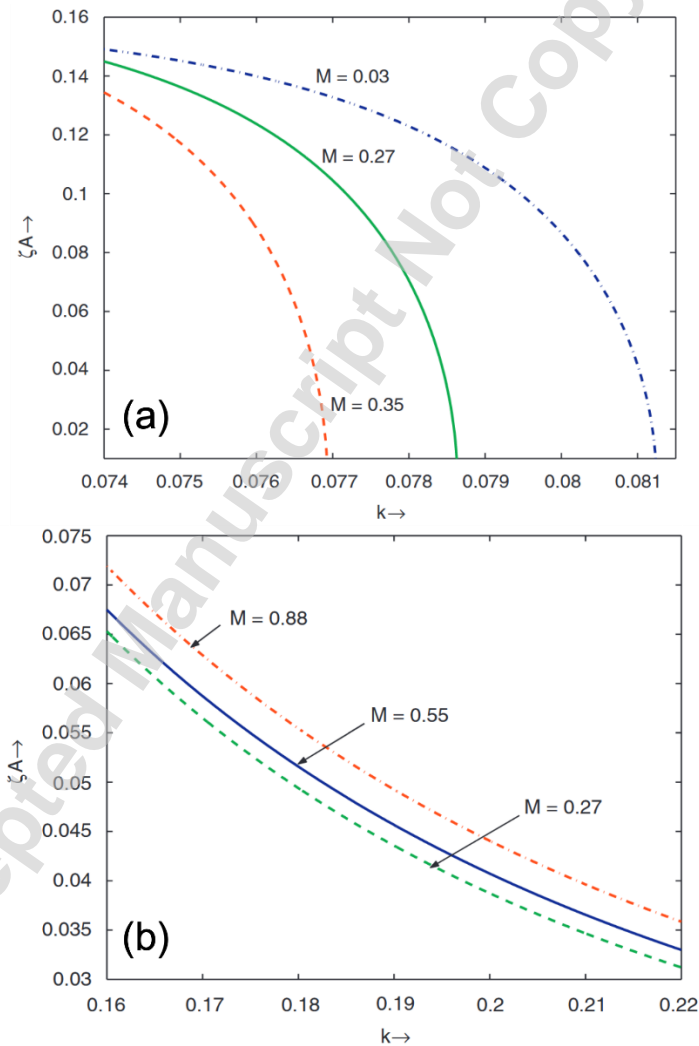
Fig. 3. Results from Gotoh [54]. (a) Neutral stability curves corresponding to different magnetic field parameters N^* under two-dimensional disturbance, (b) Relationship between critical Reynolds number Re and magnetic field parameter N^* under two-dimensional disturbance. Copyright by the Cambridge University Press (1971). Reproduced with permission.

Moreover, the direction of the MFs exerts a significant influence on the stability of the free surface. When the generated Lorentz force points towards the interior of the fluid, it will result in surface wave instability, as demonstrated by Murty [55]. When the Lorentz force is in an upward direction and exceeds the gravitational force, Rayleigh-Taylor instability will occur on the free surface, as discussed by Shercliff [56].

284 Considering the cases that the magnetic Prandtl number P_m , $P_m = \nu/\eta$, is small, which is
285 the ratio of kinematic viscosity ν to magnetic diffusivity η , the magnetic field has a
286 stabilizing effect on the flow and is less dependent on P_m [57,47]. Different magnetic
287 field components have different stabilizing effects on liquid film flow [49].

288 In the presence of a transverse non-uniform magnetic field, the amplitude of
289 surface disturbances along the flow direction is significantly enhanced, while the
290 propagation speed of these disturbances is reduced [58-60]. Moreover, under the
291 vertical magnetic field, the hard mode of free surface flow behaves similarly to the
292 unstable mode of channel Hartmann flow at low-magnetic-Prandtl-number regime
293 ($P_m \ll 10^{-4}$), and it is less affected by P_m . However, the soft mode is more sensitive to
294 changes in P_m [52]. In the inductionless limit $P_m > 0$, the critical Re of the soft mode
295 increases exponentially with Ha , while when P_m is small but non-zero, the Lorentz force
296 causes the critical Re to sub-linearly increase with Ha for insulating walls or decrease for
297 perfectly conductive walls. Weak nonlinear analysis shows that under the influence of a
298 vertical magnetic field, the flowing liquid film has both supercritical instability and
299 subcritical instability. Meanwhile, with the increase of Hartmann number, subcritical
300 and explosive unstable zones are enlarged and shrank respectively [51]. In the
301 supercritical stable region and subcritical unstable region, the threshold amplitude
302 decreases and increases respectively with the increase of Hartmann number M (Fig. 4),
303 here M is the same as Ha . In addition, the effect of the electric field on the flow stability
304 depends on its direction [51,61]. In the subcritical region, the wave velocity and
305 amplitude threshold of the nonlinear wave increase with the increase of Ha , and in the

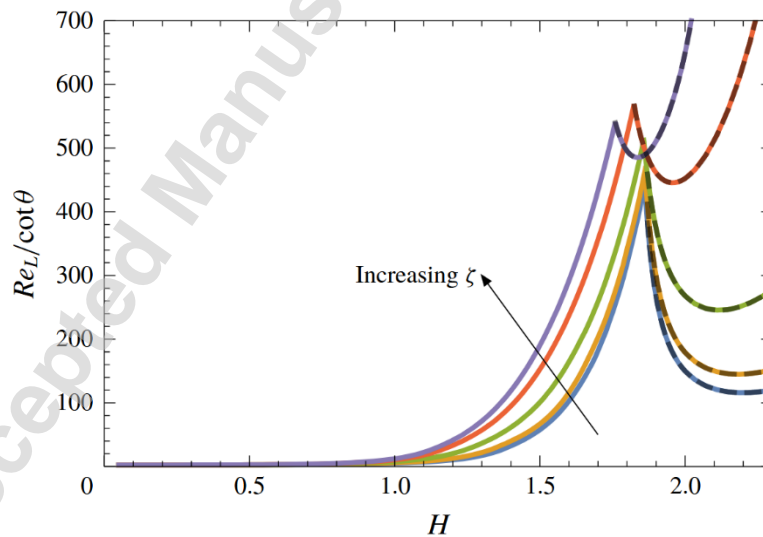
306 supercritical region, the wave velocity and amplitude threshold of the nonlinear wave
307 decrease with the increase of Ha . Cheng et al. [62] analyzed the rupture of micropolar
308 films under the effect of MFs using the long wave approximation. The findings indicated
309 that as the initial disturbance diminished, the film breakage time extended.
310 Furthermore, the micropolarity and magnetic fields exhibited a significant impact on the
311 film flow on the horizontal plate.



312
313 Fig. 4. (a) Threshold amplitude in the supercritical region for different values of
314 Hartmann number, (b) Threshold amplitude in the subcritical region for different values

315 of Hartmann number [51]. (Noting that M is the same as Ha). Copyright by the Elsevier
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317 Lunz et al. [63] analyzed the stability of the flowing liquid film on the conical
318 surface under the influence of the toroidal magnetic field. The toroidal magnetic field
319 serves to stabilize the flow, although the Hartmann layer is absent for the conical
320 surface. When the surface tension can be ignored, the flow stability is related to
321 parameters such as magnetic field intensity, local Re , liquid film thickness, and
322 inclination angle. The local critical Reynolds number increases sharply with the increase
323 of the magnetic field. According to the results from stability analysis, the critical
324 Reynolds number can be expressed as a function of other parameters (Fig. 5), so one
325 can adjust the relevant parameters to obtain the largest possible critical Reynolds
326 number to make the flow more stable.



327
328 Fig. 5. Numerically computed critical local Reynolds number $Re_L / \cot \theta$ as a
329 function of H , for ζ [63]. Copyright by the Cambridge University Press (2019).
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331 In addition to velocity disturbances and electromagnetic disturbances, factors
332 such as thermal gradients generated by local heating or cooling drive thermocapillary
333 flows [64]. The shape of the substrate [65], and gas flow or plasma [66] also affect
334 interface stabilities. Feedback control systems [67] or open-loop control systems [68]
335 can be designed to stabilize the instability of the free interface by selecting the system's
336 material, customizing the system geometry, controlling the forcing amplitude, or using
337 the thermodynamic principles. For a uniformly heated descending film, there are two
338 thermocapillary instability modes proposed by Goussis and Kelly [69]: (1) short-wave
339 thermocapillary instability caused by the interaction between the basic temperature
340 field and the perturbed velocity field, also known as the P-mode; (2) long-wave
341 thermocapillary instability due to surface temperature changes caused by free surface
342 deformation, also known as S-mode. Usually, the thermocapillary instability will cause
343 the well-known Marangoni effect. That is, when there is a wave on the surface of the
344 liquid film, the temperature of the free surface close to the heated wall is high, and the
345 temperature of the liquid film far from the wall is low. For most liquids, their surface
346 tension decreases with increasing temperature, and the fluid will move from areas with
347 low surface tension to areas with high surface tension, further aggravating the surface
348 deformation of the liquid film. Nield [70] first discovered that the critical Marangoni
349 number for the onset of Marangoni convection increases with increasing magnetic field
350 strength. However, for a deformable free surface, the magnetic field cannot stabilize all
351 disturbances [71]. Subsequently, Hasilmn et al. [72] calculated the critical parameter
352 space for the occurrence of oscillatory Marangoni convection.

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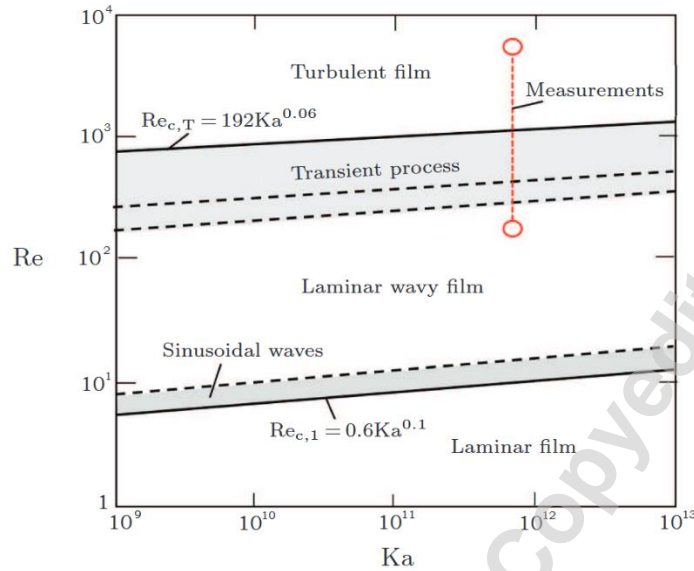
354 3.2 Morphology of surface waves

355 Surface waves can provide a clear indication of flow stability, as their behavior
356 directly reflects the underlying dynamics of the fluid. Therefore, many researchers have
357 engaged in studies related to this topic.

358 In the middle of 20 century, Alpher et al. [73] provided a detailed review of the
359 surface waves in shallow mercury flows. Without a magnetic field, surface waves in
360 mercury follow classical gravity wave dynamics, with propagation speed determined by
361 $gh_0^{1/2}$. Waves experience minimal attenuation, allowing disturbances to propagate
362 further. The prominent capillary waves form sharp "fish-line" patterns due to surface
363 tension. Hydraulic jumps in mercury flows behave similarly to those observed in classical
364 shallow-water flows. Murav'ev [74] revisited the "shallow water" approximation for
365 analyzing wave regimes on the free surface of liquid metal films. He found that the wave
366 behavior is primarily influenced by gravitational forces, slope angles, and the properties
367 of the liquid metal (e.g., viscosity and density). The corresponding equations derived for
368 wave dynamics align with classical hydrodynamic models, such as the Froude number
369 (Fr , $Fr = U/\sqrt{gh}$), which determines whether the flow regime is "fast" or "quiet".
370 Morley et al. [75-78] systematically investigated the liquid metal film flow from
371 theoretical, numerical simulation, and experiments. Their results also proved that the
372 liquid metal film flows exhibit primarily hydrodynamic behavior. The stability and
373 behavior of these waves depend on factors like Reynolds (Re , $Re = Ul/\nu$) number,
374 Weber (We , $We = \rho U^2 l / \sigma$) number, and Fr . When the flow is unstable, surface waves,

375 a common feature in such flows, can arise due to instabilities caused by inclinations,
376 high flow rates, or external disturbances caused by rough gates or nozzle misalignments.
377 Ying et al. [79] observed the irregular, three-dimensional (3D) turbulent wave structures
378 that result from hydrodynamic instabilities.

379 From the hydrodynamic aspect, the flow state of film flow can be divided into
380 three regimes: smooth laminar flow, wavy laminar flow, and turbulent flow, respectively
381 [80]. These three regimes can be divided by Ka and Re in figure 6. The Kapitza number
382 Ka is expressed as the ratio of surface tension to inertial force, which is equal to
383 $\sigma/(\rho(g \sin \beta)^{1/3} \nu^{4.3})$. For liquid flowing on steep or vertical walls, smooth laminar flow
384 only occurs at very low flow rates. Free surface waves can be subdivided into gravity
385 type, capillary type, and gravity-capillary type, with gravity or surface tension as the
386 main restoring forces, respectively. Gravity waves appear around $Fr = 1 \sim 2$, while
387 capillary waves appear around $We = 1$. The corresponding wave frequency of gravity
388 waves, capillary waves, and gravity-capillary waves can be expressed as $\omega^2 =$
389 $gk \tanh(kh)$, $\omega^2 = \gamma k^3$, and $\omega^2 = (gk + \gamma k^3) \tanh(kh)$ respectively, where ω is the
390 frequency of the wave, k is the wave number, g is gravity acceleration, h is the
391 thickness of liquid, and $\gamma = \sigma/\rho$ is the ratio of surface tension σ coefficient to density of
392 liquid. The crossover between the gravity waves and capillary waves occurs for the
393 wavelength $\lambda_{gc} = 2\pi\sqrt{\sigma/(\rho g)}$ [81]. It is important to note that the presence of waves
394 does not necessarily mean that the flow is turbulent [80].



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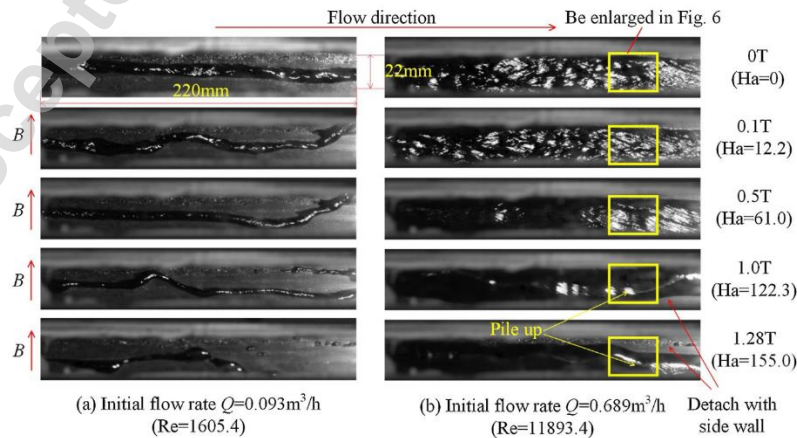
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Fig. 6. Typical flow regimes in $Ka - Re$ space, the red dashed line presents the results measured by Vieweg et al. [82] under the influence of the magnetic field. (CC BY 4.0)

However, interactions between surface waves, such as inelastic collisions, can make the waves more complex [83]. For example, in a random wave field, non-resonant interactions can generate bound waves that deviate from the linear dispersion relation [84]. In addition, intermittency occurs when the wave height increments follow a Gaussian distribution on large scales but deviate significantly from the Gaussian distribution on small scales [85]. The non-equilibrium statistical mechanics of such random nonlinear waves can be studied by using wave turbulence theory [86]. Similar to classical turbulence theory, there is energy transfer caused by wave interaction in wave turbulence, and the state of the wave system can be determined by conservation quantities such as energy flux. Through dimensional analysis, the Kolmogorov-Zakharov Spectrum of different wave systems can be obtained to reveal the distribution and

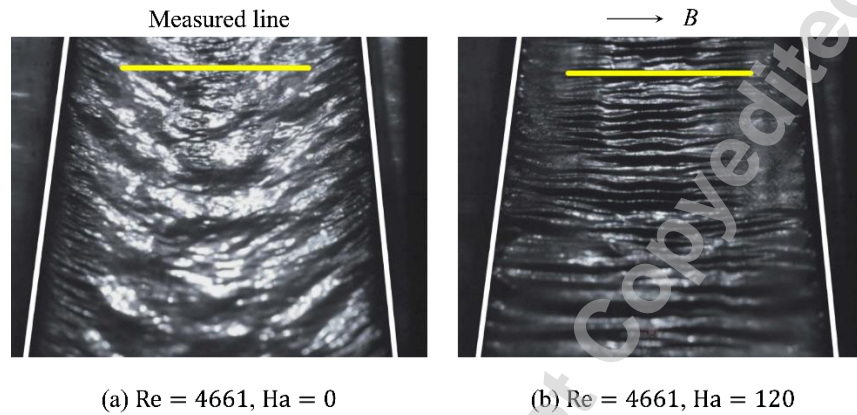
410 transfer mechanisms of energy [87]. For a general capillary wave system,
 411 $S_{\omega}^c \sim P^{1/2}(\sigma/\rho)^{1/6}\omega^{-17/6}$, while for a general capillary wave system, $S_{\omega}^g \sim P^{1/3}g\omega^{-4}$.
 412 Here S_{ω}^c and S_{ω}^g are the power spectral density of capillary wave elevation and gravity
 413 wave elevation, respectively, and P is the energy flux. Currently, there is no analytical
 414 solution for the Kolmogorov-Zakharov spectrum of gravity capillary wave turbulence, as
 415 the dispersion relation is not a pure power law [81]. Instead, it comprises two distinct
 416 power laws within the power spectral density of wave height, representing gravity
 417 waves and capillary waves, respectively.

418 Considering the liquid metal free surface flow with side walls, the surface waves
 419 containing small-scale fluctuations and vorticity can easily cause the liquid metal to
 420 pinch towards the center of the channel, leaving bare zones near the sidewalls. Such
 421 behavior is particularly evident when the liquid metal does not wet the substrate well.
 422 As shown in Fig. 7, at low Re, due to the wetting problem, the GaInSn forms an unstable
 423 liquid column flow, exhibiting random surface striations [88]. As the Reynolds number
 424 increases, the flow becomes unstable and moves to the sides of the channel, and after
 425 reaching the critical Reynolds number, the flow begins to spread.

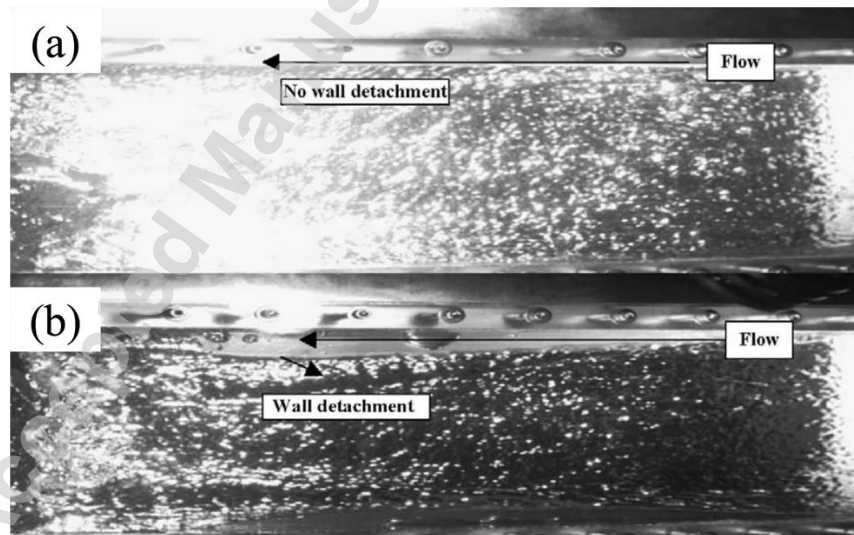


426

427 Fig. 7. Different flow patterns of liquid metal with the increase of Ha , (a) $Re =$
428 1605.4, (b) $Re = 11893.4$ [88]. Copyright by the Elsevier (2016). Reproduced with
429 permission.



430
431 Fig. 8. The surface topology observed by a high-speed camera when the
432 inclination angle of the test section is 5° without (a) and with (b) the effect of the
433 magnetic field [90].



434
435 Fig. 9. The surface waves of liquid metal film flow. (a) Flow state of the film flow
436 with a vertical magnetic field, (b) Flow state of the film flow without magnetic field [90].
437 Copyright by the Elsevier (2006). Reproduced with permission.

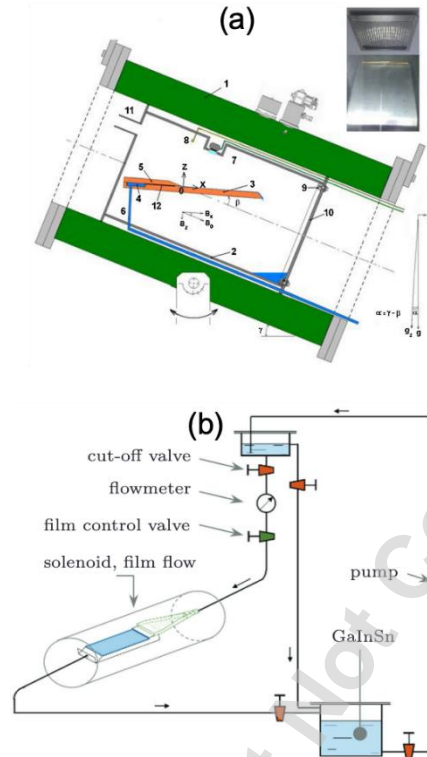
438 According to the governing equation in Sec. 2, conductive liquid metal flowing in
439 a magnetic field will produce electromagnetic induction, and the resulting MHD effect
440 makes the free surface flow of liquid metal different from ordinary free surface flow.
441 Moreover, the MHD effect is anisotropic, the velocity component in the same direction
442 as the magnetic field will not be suppressed, and the same is true for the vorticity. Ying
443 et al. [91] found that the magnetic field significantly alters the characteristics of surface
444 waves in the liquid metal flow. As shown in Fig.9, in high magnetic field strength regions,
445 a distinct departure from the ordinary hydrodynamic turbulence observed in low-field
446 regions [90]. This indicates a strong influence of the magnetic field on wave formation
447 and propagation, transitioning the wave behavior from typical hydrodynamic patterns
448 to a more structured, possibly stabilized, state at higher field strengths. The exact
449 nature of this transition and the underlying mechanisms require further investigation,
450 but the clear difference in wave structure highlights the substantial impact of the
451 magnetic field on the free surface dynamics. In the experiments from Yang et al. [88],
452 they also found that the MHD effect has an important influence on the change of the
453 flow patterns. When Re is small, the magnetic field induces a flow perturbation that
454 makes the fluid oscillate, and the flow becomes discontinuous with the increase of
455 Hartmann number Ha , which is equal to $Bl(\sigma_e/\rho\nu)^{1/2}$. Therefore, it is necessary to
456 change the wettability between the substrate and the liquid metal. Common
457 approaches include coating the substrate, printing surface microstructures on the
458 substrate [92], or using special structures such as metal mesh [65].

459 Regarding the changes in surface topologies, Yang et al. [89] observed the
460 transition of three-dimensional surface waves to quasi-two-dimensional (Q2D) surface
461 waves with wave propagated perpendicular to the direction of magnetic lines, as shown
462 in Fig. 8(b), indicating that the magnetic field suppresses waves with wave vectors
463 parallel to it. Rhoads [93] observed gravity capillary waves on the surface of a flowing
464 film and explained the effect of the magnetic field on the wave amplitude from an
465 energy perspective. In the absence of artificially created waves, the fluctuations at the
466 free surface may be caused by velocity disturbance. That is a localized rise in the fluid,
467 followed by a change in direction and fall due to continuity as it approaches the free
468 surface, resulting in localized vorticity. The restoring forces that prevent this change
469 include electromagnetic forces, surface tension, and gravity. By omitting the influence of
470 surface tension, Rhoads [93] suggested that the change in free surface amplitude should
471 overcome the sum of gravity and electromagnetic force, $(g + \alpha B^2) < h >$, where α is a
472 dimensionless parameter which is equal to $\sigma_e B^2 / \rho \omega$ and varies with the nature of the
473 surface disturbance [94]. Therefore, the average energy per unit mass supplied by the
474 velocity perturbation to the surface wave is equal to $< \tilde{u}^2 > / 2$. By making the two
475 energies equal, they obtained an equation (Eq. 15) to predict the surface amplitude.

$$< h > = \frac{< \tilde{u}^2 > / 2}{(g + \alpha B^2)} \quad (15)$$

476 From ref. [93], it can be seen that the surface amplitude predicted by Eq. 15
477 agrees well with the experimental data. Among them, the electromagnetic term is
478 caused by $|\mathbf{J} \times \mathbf{B}|$, and $\mathbf{J} = \sigma_e (\mathbf{E} + \mathbf{u} \times \mathbf{B})$. Therefore, when there is an angle between
479 the vorticity field or the velocity field and the magnetic field, the corresponding

480 electromagnetic term is non-zero, and the electromagnetic force suppresses the surface
481 disturbance. The wave component propagating along the flow direction corresponds to
482 a local vorticity in the same direction as the transverse magnetic field. Therefore, the
483 damping of the magnetic field on the stream direction wave (down-stream) is much
484 weaker than that of the wave vector propagating along the transverse direction (cross-
485 stream). Although the mechanism of magnetic field suppression is the same, compared
486 with the transverse magnetic field, a vertical magnetic field will suppress surface
487 fluctuations in all directions because the plane where the wave vector lies is always
488 perpendicular to the magnetic field. As the vertical magnetic field increases, the surface
489 wave components in the streamwise direction and the transverse direction decrease
490 with a consistent trend. In addition, the relationship between the wave amplitude and
491 the magnetic field is relatively independent of the liquid depth and flow rate, as
492 evidenced by [93]. Normally, the energy of gravity capillary waves on a free surface is
493 transferred from large scales to small scales according to a certain power law. However,
494 according to Rhoads' experimental results [93], the magnetic field not only suppresses
495 the amplitude of the waves, that is, the energy of the waves, but also changes the speed
496 of the transfer of the wave energy and even suppresses the wave energy to the noise
497 level.



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Fig. 10. Experimental systems to study the free surface flow under the influence of streamwise magnetic field. Schematic of experimental systems established by Platacis et al. [95] (a) (Copyright by the Elsevier (2014). Reproduced with permission.) and by Vieweg et al. [82] (b). (CC BY 4.0)

Concerning the influence of streamwise magnetic field on liquid metal surface waves, experimental results remain notably limited. Figure 10 shows two typical experimental systems. In the experimental system established by Platacis et al. [95] (Figure 10(a)), the magnetic field and the flow direction are not perfectly parallel but are instead slightly misaligned at an angle, causing effects in both the vertical and parallel flow directions. The induced current generated by the magnetic field in the vertical component interacts with the streamwise magnetic field component to change the pressure distribution [95]. Meanwhile, the Lorentz force in the vertical direction will also

511 disturb the flow. When only considering the streamwise magnetic field, Vieweg et al.
512 [82] observed obvious changes in the flow state under the effect of the magnetic field,
513 the red dashed line displayed in Fig. 6. This suggests the existence of an additional axis
514 perpendicular to the $Ka - Re$ space and related to Stuart number $N = Ha^2/Re$.

515 By designing a special system with controllable waves on the surface of liquid
516 metal, Ji et al. [94] studied the effect of a horizontal magnetic field (magnetic field
517 direction is parallel to wave vector) on surface waves and their dispersion relations.
518 They predicted the wave-damping rate caused by the magnetic field through linear
519 theory, which is given by

$$\exp \left[-\frac{B_{0x}^2 \omega k_r}{2\rho_e(\rho\omega^2 + 2\sigma k_r^3)} \Delta x \right] \quad (16)$$

520 where k_r is the real part of the wave number, and B_{0x} is the external streamwise
521 magnetic field, and ρ_e is electrical resistivity. The greater the magnetic field strength,
522 the faster the wave is damped. However, the observed value is much larger than the
523 theoretical prediction, because the prediction only considers the MHD effect during
524 wave propagation. Although the magnetic field does not change the dispersion relation
525 of the wave vector perpendicular to it [96], it affects the dispersion relation of parallel
526 waves. Ji et al. [94] summarized the influence of the electromagnetic field on the
527 dispersion relation as Eq. 17, where j_y is the cross-channel current density, B_x is the
528 streamwise magnetic field, and $K^2 = k^2(1 - i\sigma_e B_x^2/\rho\omega)$. The imaginary part shows
529 that the surface wave is damped due to the Joule dissipation from the streamwise
530 magnetic field component. Moreover, liquid metal oxides will change the surface

531 tension coefficient so that the dispersion relation curve of the surface wave deviates
 532 from the standard curve.

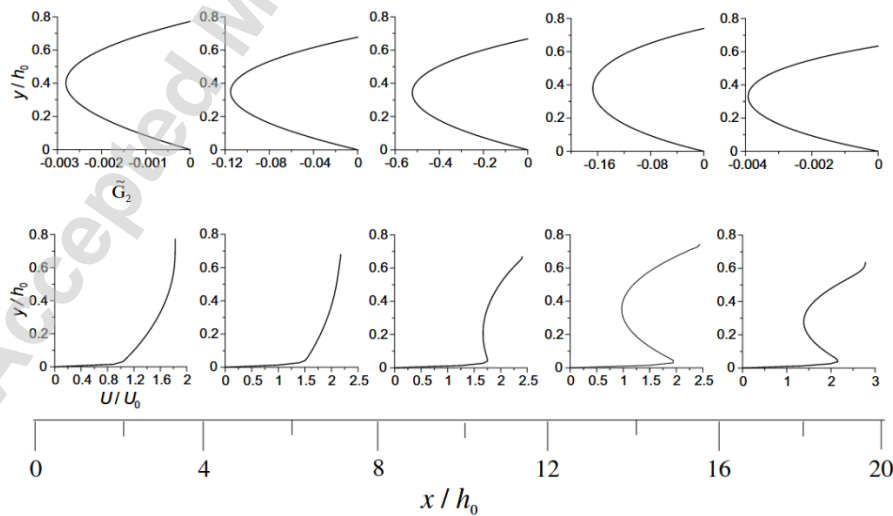
$$\rho\omega^2 = (\rho g + k^2\sigma + j_y B_x) \frac{k^2}{K} \tanh(Kh) \quad (17)$$

533

534 3.3 Thickness of liquid metal free surface flow

535 The thickness of liquid metal free surface flow, which can be influenced by the
 536 magnetic field, flow velocity, and inclination angle of the substrate, is also an important
 537 parameter. In the smooth laminar flow region without the influence of a magnetic field,
 538 there was a 1/3 scaling law (Eq. 18) between the Nusselt film thickness N_T and the
 539 Reynolds number [80], which can be derived from Navier-Stokes equations. According
 540 to the empirical equations, the scale in the turbulent region varied between 1/2 and
 541 2/3.

$$N_T = h(g\sin\beta/v^2)^{1/3} \left(\frac{\rho - \rho_c}{\rho} \right)^{1/3} = (3\text{Re})^{1/3} \quad (18)$$



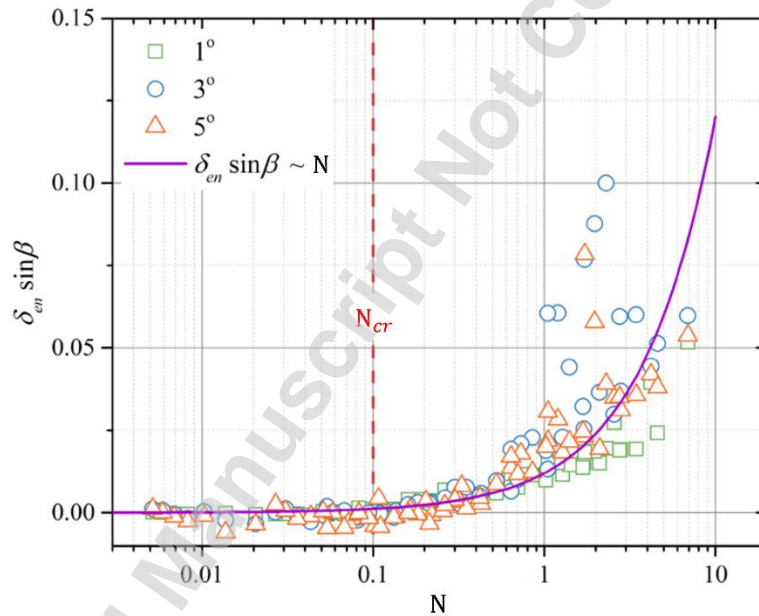
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543 Fig. 11. Downstream variations of the velocity profile (lower) and adverse
544 pressure gradient (upper) [97]. Copyright by the Elsevier (2005). Reproduced with
545 permission.

546 Under the influence of the transverse magnetic field, the Lorentz force acts as a
547 resistance, reducing the flow velocity and even forming an adverse pressure gradient in
548 the core flow, causing the vertical distribution of the velocity to change from a parabolic
549 shape to an "M" shape, similar to the velocity distribution of a conductive rectangular
550 duct under the influence of a transverse magnetic field (Fig. 11). Therefore, the
551 thickness of the liquid film will also change. Compared with the case without the
552 magnetic field, the thickness of the film increases, even accumulating at the entrance of
553 the test section [89,98,99]. Since the upstream and downstream of the test section were
554 located at the entrance and exit of the magnetic field, respectively, they were within the
555 gradient magnetic field range. Sun et al. [98] measured the distribution of the liquid film
556 thickness along the flow direction and found that the slope of the flow surface height
557 was moderate in the two regions, so the accumulation of liquid metal was mainly
558 caused by the magnetic field strength rather than the magnetic field gradient.

559 In addition, when Ha is small, the thickening effect of the magnetic field on the
560 thickness of the liquid film is not obvious. Then, the thickness increases linearly with the
561 increase of Ha , but the slope of the linear relationship is affected by Re and the
562 inclination angle β . Yang et al. [89] investigated the effects of each of the above three
563 parameters on the liquid film thickness by controlling the variables and obtained an
564 empirical correlation, $\delta_{en} \sin \beta \sim 0.012N$ (the solid line in Fig. 12), where δ_{en} is the

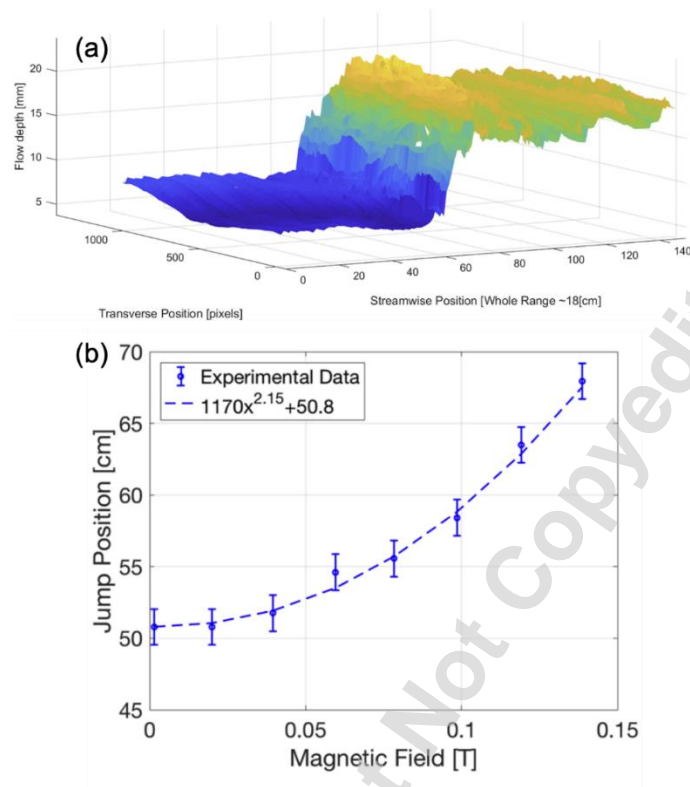
565 relative film thickening, $\delta_{en} = (h_{av} - h_0)/h_0$, and h_{av} and h_0 are the average thickness
566 and average thickness without the magnetic field, and Stuart number N . It can be seen
567 in Fig. 12 that there is a critical Stuart number N_{cr} when N is greater than N_{cr} , the MHD
568 drag force plays a dominant role in the changes in liquid film thickness and flow velocity.
569 Similar to transverse magnetic fields, vertical magnetic fields exert a force opposite to
570 the main flow direction, suppressing the flow and thickening the liquid film. Moreover,
571 the position of the Hartmann layer varies with different magnetic field directions.



572
573 Fig. 12. The film thickness changed with Stuart number, N , from Yang et al. [89].

574 In addition, as the liquid film boundary layer develops, increased flow resistance
575 leads to liquid accumulation. After surpassing the critical point, the thickness of the
576 liquid film suddenly increases, and the state pressure generated by gravity maintains the
577 process, resulting in a hydraulic jump. The formation of a hydraulic jump is also related
578 to flow separation, separation bubbles, or recirculation vortices at the bottom. After the
579 formation of the hydraulic jump, the liquid level at the hydraulic jump will suddenly

580 increase and the flow rate will slow down [100]. In the absence of a magnetic field, the
581 position of the hydraulic jump is related to factors such as flow rate, viscosity, and
582 gravity [101]. For a plane hydraulic jump [102], the position of a hydraulic jump can be
583 expressed as $x_j \propto (Q/w)^{5/3} \nu^{-1} g^{-1/3}$, where Q is the flow rate, w is the width of the
584 channel. In the experiments of Fisher et al. [103], the hydraulic jump position moved
585 downstream under the influence of the transverse magnetic field (Fig. 13), which may
586 be because the MHD drag suppressed the turbulent disturbance. In addition,
587 experiments on lithium film flow under transverse field influence have shown that liquid
588 lithium can be trapped at an inlet of an experimental setup, preventing it from flowing
589 downstream [99]. Moreover, the MHD effect reduces the height ratio across the
590 hydraulic jump. As the magnetic field increases, the type of hydraulic jump will also be
591 changed. The hydraulic jump is completely suppressed when the magnetic field intensity
592 reaches a certain value. The height ratio can reflect the energy dissipation of the
593 hydraulic jump [104].



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Fig. 13. The hydrodynamic jump [103]. (a) Sample imaging of a hydraulic jump using laser sheet diagnostic, (b) Jump position changes with a magnetic field. Copyright by the AIP Publishing (2018). Reproduced with permission.

3.4 Controlling of free surface flow

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For an engineering application such as the liquid metal film flow on PFCs in a nuclear fusion reactor, the stable and uniform thickness of liquid film flow is very important for the design of PFCs. Therefore, many efforts were exerted to reduce the MHD effects by changing the electrical conductivity of the wall and adding external force by introducing a current or thermal gradient.

605

606

Concerning the change in wall electrical conductivity, the current will be closed through the Hartmann layers and the side layers for insulating walls, while for

607 conductive walls, most of the current will pass through the wall. Therefore, the velocity
608 distribution caused by different wall conductivity is also different. According to the
609 distribution of current density of duct flow, the induced current near the side layer is
610 parallel to the magnetic field, while the current near the Hartmann layer is
611 perpendicular to the magnetic field. Therefore, the unique "M-shape" velocity
612 distribution, characterized by two velocity peaks located away from the walls but not at
613 the exact center of the channel and a lower velocity trough in the middle appeared [57].
614 Similarly, for the liquid metal film flow, the velocity near the side layers is faster than
615 the flow near the Hartmann layers and core flow. Under the influence of the transverse
616 magnetic field, the side layers are distributed near the free surface and the bottom
617 boundary layer, and the thickness is proportional to $Ha^{-1/2}$. In addition, since the
618 friction force on the free surface is much smaller than the friction force between the
619 liquid and the solid bottom wall, a free surface jet is formed [105,106]. According to
620 Sun's experimental data, the free surface velocity increased by 300%, while the average
621 velocity decreased by 45% [98]. Moreover, they also checked the influence of wall
622 conductivity on the film thickness. Both experimental and numerical simulations have
623 demonstrated that the difference in liquid film thickness between the inlet and outlet
624 grows as the wall conductance ratio increases. Consequently, to minimize MHD drag,
625 selecting materials with a lower wall conductance ratio is preferable.

626 The introduction of an external current to the liquid metal flow can also have the
627 chance to control the liquid metal free surface flow [74]. Hvasta [107] demonstrated
628 that the thickness and velocity of the liquid metal flow can be controlled by applying

629 electric and magnetic fields. Meng et al. [108] examined the impact of varying current
630 directions on the free surface flow of liquid metal within a transverse magnetic field
631 environment. For cases with a reverse current, which can generate a vertically
632 downward external Lorentz force, the liquid film becomes thinner in the middle and
633 thicker on both sides. Vice versa, when adding a forward current, which can generate a
634 vertically upward Lorentz force, the liquid column will form on the surface of the liquid
635 film when the amplitude of the current exceeds a certain value. However, due to the
636 increase in the overall thickness of the liquid film, splashing and other phenomena will
637 not occur. In particular, to get closer to the real situation in fusion reactors, Smolentsev
638 [97] investigated the influence of a constant wall-normal magnetic field and an
639 azimuthal magnetic field component that varies along the radius of the symmetric
640 geometry on the flow of a thin film which is on the surface of an axisymmetric structure.
641 The azimuthal component of the Lorentz force generated by the interaction between
642 the wall-normal magnetic field and the current in the flow plane causes secondary flow,
643 which eventually forms a swirl flow, and the direction of flow rotation depends on the
644 current distribution. The induced currents near the free surface and the substrate are in
645 opposite directions, so the fluid rotates in two directions. However, when the applied
646 current intensity is strong enough, the fluid rotates in only one direction, and the
647 rotation effect is not suppressed.

648 Moreover, the thermoelectric effect (Seebeck effect) is created due to the
649 uneven temperature distribution and can also introduce an external electric current,
650 resulting in a Lorentz force to influence the liquid metal flow [109-111]. As the

651 temperature gradient increases, the flow rate will also increase, allowing for better heat
652 exchange and reducing the impact of local high heat fluxes [112].

653

654 3.5 Free surface flow related to nuclear fusion

655 Compared with solid PFCs, liquid metal PFCs are more conducive to the
656 operation of fusion reactors at higher heating powers due to their good heat transfer
657 characteristics. Because lithium is low-Z, the core plasma has a higher tolerance for
658 lithium. At the same time, lithium can also reduce other impurity levels and
659 temperature gradients in the plasma [6].

660 The liquid film with a curved bottom wall can better adapt to the gradient
661 magnetic field, but the flow is also more difficult to control. Therefore, Zhang [65]
662 proposed the concept of a multiplayer MHD liquid film that can flow stably. This system
663 mainly consists of a base layer, an adjustment layer, and a stabilization layer. It achieves
664 stable and well-spread film flow by adjusting the distance between the metal mesh and
665 the bottom wall. Special lithium metal infused trenches (LiMIT) have also been designed
666 for liquid metal divertor, which can remove high heat fluxes by using the thermoelectric
667 magnetohydrodynamics (TEMHD) [110].

668 In the tokamak environment, the plasma can accidentally impact the surface of
669 liquid metal PFCs, resulting in the corresponding Kelvin-Helmholtz (K-H) [66], Rayleigh-
670 Taylor (R-T) [113] and Rayleigh-Plateau (R-P) [114] instabilities in free surface flow. Such
671 instabilities have the potential to induce several problematic outcomes, including
672 corrosion of the divertor target and contamination of the core plasma.

673 For Lithium-metal infused trenches (LiMIT), splashing caused by plasma also
674 depends on factors such as the wettability and the width of the trenches [115]. The
675 plasma impact not only deforms the melt surface but also gradually increases the stress
676 inside the melt over time, resulting in the movement of the melt and interface changes.
677 At the same time, the Lorentz force generated by the current from the plasma scraping
678 layer or the hot electrons emitted from the high-temperature surface and the magnetic
679 field will also cause the movement of the liquid metal [116]. In addition, under high heat
680 loads, the boiling and evaporation of the melt will also cause disturbances in the
681 interface. Waves with shorter wavelengths will be suppressed by surface tension, but
682 the amplitude of longer wavelength fluctuations will be rapidly amplified. The
683 nonuniform distributed stress in the molten metal may cause this long-wave instability.
684 In addition, the plasma-molten metal interface has the characteristics of high-density
685 difference and high surface tension, which form K-H instability under the action of
686 plasma shear stress and velocity difference, but the classic phenomenon of K-H
687 instability was not observed in the simulation and experimental results [66,117].

688 If one focuses only on the momentum transport process during plasma shock,
689 the impact effect of the plasma can be simplified to the effect of a moving pressure
690 source [118], which varies in time and space. When the applied pressure moves at a
691 velocity close to the downstream wave propagation speed, it propagates at a rate
692 similar to that of the trough it generates. This results in a continuous decrease in the
693 thickness of the liquid metal, which may lead to dangerous dewetting. Concurrently, the
694 non-local interaction of surface waves causes the troughs generated upstream to align

695 with those produced later in the oscillation cycle as they propagate downstream.
696 However, when the time scale of the pressure motion is significantly larger than the
697 response time scale of the free surface, the system approaches a quasi-steady state. In
698 contrast, if the time scale of the pressure motion is much smaller than that of the free
699 interface response, the resulting deflection of the free surface remains relatively
700 minimal.

701 The driving methods of liquid metal in Tokamak include electromagnetic drive
702 [119] and capillary drive [120,121]. Among them, EAST Tokamak uses the
703 electromagnetic control principle in Section. 3.4 to form a continuous liquid lithium film
704 flow on the surface of the divertor target plate [6]. However, due to the nonuniform
705 resistance of the pipeline, the liquid lithium cannot be spread evenly, similar to a liquid
706 column flow, as described in Section. 3.2, resulting in a surface coverage rate of only
707 30%, and the remaining uncovered surface is severely corroded. The upgraded limiter
708 improves the wetting performance of liquid lithium on the target surface by designing a
709 special capillary structure, improving the uniformity of the channel, and changing the
710 surface material [122,123], and the surface coverage rate is increased to more than
711 80%. The latest generation of lithium PFCs is designed with metal grooves, which
712 increases the surface coverage rate to more than 95%. It's similar to the LiMIT, which
713 not only enhances the thermo-electromagnetic dynamic flow of liquid lithium but also
714 improves safety [124]. Recently, Yang et al. [125] designed a kind of PFC model and
715 realized the nearly full spreading of Lithium film flow on the stainless-steel surface with
716 microgrooves with a coverage ratio of 97.32%, which is the largest one reported. With

717 the development of liquid metal PFCs, the structure of the PFCs has been continuously
718 improved, so that the liquid metal can cover the PFCs and operate stably, but there are
719 still some problems

720 In addition, when the plasma interacts with flowing lithium, a distinct bright light
721 can be seen, the temperature of the liquid lithium increases, evaporation, and droplet
722 sputtering occurs, and the lithium can be detected by spectroscopy far away from the
723 limiter. The dynamics of the droplets is the next topic that will be discussed later.

724

725 **4 LIQUID METAL DROPLETS IMPACTING**

726

727 Liquid metal droplet impacting and spreading are also typical kinds of interfacial
728 flow in fluid dynamics. These processes can be significantly influenced by factors such as
729 droplet velocity, surface tension, magnetic fields, and solidification. Understanding
730 these processes is crucial for applications in materials science, manufacturing, and
731 energy technologies. In this section, we focus on the topic of liquid metal droplets
732 wetting, spreading, impacting on solid or liquid surfaces, and colliding with each other
733 under the influence of magnetic fields.

734

735 **3.1 Wetting properties of liquid metal**

736 Wettability can be defined as the capacity of a liquid to maintain contact with a
737 solid surface and is a critical property in various scientific and engineering fields. For
738 example, the liquid metal PFCs in Tokamak need a free surface flow, as discussed in Sec.
739 3 cover on the solid surface. Therefore, achieving good liquid metal wetting on the solid

740 surface (ensuring complete coverage) is a crucial initial step. Here, we introduce the
 741 wettability of several typical liquid metals commonly adopted in engineering
 742 applications or fundamental studies.

743 From the fundamental aspect, mercury is often selected as a modeling liquid to
 744 study MHD problems in experiments in earlier years. Ellison et al. [126] studied the
 745 contact angle of mercury on six different materials as a function of temperature. As
 746 shown in Table 1, the temperature range is 25°C to 150°C. Results indicated that contact
 747 angles were very high on substrates, ranging at equilibrium from about 130 °C to 145 °C
 748 at 25 °C. Although there were significant differences between the dynamic advancing
 749 and receding angles, equilibrium was rapidly achieved in all systems, and the residual
 750 hysteresis was negligible. Additionally, the temperature coefficient of the contact angle
 751 was consistently small and positive across all systems.

752
 753 Table 1. Contact angle of mercury on polished solid substrates at various temperatures
 754 [126]. Here, $\theta_{a,max}$ and $\theta_{a,eq}$ stand for the maximum and equilibrium advanced contact
 755 angle, while $\theta_{r,min}$ and $\theta_{r,eq}$ stand for the minimum and equilibrium receded contact
 756 angle.

| Substrates | Temperature, °C | | | | | |
|------------------|-----------------|-----|-----|-----|-----|-----|
| | 20 | 50 | 75 | 100 | 125 | 150 |
| Tungsten | | | | | | |
| $\theta_{a,max}$ | 142 | 150 | 155 | 155 | 159 | 160 |
| $\theta_{a,eq}$ | 130 | 130 | 140 | 137 | 140 | 137 |

| | | | | | | |
|------------------|-----|-----|-----|-----|-----|-----|
| $\theta_{r,eq}$ | 132 | 132 | 137 | 135 | 141 | 137 |
| $\theta_{r,min}$ | 121 | 110 | 120 | 119 | 123 | 124 |
| Stainless steel | | | | | | |
| $\theta_{a,max}$ | 146 | 150 | 151 | 158 | 162 | 157 |
| $\theta_{a,eq}$ | 133 | 134 | 136 | 146 | 142 | 141 |
| $\theta_{r,eq}$ | 132 | 134 | 138 | 143 | 144 | 141 |
| $\theta_{r,min}$ | 124 | 121 | 120 | 130 | 132 | 130 |
| Nickel | | | | | | |
| $\theta_{a,max}$ | 148 | 150 | 156 | 161 | 160 | 165 |
| $\theta_{a,eq}$ | 139 | 135 | 138 | 146 | 145 | 145 |
| $\theta_{r,eq}$ | 138 | 134 | 137 | 146 | 146 | 145 |
| $\theta_{r,min}$ | 123 | 116 | 124 | 129 | 133 | 130 |
| Quartz | | | | | | |
| $\theta_{a,max}$ | 147 | 144 | 150 | 152 | 155 | 159 |
| $\theta_{a,eq}$ | 132 | 132 | 124 | 130 | 132 | 136 |
| $\theta_{r,eq}$ | 134 | 132 | 122 | 131 | 132 | 136 |
| $\theta_{r,min}$ | 115 | 122 | 110 | 122 | 118 | 114 |
| Glass | | | | | | |
| $\theta_{a,max}$ | 147 | 147 | 152 | 156 | 154 | 159 |
| $\theta_{a,eq}$ | 133 | 132 | 132 | 136 | 139 | 140 |
| $\theta_{r,eq}$ | 134 | 130 | 131 | 137 | 137 | 140 |

| | | | | | | | |
|-----|------------------|-----|-----|-----|-----|-----|-----|
| 757 | $\theta_{r,min}$ | 122 | 112 | 120 | 125 | 126 | 125 |
| 758 | Teflon | | | | | | |
| 759 | $\theta_{a,max}$ | 157 | 162 | 166 | 169 | 175 | 178 |
| 760 | $\theta_{a,eq}$ | 134 | 135 | 137 | 142 | 146 | 147 |
| 761 | $\theta_{r,eq}$ | 132 | 132 | 134 | 145 | 145 | 146 |
| 762 | $\theta_{r,min}$ | 104 | 98 | 109 | 119 | 119 | 116 |

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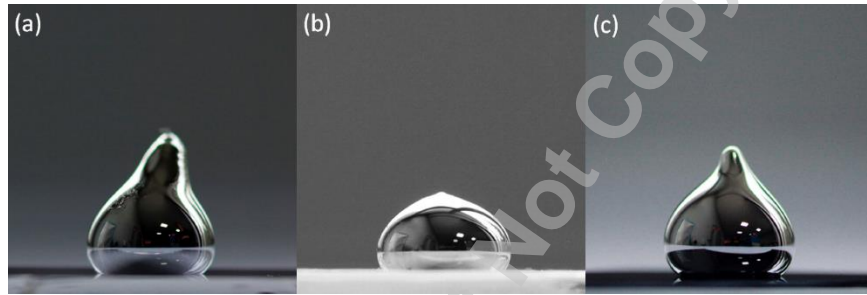
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Recently, gallium-based alloys (GBAs) have replaced mercury in the fundamental studies of MHD effects due to their excellent properties, such as low toxicity and liquid state at room temperature. Nevertheless, the complex wetting properties of GBAs present a significant barrier to their practical development and utilization. Unlike the wetting behavior of Hg, it is generally found that when exposed to oxygen concentrations above 1ppm, GBAs will quickly form a solid oxide layer of several nanometers on the surface [127, 128]. This oxide layer will provide a certain rigidity to the liquid metal surface, making the GBAs maintain their shape over a large surface area [129-133]. Nonetheless, some experimental results showed that the presence of the oxide film makes GBAs easy to adhere to the surface of almost all materials, such as metals, glass, and various polymers [134-140]. Lin et al. [141] measured the contact angle of E-GaIn on the Cu and graphite surface. For E-GaIn/Graphite, the contact angle was about 150°, for E-GaIn/Cu, the contact angle decreased to approximately 85°. Dobosz et al. [142] have obtained the contact angle between the Ga-Sn-Zn eutectic and

779 the substrates, with contact angles $132 \pm 1^\circ$ for glass, $118 \pm 2^\circ$ for quartz, and $134 \pm 1^\circ$
 780 for silicon, as shown in Fig. 14. The results indicated that the wetting behavior of Ga-Sn-
 781 Zn eutectic alloy is similar in the cases of glass and silicon. The obtained values in the
 782 case of silicon are similar to the value obtained in reference [143] for Galinstan on
 783 silicon (139° at 303 K).

784



785

786 Fig. 14. Contact angles of Ga-Sn-Zn on three material surfaces: (a) glass, (b)
 787 quartz, and (c) silicon [142]. (CC BY 4.0)

788

789 Table 2. Wetting angles of PbBi on the RAFM steel surface based on the ellipse fitting
 790 [150].

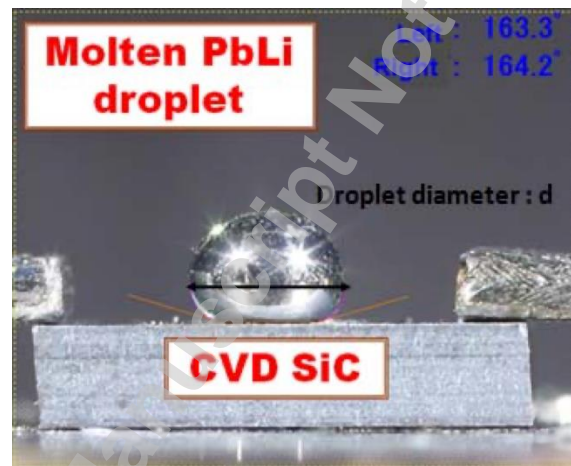
| Temperature (°C) | Wetting angles (°) | |
|---------------------|--------------------|------------|
| | Left side | Right side |
| 350 | 148.9073 | 146.2145 |
| 400 | 150.7129 | 148.2486 |
| 450 | 145.9786 | 142.2444 |
| 500 | 144.9476 | 141.1075 |

791

792 According to existing research, two categories are roughly classified to adjust the
793 wettability of GBAs: (i) Formation and removal of the gallium-based surface oxide film
794 and (ii) surface treatment of the LM droplets of the substrates. Considering gallium-
795 based oxide as a pivotal role in liquid metal wetting behavior, researchers tried to
796 facilitate or suppress the oxide formation to enhance or restrain liquid metal wetting
797 and adhesio [133-145]. Guo et al. [146, 147, 148] dispersed Ni, Cu, or Fe nanoparticles
798 evenly into the gallium-based liquid metal, improving its conductivity meanwhile
799 increasing the amount of gallium-based oxide wrapped around the nanoparticles. They
800 found that the obtained semi-solid material had much larger adhesion to many
801 materials and could be easily transferred to paper, PU(Polyurethane), and other
802 substrates. In contrast, Bilodeau et al. [135] eliminated the surface oxide film to reduce
803 liquid metal wetting by a certain concentration of acid or base solution, and Handschuh-
804 Wang et al. [133] found the base more powerful than acid in gallium-based oxide
805 removal. Khan et al. [144] adopted an electrochemical method to deposit or remove the
806 oxide layer on a drop of GBAs in NaOH solution, dramatically changing its wetting
807 behavior.

808 From aspects of engineering applications, liquid metals with high melting
809 temperatures, such as Lithium lead (PbLi), lithium (Li), and sodium (Na), are commonly
810 adopted. The wetting properties between the liquid metal and the structural materials
811 are important. Ueki et al. [149] measured the contact angle of a molten PbLi droplet on
812 a silicon carbide (SiC) wall at different temperatures (from 250°C to 400°C). Results
813 indicated that the contact angles of all cases are greater than 150°, which could be

814 classified as super-hydrophobic (Fig. 15). Nevertheless, if corrosion of the wall by PbLi
815 occurs, it can be reasonably anticipated that the contact angle will change. Liquid lead-
816 bismuth (PbBi) is a kind of coolant in the Lead-cooled Fast Reactor (LFR). Pan et al. [150]
817 measured the wetting angle of PbBi at reduced activation ferritic/martensitic steel
818 (RAFM steel) surface using the sessile drop method, which was carried out under the
819 protection of high-purity argon. As listed in table 2, experimental results show that the
820 wetting angles of liquid PbBi on RAFM steel are greater than 140° at 350–600°C.
821 Therefore the PbBi also showed the characteristic of superhydrophobic.



822
823 Fig. 15. A molten PbLi droplet on a CvD SiC (the droplet diameter: 3.2 mm)
824 [149]. Copyright by the Elsevier (2011). Reproduced with permission.

825 Liquid Na is commonly used as a coolant in fast breeder reactors (FBRs) because
826 of its high thermal conductivity, specific heat, low viscosity, low melting temperature,
827 high boiling temperature, and good compatibility with structural materials. In FBR
828 plants, the wettability of structure material owing to the liquid Na content is an
829 important property for safe operation and high efficiency. Saito et al. [151] have
830 investigated the relationship between the contact angle of Na on Cu and its alloys. The

831 results show that the contact angle of pure Cu is smaller than that of the Cu alloys. The
832 range of the contact angle of all the substrate metals is 110–120°, which is within the
833 standard deviations. Therefore, only slight deviations were noted for the contact angle
834 between the Cu alloys.

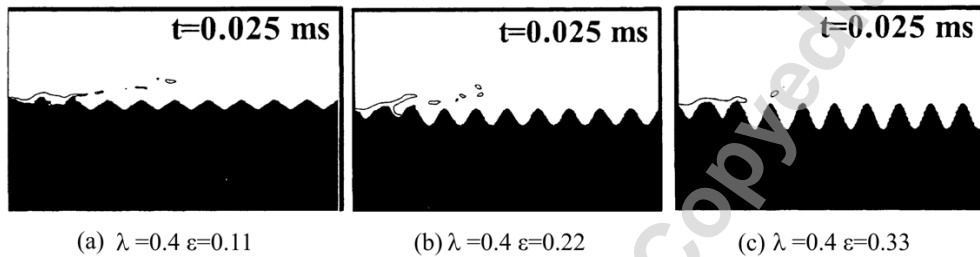
835

836 4.2 Impingement of droplets on a solid surface

837 When a liquid droplet impacts the solid surface, various phenomena such as
838 spreading, bouncing, splashing, or sticking can occur due to different impact velocities,
839 surface characteristics, and the physical properties of the liquid. This section focuses on
840 the spreading phenomenon of a class of high-surface-tension liquid metal droplets
841 with/without a magnetic field.

842 In the absence of MFs, the spreading of metal droplets behaves similarly to that
843 of conventional fluids. This similarity arises because the ratios of physical properties
844 ρ/σ , represented in the dimensionless Weber number, We , $We = \rho U^2 D / \sigma$, are
845 essentially comparable between liquid metals and conventional fluids like water, here ρ
846 and σ are the density and surface tension of the liquid, U is the impact velocity, D is the
847 diameter of a droplet. However, the spreading behavior of liquid metals is more
848 susceptible to their specific melting points and oxidizing properties. Ahmed et al. [152]
849 investigated the effect of substrate morphology on metal droplet impact behavior. The
850 roughness of the substrate was changed to form different uneven substrates, as shown
851 in Fig. 16. Results revealed that droplet impingement on uneven substrates consistently
852 leads to splashing. However, the intensity of splashing decreased as surface roughness

853 increased since the rougher textures captured and retained more fluid in their gaps,
854 where it then solidified. In contrast, impacts on smoother substrates, which have less
855 roughness, were marked by enhanced splashing due to the liquid being more prone to
856 jetting outward, propelled by its momentum.



857 (a) $\lambda=0.4$ $\epsilon=0.11$ (b) $\lambda=0.4$ $\epsilon=0.22$ (c) $\lambda=0.4$ $\epsilon=0.33$
858 Fig. 16. Liquid metal droplet spreading on substrates with different roughness
859 [152]. Copyright by the Elsevier (2002). Reproduced with permission.

860 Solonenko et al. [153, 154] performed a comparative analysis of how hollow and
861 solid zirconium oxide droplets deposit on surfaces. The study showed that hollow
862 droplets tend to form smoother films than solid droplets. Kumar and Gu [155] employed
863 the VOF method to examine the impact of hollow tin droplets on a wall, revealing a
864 significant correlation between the gas phase distribution and the dynamic morphology
865 of the droplets upon impact. Dhiman and Chandra [156] observed that at elevated wall
866 temperatures, the edge of the tin droplet solidifies at a slower rate, allowing for more
867 extensive spreading with minimal resistance. Conversely, at lower wall temperatures,
868 the solidification rate of the droplet edge increases significantly, substantially inhibiting
869 its spread. Ogura et al. [157] investigated the spreading behavior and deposition
870 thickness of molten copper using a 0.5 kg sample. Their findings suggest that the melt
871 mass is a crucial factor influencing the spreading area. Several recent studies [158, 159,

872 160] on 3D printing used metal droplets with high surface tensions, but their impact
873 speeds were very low. To simulate a more realistic engine environment, Li et al. [161,
874 162] developed an experimental system to study the impact of high-temperature
875 aluminum droplets on solid surfaces. By comparing the dynamic behavior of droplets on
876 surfaces with varying inclinations, they investigated the rebound characteristics of the
877 aluminum droplets. Based on comprehensive experimental data and the principle of
878 energy conservation, they developed a model to calculate the rebound coefficient of
879 aluminum droplets and found a method to quantitatively determine the residual energy
880 of the droplets during the collision process. Furthermore, Li et al. [163] investigated the
881 heat transfer processes during the collision of high-temperature aluminum droplets on
882 an inclined wall. Their findings indicated that the temperature at the impact point
883 undergoes a two-stage reduction: an initial rapid decrease followed by a slower
884 decrease as the droplet spreads along the wall.

885 For gallium-based metal droplets, which are liquid at room temperature, much
886 research has focused on their spreading properties on different material surfaces, such
887 as Au, PU, diamond, and graphene [164-166]. Moreover, by changing the surface
888 roughness, it has been demonstrated that the rebound properties of gallium-based
889 metal droplets after impacting were enhanced [128, 140]. Meanwhile, results also
890 proved that the applied SiO₂ microparticles to solid surfaces could adjust the adhesion
891 of gallium-based metal droplets on solid surfaces [138]. Zhang et al. [167] created nano-
892 scale surface roughness on PDMS, copper, and stainless steel using femtosecond laser
893 (fs-laser) ablation, achieving surfaces with significant hydrophobicity against liquid

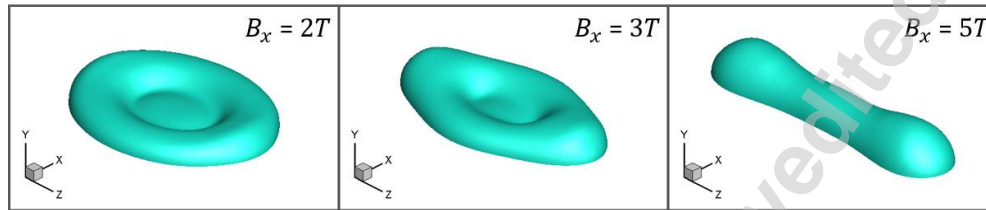
894 metal. Nano-second laser (ns-laser) is also effective in creating fine surface
895 microstructures, imparting properties such as super-hydrophobicity [168, 169],
896 superhydrophobicity [170], and ultra-anti-reflection [171], and is more accessible and
897 cost-effective. Therefore, ns-laser was selected to prepare structured metal surfaces for
898 this study. The wetting behaviors of eutectic gallium-indium alloys (GaInSn, with a
899 melting point of 15.5°C) on ns-laser ablated structured metal surfaces were
900 investigated. Appropriate ns-laser processing methods for H13 steel, copper, 304
901 stainless steel (SS), and tantalum were explored. Surfaces with a wide range of surface
902 roughness, compositions, and various micro-patterns were prepared, and the
903 relationships between these factors and the wetting behavior of liquid metal were
904 thoroughly examined.

905 When considering the effect of the magnetic field on the spreading
906 characteristics of metallic droplets, the outcomes become highly complex due to the
907 anisotropic nature of the magnetic field, which varies depending on the angle between
908 the magnetic field and gravity. Tagawa [172] carried out numerical simulations on an
909 initially cubic droplet falling onto a flat wall in a uniform static magnetic field. Their
910 findings revealed that when a horizontal magnetic field was applied, droplet spreading
911 tended to become two-dimensional (2D), while a vertical magnetic field application
912 resulted in the electromagnetic force dampening the spread. However, his analysis
913 remains limited to a qualitative description due to the lack of a clear explanation of
914 droplet deformation. Yang et al. [173] obtained the experimental data of a GaInSn
915 droplet spreading on a glass surface in a horizontal magnetic field using a high-speed

916 imaging method. The impact velocity ranges from 0.1 m/s to 1.2 m/s and the magnetic
917 field can adjust from 0 T to 1.6 T. As shown in Fig. 17, an elliptical spreading pattern of
918 the droplet is first observed by experiments. To explain this phenomenon in more detail
919 from a mechanistic perspective and verify the existence of this phenomenon in a wider
920 parameter range. The scaling law for the maximum spreading factor β_m and aspect ratio
921 of the maximum spreading factor κ_m with the relationship of $\beta_m \sim WeN$ and
922 $\kappa_m \sim (WeN)^{1/2}$ were obtained. Zhou et al. [174] conducted an experimental
923 investigation into the spreading characteristics of a GaInSn droplet on a solid substrate
924 under a strong vertical magnetic field, with intensities ranging from 0 to 2.5 T. Initially,
925 several droplet parameters, such as shape, spreading factor, dynamic contact angle,
926 spreading velocity, and rebound behavior post-impact, were analyzed without a
927 magnetic field. The fitting relationship between the maximum spreading factor (β_m) and
928 the Weber number was obtained and compared with existing scaling laws. Additionally,
929 the effect of the vertical magnetic field on these parameters was systematically
930 examined. Quantitative results on β_m and maximum spreading time (t_{Dmax}), varied with
931 the Ha and We number, provide a comprehensive understanding of the spreading
932 dynamics. The relationship between β_m and We under different magnetic field
933 intensities demonstrates that a vertical magnetic field significantly inhibits the spreading
934 of liquid metal droplets.

| | C1 | C2 | C3 | C4 | C5 | C6 |
|-----|--------------|------------|----------------|---------------|----------------|----------------|
| R1 | Time (ms) | Ha=0 OV | Ha=135.5 OV | Ha=0 FV/SV | Ha=135.5 FV | Ha=135.5 SV |
| R2 | | | | | | |
| R3 | 0 | | | | | |
| R4 | 1 | | | | | |
| R5 | 2 | | | | | |
| R6 | 3 | | | | | |
| R7 | 4 | | | | | |
| R8 | 5 | | | | | |
| R9 | 6 | | | | | |
| R10 | 7 | | | | | |
| R11 | 8 | | | | | |

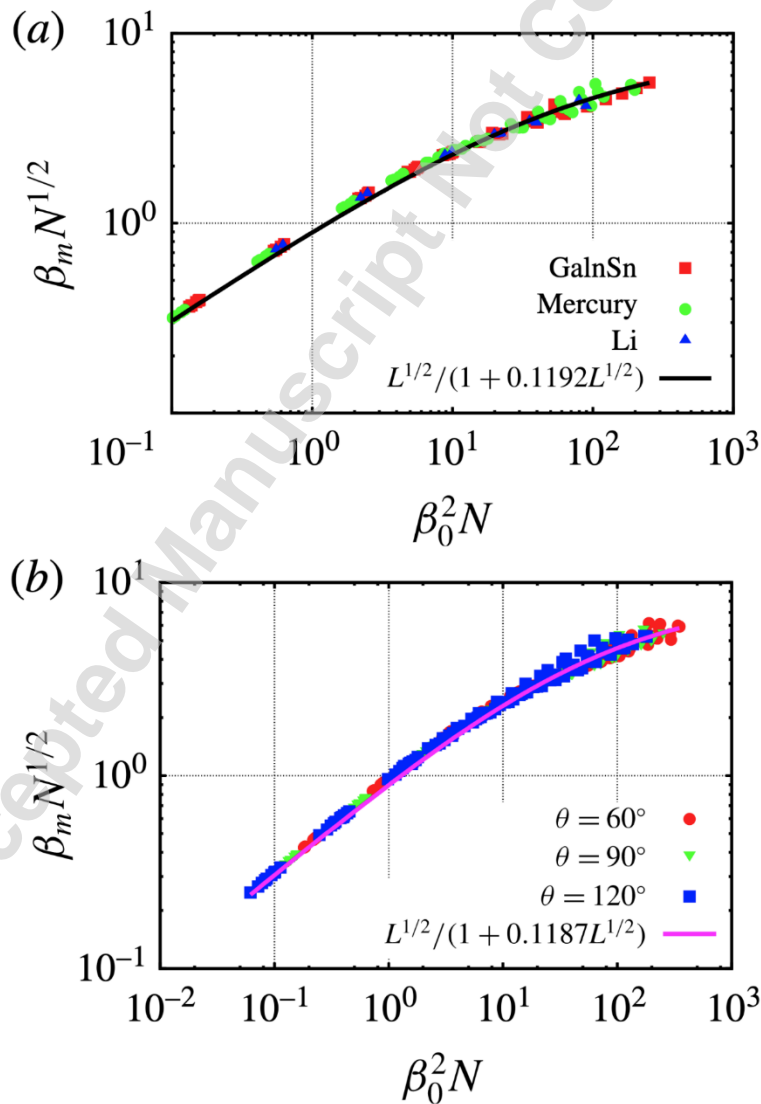
936 Fig. 17. Snapshots of the spreading process for cases with and without a
937 magnetic field [173]. Copyright by the AIP Publishing (2018). Reproduced with
938 permission.



939
940 Fig. 18. Droplet Shapes at the moment of the maximum spreading in the z
941 direction for the three typical cases [175]. Copyright by the Taylor & Francis (2018).
942 Reproduced with permission.

943 Han et al. [175] used direct numerical simulation (DNS) to investigate the
944 spreading process of a liquid metal droplet on a flat solid surface in the presence of a
945 horizontal magnetic field. Their simulation results validated the experimental
946 observations by Yang et al. [173], i.e., the elliptical spreading pattern, with the droplet
947 elongated perpendicular to the magnetic field. By extending the experimental
948 parameter, Han et al. [175] found that the droplet looks more like a slender stick than
949 an ellipse when it reaches the maximum spreading, as shown in Fig. 18. Furthermore,
950 given the apparent high surface tension of liquid metals, existing models are not
951 suitable for predicting spreading coefficients. Meng et al. [176] experimentally studied
952 the spreading and rebound characteristics of liquid gallium (Ga) droplets when
953 impacting isothermal and supercooled surfaces under the action of a horizontal
954 magnetic field. Experimental results showed that the magnetic field inhibits the
955 generation of secondary droplets during the spreading and rebound of the droplets

956 parallel to the magnetic field direction and has a stretching effect on the droplets in the
 957 direction parallel to the magnetic field during the rebound process. When the droplet
 958 impacts the supercooled wall within a certain range of We values, secondary droplet
 959 separation will also occur, and the speed of the secondary droplets generated at this
 960 time is relatively small. The intensification of the magnetic field and the increase in the
 961 We number contribute to the reduction of droplet oscillation in the vertical direction,
 962 thereby accelerating the solidification process.



963

964 Fig. 19. The rescaled maximum spreading ratio as a function of $\beta_0^2 N$, with the
965 solid line showing the interpolating function $\beta_m N^{1/2} = L^{1/2} / (1 + BL^{1/2})$ fitted to the
966 numerical results. (a) Numerical results obtained within fixed θ of 90° when using
967 different liquid metals; (b) Numerical results obtained within different θ of 60° , 90° and
968 120° [40]. Copyright by the Cambridge University Press (2016). Reproduced with
969 permission.

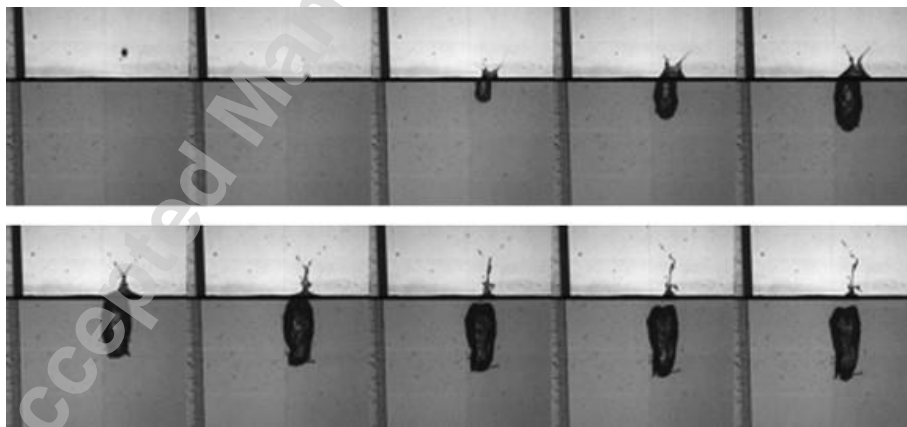
970 More recently, we should stress that there are several works to provide
971 quantitative results about the maximum spreading of the impacting droplets under the
972 influence of external magnetic fields. Concerning the influence of vertical magnetic field
973 on liquid metal droplet impacting, Zhang et al. [40] developed a theoretical model to
974 predict the maximum spreading factor by using the principle of energy conversion. The
975 results show that when there is no magnetic field, the maximum factor can be well
976 predicted by an interpolation scheme between the viscous and capillary effects. When
977 gradually increasing the magnetic field strength, the induced Lorentz force dominates
978 over the viscous and capillary forces, causing the spreading behavior to enter the Joule
979 regime. By interpolating it with the viscous dissipation and the capillary effects, a
980 universal scaling law of $\beta_m \propto N^{-1/2} f_c \beta_0^2 N$ is proposed to predict the maximum
981 spreading diameter. All the numerical results are found to rescale onto a single curve
982 predicted by $\beta_m N^{1/2} = L^{1/2} / (1 + BL^{1/2})$, as shown in Fig. 19, proving that the
983 theoretical model is correct in predicting the maximum spreading factor by constructing
984 a balanced formula between kinetic energy, capillary energy, viscous dissipation energy
985 and Joule dissipation energy. After that, under the influence of a horizontal magnetic

986 field, Han et al. [177] developed a new model that considered the anisotropic spreading
987 behaviors produced by the horizontal magnetic fields. In the context of that model, they
988 evaluate the residual kinetic energy due to high surface tension exhibited by the liquid
989 metal and give a more precise estimation of the spreading time than previous works
990 designed for ordinary fluids. Then they incorporate the work of Lorentz force into the
991 energy balance equation, and the theoretical predictions exhibit a very good agreement
992 with the numerical results. Based on these theoretical arguments, Sun et al. [178]
993 conducted numerical simulations to study the solidification behavior of a liquid metal
994 droplet impacting a cold substrate under the influence of a vertical magnetic field, and
995 an interpolation scheme that integrates impact, thermal, and MHD effects is proposed
996 to predict the maximum spreading factor, showing strong agreement with the numerical
997 results. For non-isothermal conditions, solidification transitions the droplet from full
998 rebound to adhesion on the cold substrate, while the magnetic field induces a pinch-off
999 phenomenon during droplet retraction. The mechanisms underlying the transition
1000 between different impact regimes are analyzed and compared with existing
1001 experimental data and analytical models. Lastly, it is demonstrated that the thickness
1002 growth of the solidified sheet can be predicted by solving a simple one dimensional
1003 Stefan problem, highlighting the dominance of thermal dynamics over hydrodynamic or
1004 MHD effects during the melting of the spreading droplet.

1005

1006 4.3 Impingement of droplets on liquid surface

1007 When a liquid metal droplet impacts a liquid surface instead of a solid one, the
1008 outcomes are significantly different due to the change in boundary condition. Li et al.
1009 [179] investigated the interaction between a high temperature zirconium dioxide sphere
1010 and cold water, discovering that the velocity of the sphere increases with its
1011 temperature. Similarly, Zhan et al. [180] investigated the dynamics of high-temperature
1012 steel spheres descending into cold water, also concluding that the velocity of these hot
1013 spheres rises with their temperature. Li et al. [181] experimentally investigated the
1014 moving characteristics of molten metal droplets impact coolant (Fig. 20). Lead-bismuth
1015 was chosen as the metal because of its low melting point and water as a cooling coolant.
1016 Results showed that the falling velocity of the droplet increases with the droplet's initial
1017 temperature when the coolant temperature is invariable. The falling velocity of the
1018 droplet increases with the coolant temperature when the droplet's initial temperature is
1019 invariable.



1020
1021 Fig. 20. Experimental images of the experiment condition H790Tw22-Tp334-
1022 2000. H: falling distance of molten droplet before it touches the coolant surface, mm;
1023 Tw: water temperature, C; Tp: molten droplet temperature, C; 2000: photo-frequency of

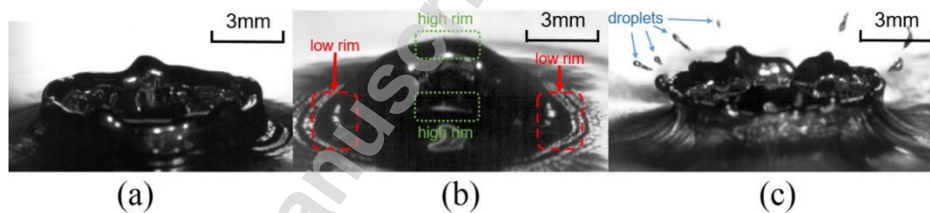
1024 the high-speed camera, fps (the time interval between two adjacent photos in the figure
1025 is 3 ms) [181]. Copyright by the Elsevier (2008). Reproduced with permission.

1026 The impact of a three-dimensional liquid metal drop onto a film of the same
1027 liquid is numerically investigated by Wang et al. [182]. The VOF method combined with
1028 adaptive mesh refinement is used for interface tracking, while a consistent and
1029 conservative scheme is adopted to calculate the induced current density and Lorentz
1030 force. It was found that the perpendicular magnetic field confines the sputtering flow,
1031 reduces the crown diameter, and tends to suppress the sputtering flow. However, at
1032 sufficiently high falling velocities, the Vertical Magnetic Field (VMF) fails to mitigate the
1033 instability, resulting in the ejection of secondary droplets from the crown rim.

1034 Hadj-Achour et al. [183] experimentally investigated the behavior of a molten
1035 droplet impact perpendicular to the interface of a deep water pool to characterize the
1036 different fragmentation regimes and to develop a relationship for the characteristic size
1037 of fragments that can be used in nuclear safety modeling codes. The We number is
1038 changed from 10 to 118 with a constant water temperature of 75°C. Their findings
1039 indicated that in liquid gas systems, the droplet undergoes various fragmentation
1040 regimes depending on its Weber number: oscillations, bag oscillations, prolate drop
1041 stretching breakup, and finally, bowl-shaped bag breakup. Additionally, a
1042 straightforward relationship between the Sauter Mean Diameter (SMD) and the We
1043 number is proposed, grounded in energy and mass balance considerations.

1044 Some researchers also focus on liquid metal droplets impacting the pool of the
1045 same materials. Li et al. [184] experimentally investigated the fluid dynamics of room

1046 temperature liquid metal droplet (RTLMD) impact onto a pool of the same liquid in
1047 ambient air. The experiments varied the impact Fr from 20 to 26, and the We from 479
1048 to 763. The variables set for the study included droplet diameter, impact velocity, and
1049 pool temperature. The results indicated that droplet size and impact velocity exhibit
1050 similar trends in their effects on splash height, with no secondary droplet separation in
1051 either case. However, an increase in pool temperature significantly enhances the splash
1052 effect, resulting in the formation of shuttle-shaped secondary droplets detaching from
1053 the central jet. This difference is likely due to the oxide scale of the RTLMD droplets,
1054 which significantly influences the droplet appearance and splash morphology. Energy
1055 analysis further supported this hypothesis, clarifying that the oxidation effect primarily
1056 alters the viscosity of RTLMD.



1057 (a) Symmetric crown, $We = 75$ and $Ha = 0$, (b) asymmetric crown, $We =$
1058 75 and $Ha = 174$, and (c) prompt splashing, $We = 94$ and $Ha = 0$ [185]. Copyright by the
1059 Elsevier (2020). Reproduced with permission.

1062 Ren et al. [185] experimentally conducted a study on the impact of high surface
1063 tension liquid metal droplets on the same liquid film in the presence of a horizontal
1064 magnetic field. They observed three distinct outcomes: symmetric crown, asymmetric
1065 crown, and instantaneous splash, with the asymmetric crown being newly identified in

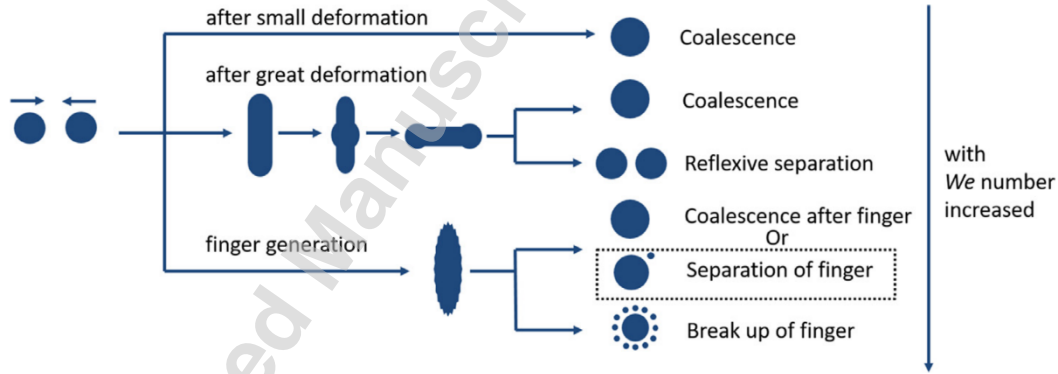
1066 this study (Fig. 21). They systematically examined the influence of variables such as
1067 droplet size, impact velocity, initial film thickness, and magnetic field intensity on these
1068 results. For crown formation, the external horizontal magnetic field altered the crown
1069 shape from symmetric to asymmetric during expansion. In the case of the instantaneous
1070 splash, the magnetic field enhanced the splashing effect and reduced the critical Weber
1071 number required for instantaneous splashing. This research ultimately demonstrated
1072 that the Lorentz force, under the influence of an external horizontal magnetic field, is
1073 the primary factor affecting the splashing behavior of liquid metal droplets.

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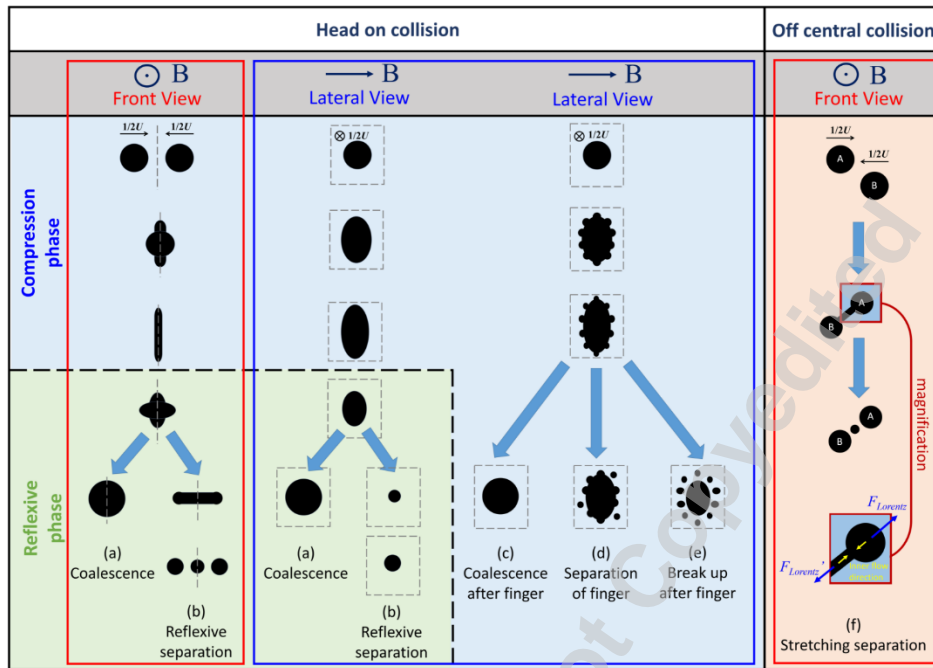
1075 4.4 Collisions between two droplets

1076 Regarding collisions between metal droplets, fewer studies have been carried
1077 out due to the difficulty of controlling the droplets and their extreme susceptibility to
1078 oxidation, especially including the effects of magnetic field. Menchaca-Rocha et al. [186]
1079 conducted experimental investigations on the collision of two mercury droplets in the
1080 absence of a magnetic field. The droplets, falling from two 30° ramps, impacted a
1081 specially treated horizontal glass surface. They proposed a surface-dynamic nuclear
1082 fragmentation model for macroscopic droplet collisions (with diameters up to 5 mm).
1083 However, the initial nonspherical shape of the droplets introduced ambiguity regarding
1084 whether the observed deformation resulted from the initial shape or the collision itself.
1085 Besides, Xia et al. [187, 188] numerically investigated head-on and off center collisions
1086 of alumina droplets at 3387 K, conditions under which experimental studies are
1087 unfeasible due to the extremely high temperatures.

1088 Jia et al. [189, 190] performed comprehensive experiments on the dynamics of
1089 GaInSn droplet collisions without and with the influence of the magnetic field. The
1090 experiments covered a broad range of We numbers from 5 to 400 and impact
1091 parameters (X , $X = x/D$ where x is the distance between two droplet centers before
1092 the moment of impact in the plane perpendicular to the relative velocity vector) from 0
1093 to 1. As shown in Fig. 22, They identified six distinct collision types: coalescence,
1094 stretching separation, coalescence after finger formation, separation of finger, break up
1095 after finger formation, and stretching separation with finger pinching. Our research
1096 revealed that both collision regimes, in the low We number range without finger
1097 generation and in the high We range with finger generation, were observable within a
1098 relatively narrow range ($We < 400$) compared to other ordinary liquids.



1099
1100 Fig. 22. Schematic of collision regimes for head-on collision with We number
1101 increased [189]. Copyright by the Elsevier (2019). Reproduced with permission.



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Fig. 23. Possible collisional process for the large intensity of horizontal magnetic

field ($B > 4T$) (the grey dotted squares in lateral view are the cross-sectional drawing of

grey dotted lines in front view) [190]. Copyright by the APS (2021). Reproduced with

permission.

Additionally, Jia et al. [190] investigated the influence of a horizontal magnetic

field on the binary impact dynamics of GaInSn droplets, with magnetic field intensities

up to 1.5 T and We numbers ranging from 5 to 200. The results indicate that for a small

magnetic interaction parameter ($N < 0.4$), the collision regime known as reflexive

separation is significantly enhanced by the presence of a magnetic field compared to

scenarios without a magnetic field (Fig. 23). For collision regimes other than reflexive

separation, a correlation that is $N \sim f(B/B_0, We/48)$ was proposed. This correlation can

help interpret the minimal influence of the magnetic field on these regimes and predict

significant effects when the magnetic field strength exceeds a critical value of 4 T.

1116 Furthermore, they delineated the potential geometrical morphology of droplets post-
1117 collision, informed by the understanding of the physical processes governing liquid
1118 metal droplet collisions under a horizontal magnetic field.

1119

1120 **5 BUBBLE MOTION IN LIQUID METAL**

1121

1122 To effectively study the influence of magnetic fields on bubbles, it is essential to
1123 start with a single bubble, where the key scientific issues include the need to elucidate
1124 the mechanisms underlying MF-induced changes in bubble dynamics, particularly in
1125 terms of wake modification, velocity modulation, and path transitions. Then, the study
1126 of multiple bubbles introduces additional complexities, such as inter-bubble
1127 interactions, collective behavior, and their influence on flow structures under magnetic
1128 fields. Consequently, this part will draw on insights from both experimental
1129 investigations and numerical simulation studies.

1130 The problem of a single bubble rising in liquid metal due to the effect of
1131 buoyancy is governed by different non-dimensional parameters, which arise from the
1132 governing equations Eqs. (1)~(3). They are the Galilei number, the Bond number, and
1133 the magnetic interaction number, expressed as

$$Ga = \frac{\rho\sqrt{gD}D}{\mu}, \quad Bo = \frac{\rho g D^2}{\sigma}, \quad Ne = \frac{\sigma_e B_0^2 D}{\rho\sqrt{gD}}, \quad (19)$$

1134 where D characterizes the bubble's diameter, and other physical properties all belong to
1135 the liquid metal. Ga describes the ratio of the buoyant force to the viscous force, Bo is
1136 the competition between the buoyant force and the surface tension force, and Ne is the

1137 dimensionless Lorentz force compared to gravity. Besides, the ratio of the physical
1138 properties, i.e., the density, the viscosity, and the electric conductivity, between the gas
1139 and the liquid metal are also important in determining the rising characteristics of the
1140 bubble. However, their values are far smaller than one ($< 10^{-3}$) in the present scenario,
1141 and thus the bubble is treated as a massless, slippery, and insulated phase compared to
1142 the liquid. In addition, if the terminal velocity u_t of the rising bubble is known as a
1143 result, some other non-dimensional parameters can also be defined as

$$\text{Re} = \frac{\rho u_t D}{\mu}, \text{We} = \frac{\rho u_t^2 D}{\sigma}, \text{N} = \frac{\sigma_e B_0^2 D}{\rho u_t}, \text{Ha} = \sqrt{\text{NRe}} \quad (20)$$

1144 which are the Reynolds number, the Weber number, the magnetic interaction number,
1145 and the Hartmann number respectively, with the characteristic velocity \sqrt{gD} in Eq. 19
1146 replaced by u_t . The remaining important dimensionless parameter to characterize the
1147 bubble motion is the Morton number, defined as $\text{Mo} = \mu^4 \rho g / \rho^2 \sigma^3$ which fully relies
1148 on the fluid properties. For the liquid metal, note that the density is much greater than
1149 that of water (six times for liquid GaInSn and thirteen times for liquid mercury) while
1150 their dynamic viscosities are comparable. Essential material properties of liquid GaInSn,
1151 which is usually employed in experiments owing to its liquid status at room
1152 temperature, and the range of corresponding non-dimensional parameters for a $D =$
1153 5mm sized bubble are shown in Table 3, and for comparison this table also contains the
1154 corresponding values for water.

1155 Table 3. Physical properties of the working medium of liquid GaInSn and water, and
 1156 typical values of dimensionless parameters at a temperature of 20°C for a $D = 5\text{mm}$
 1157 sized bubble.

| | Water | GaInSn |
|---|-----------------------|-----------------------|
| Density, $\rho(\text{kg}/\text{m}^3)$ | 998 | 6361 |
| Dynamics viscosity, $\mu(\text{kg}/\text{ms})$ | 9.8×10^{-4} | 2.2×10^{-3} |
| Electric conductivity, $\sigma_e(\Omega^{-1}\text{m}^{-1})$ | 0.1 | 3.27×10^6 |
| Surface tension, $\sigma(\text{N}/\text{m})$ | 0.073 | 0.553 |
| Morton number, Mo | 2.3×10^{-11} | 2.4×10^{-13} |
| Galilei number, Ga | 160 | 340 |
| Bond number, Bo | 3.2 | 2.9 |
| Reynolds number, Re | ≈ 1300 | ≈ 3300 |
| Weber number, We | 4.7 | 2.8 |

1158

1159

1160 5.1 Rising characteristics of single bubble

1161 Depending on the physical properties of the liquids, the rising trajectory of a
 1162 single bubble is probably unstable that it may present a zigzag, a spiral, or even a chaos
 1163 rising behavior [17, 18, 191]. Numerous experimental and numerical studies have been
 1164 carried out to determine the relationships between the wake structures and the path
 1165 instabilities of the rising bubble. Particularly, the spiral path was observed to associate

1166 with a continuous wake composed of double-threaded vortex pairs, while the zigzag
1167 motion was accompanied by the shedding of hairpin vorticities [18, 192-199]. Moreover,
1168 by using different silicon oils, which had a considerable range of viscosity and surface
1169 tension, Zenit et al. [200] found that it was actually the aspect ratio of the deformed
1170 bubble rather than the Reynolds number that triggered the path instability. Table 3
1171 reveals that the terminal Reynolds number of the rising bubble in liquid metal is
1172 significantly higher than that reported in water. As a consequence, the wake structures
1173 and the shape evolution of the bubble are significantly more complex in liquid metal
1174 than those in water, as identified by Zhang and Ni [39]. However, there are numerous
1175 difficulties with experimental measurements of bubble rising in liquid metal. The first
1176 challenge is associated with the opacity of these fluid materials, and thus the optical
1177 measurement techniques specified for ordinary fluid flow hence cannot be used to get
1178 detailed insight into bubble motion. The second problem is the contamination mainly
1179 from oxidation, which is known to have a significant impact on the dynamics of rising
1180 bubble [201, 202], because the rising bubble no longer persists in fully slippery
1181 interfacial condition but behaves more or less like a rigid particle in the presence of
1182 contamination [17]. The last difficulty arises because the liquid metals typically have a
1183 large surface tension compared to the ordinary fluid, e.g. the surface tension of mercury
1184 is 0.487N/m compared to 0.073N/m of water at the room temperature, and thus it is
1185 rather difficult to experimentally control the size of the generated bubble [9, 13].

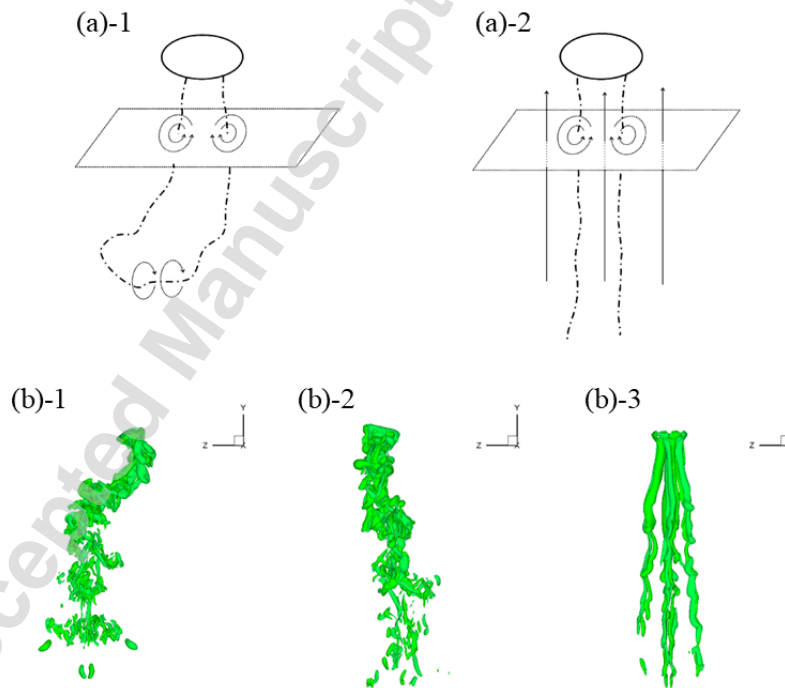
1186 In recent years, different measurement methods have been developed to detect
1187 the bubble path and velocity in liquid metals in the presence of external magnetic fields,

1188 essentially to overcome those opacity-induced problems. The measurement techniques
1189 can be categorized into intrusive and non-intrusive methods. For the first measurement
1190 technique, local conductivity probes [10-12, 203, 204] are used to determine quantities
1191 such as the bubble velocity and the bubble size. However, measurements with any local
1192 probe disturb the flow in a significant way, especially if the structures to be investigated
1193 reach dimensions comparable to the probes. In addition, the Ultrasonic Doppler
1194 Velocimetry (UDV) technique, as a typically non-intrusive method, offers a possibility to
1195 get information about the flow structure and bubble quantities without flow disruption
1196 [9, 205, 206]. UDV overcomes the limitations of traditional probes which can
1197 significantly alter the flow field, however, it has to rely on the occurrence of small
1198 particles moving with the flow so that the velocity of the bubble can be calculated from
1199 the particle position from the time delay between the emitted pulse and the
1200 corresponding echo. Consequently, the contamination of those small particles may
1201 significantly alter the rising dynamics of the bubble. X-ray radiography [207-209], as
1202 another means of non-intrusive approach, has also proven to be an efficient method for
1203 bubble detection in liquid metals that is based on the absorption contrast between the
1204 liquid and gas phase. However, the main limitation of this technique is the thickness of
1205 the fluid volume that can be investigated due to high X-ray absorption coefficients for
1206 liquid metals. The other irradiation method relying on neutron radiography is also
1207 developed, and it allows for thicker measuring volumes than the X-ray radiograph but at
1208 the expense of image contrast [210]. These above-mentioned methods can obtain some
1209 information about the rising bubble from a projected two-dimensional (2D) plane but

1210 lack the resolution and adaptability needed for dynamic three-dimensional (3D)
1211 analyses. Recent advancements have introduced more powerful techniques, with the
1212 Ultrasonic Phased Array (UPA) emerging as a cornerstone in the field [13, 211, 212]. UPA
1213 systems use arrays of ultrasonic transducers to generate and detect high-frequency
1214 sound waves, enabling detailed tracking of bubble motion in opaque and dynamic
1215 environments. They provide real-time 3D visualizations of trajectories, oscillation
1216 modes, and distribution patterns of bubbles under various magnetic field conditions. For
1217 instance, UPA has been successfully applied to study bubble dynamics in gallium indium
1218 tin (GaInSn) alloys, revealing the anisotropic effects of magnetic fields on bubble
1219 trajectories and terminal velocities [13, 14]. Another critical advancement is the
1220 combination of UPA with other methods such as the Ultrasound Transit Time Technique
1221 (UTTT), which provides high-precision measurements of bubble diameters and void
1222 distributions in bubble plumes. UTTT resolves the bubble size with an accuracy of less
1223 than 7%, making it an indispensable tool for studying bubbles in liquid metals [12, 213].
1224 These techniques have expanded the scope and depth of experimental research,
1225 enabling more nuanced explorations of magnetic field effects on bubble behavior.

1226 On the other hand, direct numerical simulation of bubble motion in liquid
1227 metals, even for isolated ones, is challenging due to the large differences in density and
1228 viscosity between the phases and the high bubble Reynolds number typically
1229 encountered, as presented in Table 3. As a result, there are only very few phase-
1230 resolving simulations of bubbles in liquid metal under the influence of an MF. Schwarz
1231 and Fröhlich [32, 213] proposed an Euler-Lagrange approach for the phase-resolving

1232 simulation of the ascent of a single bubble in liquid metal that an immersed boundary
1233 method was implemented to numerically represent the bubble. Such a method enabled
1234 them to investigate the bubble motion exposed to a vertical magnetic field (VMF). In the
1235 framework of the Eulerian system, Zhang and Ni [19, 39] developed a VOF method to
1236 investigate the mechanism of the MHD influence on bubble motion. After that, the
1237 method was further developed into a more conservative form by treating the deforming
1238 bubble interface as an insulated boundary [25]. Utilizing a similar VOF approach, Jin et
1239 al. [20] implemented a ghost fluid method to reduce the spurious flow in the vicinity of
1240 the bubble, and the MHD effect of a horizontal magnetic field (HMF) on the bubble
1241 motion is investigated.



1242
1243 Fig. 24. Variations of vortex structures behind the bubble under vertical
1244 magnetic fields. (a) Experimental sketch, the left is without MF while the right is with MF
1245 [9], Copyright by the Elsevier (2005). Reproduced with permission.; (b) numerical

1246 results, with $N = 0, 0.28$ and 0.87 from left to right [39]. Copyright by the AIP Publishing
1247 (2014). Reproduced with permission.

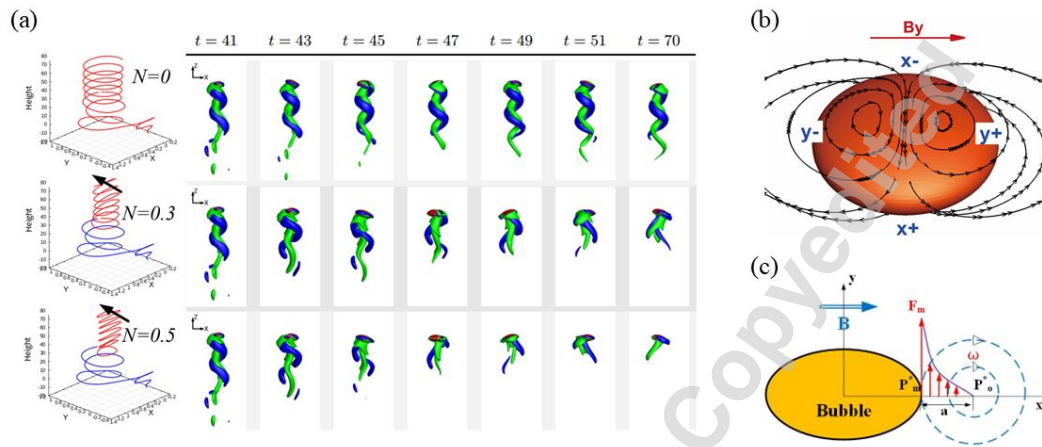
1248 Using these advanced measurement techniques and numerical methods
1249 designed for bubble motion in liquid metals, experimental studies have yielded valuable
1250 insights into how magnetic fields influence the rising characteristics of a single bubble.
1251 Under VMFs, Zhang et al. [9] have experimentally reported diverse impacts of VMFs on
1252 the terminal velocity of the rising bubble, which depends on the bubble size. For small
1253 bubbles ($D \leq 4.6\text{mm}$) a decrease of the terminal velocity with increasing magnetic
1254 interaction parameter N was observed, whereas for larger bubbles ($D \geq 5.4\text{mm}$) the
1255 application of the magnetic fields increased the terminal velocity. They thought it was
1256 because the electromagnetic force would dampen the wake vortices, leading to more
1257 rectilinear bubble trajectories when the magnetic field was applied. Such VMF-induced
1258 influences on wake structure are sketched in Fig. 24(a), where scenarios without and
1259 with magnetic fields are portrayed for comparison. Very recently, the experimental
1260 findings reported by Gou et al. [211] further supported such influences caused by VMFs
1261 using a UPA technique. They observed that the originally zigzagging oscillation of the
1262 bubble was suppressed in a weak VMF ($N < 1$), while the path became an oblique
1263 straight-line in moderate VMF ($N \approx 1$), and meanwhile, the terminal velocity of the
1264 rising bubble firstly increased and then decreased as the field strength was intensified. A
1265 more detailed investigation of the mechanisms caused by VMFs was reported by Zhang
1266 et al. [39] utilizing numerical simulations. They found the Lorentz force interacts with
1267 bubble-induced electric currents, aligning wake vortices parallel to the field, as shown in

1268 Fig. 24(b). This alignment suppresses wake instabilities and dampens oscillatory forces,
1269 leading to more rectilinear and stabilized bubble trajectories, aligning with those
1270 experimental measurements. VMFs also deform bubbles, compressing them along the
1271 vertical axis while elongating them horizontally due to the pressure distribution
1272 modified by the Lorentz force. As the magnetic field strength increases, these effects
1273 intensify, further suppressing turbulence in the wake and reducing the bubble's terminal
1274 velocity. The magnitude of stabilization and deformation is determined by the
1275 interaction parameter, a dimensionless quantity representing the ratio of Lorentz force
1276 to inertial forces, with higher values corresponding to greater magnetic field influence.
1277 Besides, their numerical study also reported a 'secondary path instability' at sufficiently
1278 high VMF strengths, owing to the fact that the field causes a disturbance around the
1279 rectilinearly rising bubble, driving the bubble to rise obliquely. This unexpected scenario
1280 is recently verified by using a linear stability analysis carried out on a moving sphere in
1281 the presence of external VMF [214].

1282 On the other hand, experiments corresponding to HMFs demonstrate that
1283 bubble motion transitions from zigzag or spiral patterns to more rectilinear paths at
1284 moderate field strengths, while weak fields suppress oscillatory behaviors [16, 206,
1285 215]. Additionally, the most impressive scenario induced by HMFs is the anisotropic in
1286 fluence on the rising path of the bubble, and particularly, the oscillatory trajectory
1287 persists in the vertical plane perpendicular to the field while it is dampened quickly in
1288 the vertical plane parallel to the field. Such anisotropic effects were carefully examined
1289 in numerical simulations by Zhang et al. [216, 217] and Jin et al. [20], and the results

1290 reveal that bubble motion becomes anisotropic due to the modification on vortices
1291 structures, as shown in Fig. 25(a). The figure shows that in the absence of the magnetic
1292 field ($N = 0$), the twisted vortex structures in the wake region of the bubble are evident,
1293 corresponding to a spiral motion of the rising bubble. For $N = 0.3$ the twisting patterns
1294 are decreased and the positive (green) and negative (blue) vortices appear to be more
1295 parallel to each other at later times (see $t = 51$ and 70). As the strength of the magnetic
1296 field continues to increase (i.e. for $N = 0.5$), it can be seen that the twisted vortex
1297 structures are fully disintegrated and become mostly parallel due to the anisotropic
1298 effect of the HMF on the vortex structures [218, 219]. In other words, the HMFs tend to
1299 make the flow field more homogeneous along the direction of the HMF while keeping
1300 its structure uninfluenced in the transverse orthogonal direction. In-depth analysis
1301 reveals that it is because of the directional electromagnetic forces induced by the MHD
1302 effects, which can be described by the current density lines around the bubble in Fig.
1303 25(b). Owing to the vertically electromagnetic force in the vertical plane parallel to the
1304 field, as depicted in Fig. 25(c), a Lorentz torque can be produced to offset the pressure
1305 gradient along the field direction, and thus the vorticity strength as well as the velocity
1306 inhomogeneity are reduced correspondingly. Recent experimental studies carried out by
1307 Gou et al. [13], with UPA being applied for accurate exploration of bubble path, revealed
1308 transitions in bubble motion modes from random oscillation (R-mode) to perpendicular
1309 motion (V-mode), parallel motion (P-mode), and finally straight-line motion (S-mode)
1310 with increased HMF intensity. The study also highlighted the correlation between
1311 motion modes and terminal velocities, demonstrating that the suppression of

1312 oscillations leads to velocity increases at moderate field strengths but declines at higher
 1313 intensities due to increased drag.



1314
 1315 Fig. 25. Anisotropic influences of the HMF on the bubble motion [216, 217]. (a)
 1316 Suppression of path oscillation in the vertical plane perpendicular to the HMF and the
 1317 corresponding evolution of the vortices structures; (b) current density lines around the
 1318 bubble; (c) schematic illustration of the competition between Lorentz force and
 1319 pressure gradient, generating vortices at both ends of the bubble. Copyright by the AIP
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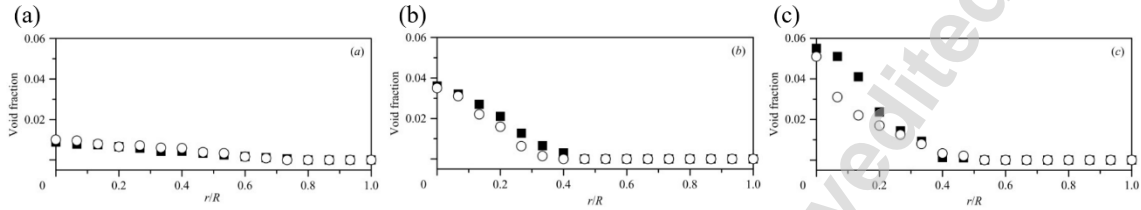
1321

1322 5.2 Rising characteristics of bubble plume

1323 While the dynamics of a single bubble in the presence of external MFs have been
 1324 extensively investigated, the study of multiple bubbles introduces additional
 1325 complexities such as inter-bubble interactions, collective behavior, and their influence
 1326 on turbulent flow structures under magnetic fields [14, 212, 7]. Key scientific questions
 1327 include understanding the mechanisms of inter-bubble interactions and exploring
 1328 anisotropic turbulence effects on bubble behavior. Most of the experimental studies

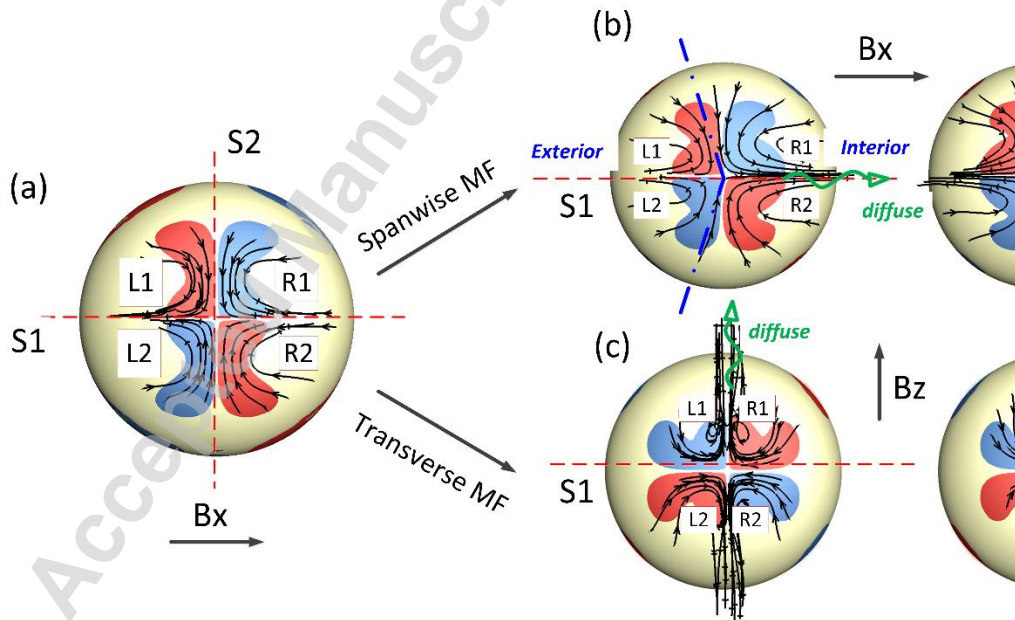
1329 consider the HMFs owing to the restriction of the MF device. Pioneering experimental
1330 studies by Eckert et al. [10, 11] were devoted to the lateral dispersion of a bubble
1331 swarm in a turbulent liquid metal channel flow in an imposed HMF. The gas phase was
1332 injected by a single orifice positioned in the center of the cross-section. An anisotropic
1333 distribution of the void fraction arose from a drastic decrease of the bubble dispersion
1334 coefficient along the MF lines, whereas moderate damping of the mass transfer
1335 properties was observed in the direction perpendicular to the field. The local mass
1336 transfer is governed by the anisotropic character of turbulent MHD flows dominated by
1337 the existence of quasi-two-dimensional vortices at $Ha \gg 1$. Such anisotropic findings
1338 under the influence of the HMFs are consistent with the scenario reported on a single
1339 rising bubble. Later on, Zhang et al. [7] made a quantitative assessment of the bubble
1340 plume behavior in the HMF experimentally. They found that in the conventional case
1341 without a magnetic field gas bubbles emerge isotropically all over the cross-section,
1342 whereas the maximum bubble number was observed at the center. The application of
1343 the magnetic field causes a concentration of the bubbles inside a smaller bounded spot.
1344 The position of this spot is not fixed but shows a low-frequency oscillation along the
1345 diameter of the liquid metal column perpendicular to the direction of the magnetic
1346 field. Besides, the original axisymmetric distribution of bubbles became anisotropic with
1347 respect to the direction of the MF lines, as shown in Fig. 26. An upwards flow dominated
1348 in a plane parallel to the MF, whereas the recirculating motion was enforced in the
1349 orthogonal plane. This finding conforms in principle to the results from their previous
1350 experimental results [10, 11]. In contrast, VMFs primarily elongate wake structures in

1351 the vertical direction, suppressing small-scale turbulence more effectively than HMFs.
 1352 This effect creates distinct isotropies in bubble trajectories, as observed in studies
 1353 where VMFs align bubble paths more strictly along the field lines [11, 212].



1354

1355 Fig. 26. Time-averaged radial distributions of the local void fraction measured by
 1356 a conductance probe at various HMFs [7]. (a) $Ha = 0$; (b) $Ha = 271$; (c) $Ha = 484$, closed
 1357 squares denote the distribution in the vertical plane parallel to the field lines while the
 1358 open circles are the distribution in the orthogonally vertical plane. Copyright by the
 1359 Cambridge University Press (2007). Reproduced with permission.



1360

1361 Fig. 27. Sketch showing how the Lorentz diffusion induced vortices evolve in the
 1362 presence of a neighboring bubble [15]. (a) Single bubble case in a HMF, R1(2) and L1(2)

1363 are the Lorentz diffusion induced streamwise vortices, the black arrow lines are the
1364 velocity streamlines, and the green dashed lines denote the double symmetrical planes
1365 S1 and S2. (b,c) Bubble pair cases under a spanwise MF and a transverse MF,
1366 respectively, and the red arrow lines highlight the diffusion effect along the field lines.
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1368 Numerical simulations offer significant advantages over experimental studies in
1369 analyzing the motion behavior of multiple bubbles in magnetic fluids because they
1370 provide precise control over parameters that are often difficult to manipulate in
1371 experimental setups. However, the interplay between the bubble plume and the
1372 turbulence flows challenges the accuracy of the numerical simulations, especially in
1373 resolving the thin liquid film during bubble-bubble interaction and capturing the very
1374 thin boundary layer and the Kolmogorov scale. Miao et al. [220] implemented an Euler-
1375 Euler multiphase model into the commercial flow solver CFX, while the turbulent flow
1376 was considered by a RANS-CSST turbulence model. Their numerical results reported the
1377 destabilization effect of the global flow due to the HMF, which was consistent with the
1378 experimental observation mentioned above [7]. Depending on the gas flow rate and the
1379 MF strength different transient flow patterns were observed showing strong velocity
1380 oscillations with predominant frequencies. This finding appears to be contradictory with
1381 respect to the classical studies of the MHD duct flow and is a strong indicator that the
1382 relaminarization process of an originally turbulent flow by an applied MF is not fully
1383 understood yet for flow configurations deviating from the standard duct flow. Relevant
1384 numerical investigations were bubble pair interactions under MFs, as reported by Zhang

1385 and Ni [15], and the results show that bubble interaction and vortex formation were
1386 strongly influenced by field strength and orientation. The interaction between a bubble
1387 pair is weakened under VMFs, and a transition from bounce to coalescence is observed.
1388 In the presence of an HMF, its influences are rather direction dependent and
1389 anisotropic: a transverse MF (perpendicular to the bubble-bubble line) always leads the
1390 bouncing bubble pair to coalescence, while a spanwise MF (parallel to the bubble-
1391 bubble line) has more complex impacts depending on the field strength. Under a
1392 spanwise MF, although the flow field is homogenized along the field lines which produce
1393 a smaller attractive force by declining the pressure difference at the two sides of the
1394 bubble, nevertheless, the Lorentz diffusion also produces double streamwise vortex
1395 pairs at the rear of the bubble and the interior pair are stronger in the presence of a
1396 neighboring bubble so that a repulsive lift force is induced. As a consequence, the two
1397 effects compete with each other on varying the magnetic strength, and thus a weak
1398 spanwise MF makes the bubble coalesce while a strong MF causes them to bounce or
1399 even to repel. On the other hand, a transverse MF always promotes the coalescence
1400 between the bubble pair because the Lorentz diffusion induces a pair of vortices at an
1401 interior position to offset the collision induced vortex pair, and thus the collision
1402 between the two bubbles is dampened so that a coalescence is always favored. Such
1403 distinctive effects induced by transverse and spanwise HMFs are sketched in Fig. 27.
1404 Such bubble pair results also bridge the gap between single bubble motion and bubbly
1405 plume in MHD flow, they explain the experimental scenarios why VMFs promote the

1406 coalescence between bubbles and HMFs produce anisotropic distribution between the
1407 bubbles.

1408

1409 **6 CONCLUSION**

1410

1411 6.1 Summary

1412 In the present review, we focus on the interfacial flow of liquid metal with high
1413 surface tension related to the PFCs in the Tokamak system and electrometallurgy.
1414 Recent progress on the corresponding liquid metal free surface flow, droplet
1415 impingement, and bubble motions in liquid metal under the influence of a magnetic
1416 field are summarized. According to the MHD theory, the anisotropy of the magnetic
1417 field significantly influences the dynamics, transport properties, wave behavior, and
1418 energy dissipation rates in liquid metal flow, impacting their stability and efficiency in
1419 various applications. When the magnetic field orientation varies in angle relative to the
1420 flow direction, its impact on interfacial flow is distinctly different.

1421 *MHD effects of free surface liquid metal flow.* For the free surface liquid metal
1422 flow, the stability of the flow is easily affected by the wettability of solid walls,
1423 temperature difference, and various disturbances at the interface. The introduction of
1424 electromagnetic forces by the presence of a magnetic field makes the flow more
1425 complex. The magnetic field has a generally stabilizing effect on the free surface liquid
1426 metal flow by changing the 3D surface waves to Q2D surface waves and ultimately
1427 removing all surface waves at a very strong magnetic field. Meanwhile, due to the
1428 presence of MHD drag, the thickness of the liquid film increases dramatically with the

1429 increase of Stuart number, N , showing a linear relationship. Results also indicated that
1430 by introducing the external current, temperature differences, and changing the electric
1431 conductivities of walls, it is possible to find a good solution to drive the flow, reduce the
1432 MHD drag, and therefore control the free surface of liquid metal. Till now, many kinds of
1433 liquid metal PFCs are under testing.

1434 *MHD effects of liquid metal droplet dynamics.* When the direction of magnetic
1435 field lines aligns with the direction of droplet impact, the magnetic field inhibits the
1436 radial spreading of the metal droplet, characterized by a scaling law relationship for the
1437 maximum spreading coefficient, $\beta_{max} \propto N^{-1/2} f_c(\beta_0^2 N)$. However, when the direction of
1438 the magnetic field is perpendicular to the direction of droplet impact, the spreading of
1439 the droplet exhibits anisotropy. Specifically, the spreading along the direction of the
1440 magnetic field lines is compressed while it is stretched in the perpendicular direction.
1441 Further research has also established a relationship between the average maximum
1442 spreading coefficient and the strength of the magnetic field.

1443 *Bubble dynamics.* The MHD influence on the rising dynamics of the bubble
1444 motion significantly depends on the MFs strength and orientation. Either the VFM or
1445 HMF tends to homogenize the flows along the field lines, hence the vorticity magnitude
1446 and the path oscillation inclined to that direction are reduced correspondingly, leading
1447 to a larger terminal rising velocity at weak to-moderate MF but a smaller terminal rising
1448 velocity at moderate-to-strong MF. Owing to such influence on the vortice structures,
1449 the distinctive distribution of volume fraction can also be expected and understood for
1450 bubbly plumes under different MFs.

1451 6.2 Open questions

1452 *There is still a significant amount of work to be carried out for the free surface*
1453 *liquid metal flow from both engineering and fundamental aspects to promote the*
1454 *realization of liquid metal PFCs.* Since the magnetic field lines in the Tokamak system are
1455 not in a single direction but comprise multiple components, future research could
1456 explore the stability of liquid metal free surface flows under complex magnetic field
1457 conditions. When high-temperature plasma interacts with the free surface of liquid
1458 metal, the induced surface pressure and significant temperature differences can cause
1459 extreme instabilities on the liquid metal surface, leading to phenomena such as liquid
1460 metal evaporation and splashing. The stability of free surface liquid metal flows under
1461 coupled magnetic and thermal conditions will be a particularly challenging area of
1462 research in the future. From the aspect of engineering applications of liquid metal PFCs,
1463 more efforts should be put into achieving stable liquid metal film flow by improving the
1464 wetting of liquid metal on solid surfaces since most experimental results showed that
1465 liquid metals exhibit hydrophobic characteristics on solid surfaces, with corresponding
1466 contact angles well over 90°. To overcome the MHD drag, which may stop the liquid
1467 metal flow, suitable insulating material should be used to produce the PFC.

1468 *The understanding of the MHD effects in realistically complex systems of the*
1469 *bubbly plume is far from complete.* Our understanding of the MHD effects on bubbly
1470 flows is largely limited to flows with a moderate number of bubbles, while real liquid
1471 metal flows in fusion reactors and metallurgic engineering are much more complex that
1472 the number of bubbles is nearly $O(10^6)$ or even more. More realistic and efficient

1473 numerical methods should be developed to accurately simulate the bubbly flows under
1474 the influence of the MFs, to provide more insights into how the bubbles' motion
1475 interacts with the MHD turbulence.

1476

1477 **ACKNOWLEDGMENT**

1478

1479 We would like to thank Dr. Jiandong Zhou, Dr. Xiaotong Ma and PhD student
1480 Linling Li of Xi'an Jiaotong University for their help in collecting materials for the review.

1481 **FUNDING**

1482 National Science Foundation of China with grants 51927812, 5222260 and
1483 12222208.

1484 National Key Research and Development Program of China with grant
1485 2022YFE03130000.

1486 The Strategic Priority Research Program of Chinese Academy of Sciences with
1487 grant XDB0790103.

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1490 **NOMENCLATURE**

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| | |
|------------|--|
| ρ | density of the immiscible fluids |
| μ | dynamic viscosity of the immiscible fluids |
| ν | kinematic viscosity of the fluid |
| F_l | Lorentz force |
| F_s | surface tension force |
| J | the induced current density |
| B | magnetic field |
| B_0 | external magnetic field (constant), T |
| B_{0x} | external streamwise magnetic field |
| μ_m | magnetic permeability |
| η | magnetic diffusivity of the fluid |
| σ_e | electrical conductivity |
| ρ_e | electrical resistivity |
| σ | surface tension coefficient |
| u | fluid's velocity |
| U | characteristic fluid velocity |
| φ | induced electric potential |
| κ | local curvature of the interface |

| | |
|---------------|---|
| k_r | real part of the wave number |
| \mathbf{n} | normal direction of the interface |
| δ_s | Dirac distribution function |
| \mathcal{S} | remaining source term in Eq. (1) |
| p | pressure |
| x_I^k | the position of the interface in the k -th grid |
| x_I | the position of the interface |
| \mathcal{F} | volume fraction the continuous or disperse phase |
| K | wave number in the flow direction |
| N_T | Nusselt film thickness |
| Re | Reynolds number |
| M | is the same as Hartmann number |
| Ha | Hartmann number |
| N | Stuart number |
| N^* | is defined as Ha^2/Re^2 |
| Fr | Froude number |
| We | Weber number |
| Ga | Galilei number |
| Bo | Bond number |

| | |
|----------------|--|
| Ne | dimensionless Lorentz force compared to gravity |
| Mo | Morton number |
| N_{cr} | critical Stuart number |
| P_m | magnetic Prandtl number |
| Re_m | magnetic Reynolds number |
| g | gravity acceleration |
| α | dimensionless parameter equals to $\sigma_e B^2 / \rho \omega$ |
| β | Inclination angle of free surface flow |
| ω | frequency of the wave |
| k | wave number |
| h | the thickness of liquid |
| h_{av} | average thickness under magnetic field |
| h_0 | average thickness without the magnetic field |
| γ | the ratio of surface tension coefficient to density of liquid |
| S_{ω}^c | power spectral density of capillary wave elevation |
| S_{ω}^g | power spectral density of gravity wave elevation |
| P | energy flux |
| \tilde{u} | velocity perturbation |
| u_t | terminal velocity during the bubble rising |

| | |
|------------------|---|
| δ_{en} | relative film thickening |
| Q | flow rate |
| w | width of the channel |
| $\theta_{a,max}$ | maximum advanced contact angle |
| $\theta_{a,eq}$ | equilibrium advanced contact angle |
| $\theta_{r,min}$ | minimum receded contact angle |
| $\theta_{r,eq}$ | equilibrium receded contact angle |
| D | diameter of a droplet |
| β_m | maximum spreading factor during the droplet spreading |
| κ_m | aspect ratio of the maximum spreading factor |
| t_{Dmax} | time at maximum spreading |
| X | impact parameters of two droplets |

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