Theory of Flow Access With Apparent Obstacles: Cascades, Jumps, Roll Waves, and Turbulence

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This article addresses the fundamental question of whether a flow system that has freedom to morph also exhibits the natural tendency to configure itself to provide greater access to its currents. The reason for questioning this evolutionary tendency in nature is that there are several flow architectures that seem to contradict the tendency. First, the flow of rain water on an inclined street covered with gravel does not carve a smooth channel with uniform depth and width. Instead, it carves a straight path with cataracts, which are periodic "dams" interspaced with pools and waterfalls. The pools are most evident after the rain. Second, sheet flow that is smooth and rectilinear terminates itself into a "wall" posed by a thicker layer of fluid that flows more slowly. This is known as a "hydraulic jump." Are these abrupt morphological changes hindering or liberating the flow? By questioning the directionality of these changes we have an excellent opportunity to check and validate the constructal law of evolutionary design toward greater access.
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1 Introduction

In this article I question of whether a flow system that has freedom to morph also exhibits the natural tendency to configure itself to provide greater access to its currents. In other words, I test the validity of the constructal law. There is considerable empirical evidence in support of the natural tendency toward greater flow access. Examples are reviewed briefly in the next section.

The reason for questioning this evolutionary tendency is that there are flow architectures that seem to contradict the tendency. Three examples are mounted together in Fig. 1, and they are highly familiar.

First, the flow of rain water on an inclined street covered with gravel does not carve a smooth channel with uniform depth and width. Instead, it carves a straight path with cataracts, which are periodic "dams" interspaced with pools followed by "cascades." The pools are most evident after the rain (Fig. 1(*a*), right).

Second, a sheet flow that is smooth and rectilinear terminates itself into a "wall" posed by a thicker layer of fluid that flows more slowly. This is known as a "hydraulic jump," Fig. 1(*b*). It is considered classic and understood.

Third, the sheet of rain water that slides on smooth pavement exhibits periodic water falls. The flow that precedes the fall is inclined (it is not a pool), Fig. 1(*c*). The phenomenon is known by several names (roll waves, ripples, ridges).

In sum, are these abrupt morphological changes hindering or liberating the flow? I do not know, but I want to know. By questioning the directionality of these changes we have an excellent opportunity to check the validity of the constructal law of evolutionary design toward greater access. For now, the three examples of Fig. 1 give the impression that the falls and the jumps are obstructing the flows, like the fences and the water trap in the 3000 m steeplechase run in track and field.

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2 Background

The straight line is the shortest path between two points, and the straight duct with round cross section poses minimum resistance to flow. These truths are known scientifically since antiquity [1,2], and arguably are much older. Today, we know them intuitively even when we encounter them in homework problems of fluid dynamics.

Why should the intuitive truths be questioned? Here are three reasons:

The first is a general trend in science: what is intuitively correct is taken for granted (i.e., understood), and so it escapes scrutiny.

The second is that the straight path and the round cross section are about flow *architecture*. The science of configuration, or design, is a relatively new domain that hovers over many of the sciences (geophysics, physiology, engineering, urbanism). With the advent of fire, boating, wind power, steam power, and aviation, the science of flow architecture covers fluid dynamics as well [3–5].

Third, in scientific thought the straight line and the round cross section emerge along deductive sequences of imagined *changes* in configuration. The changes are oriented with purpose (with directionality) toward providing easier access from point to point. Observable changes with discernible direction in time are the phenomenon of *evolution*, or evolutionary design. Evolution is the phenomenon that unites everything that is animate and inanimate.

The three reasons escape scrutiny because they are known, invoked, and accepted intuitively. They should not be. Evolutionary design is the mother of all engineering science, innovation, and evolution. With Fig. 2, for example, we showed that the number of possible flow architectures that connect two points is infinite [6]. With direction in the changes that we contemplated, we eliminated one class of architecture at a time, in favor of better classes.

Specifically, the physics phenomenon of "economies of scale" [7] is why two ducts in parallel are inferior to a single duct with the same volume as the two ducts combined. Next, the many curved ducts between the two points are inferior (as a class) to the straight duct. The many cross sections that have sharp corners are inferior (as a class) to those that do not have sharp corners.

The numerous polygonal cross sections are inferior to the few that are shaped as regular polygons. This last stage of evolution

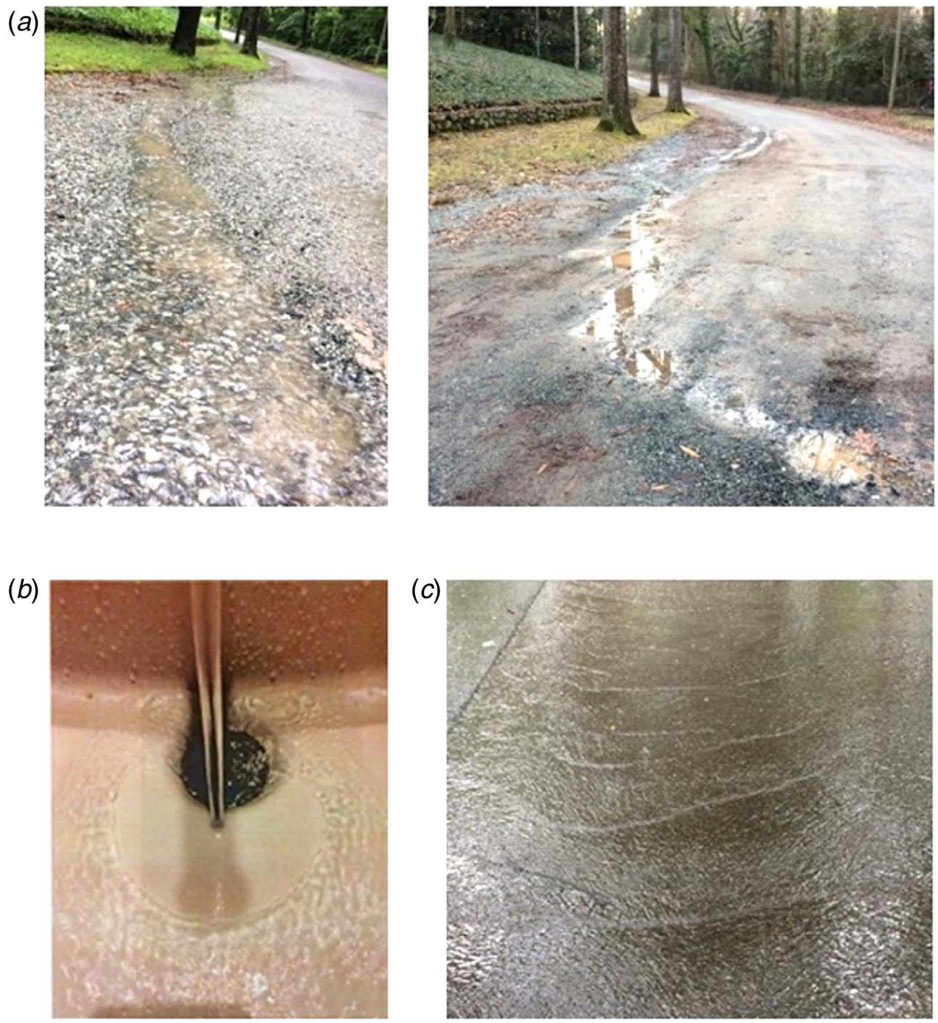

Fig. 1 Flows with apparent obstacles: (*a*) cascades or cataracts, (*b*) hydraulic jumps, and (*c*) roll waves or ripples

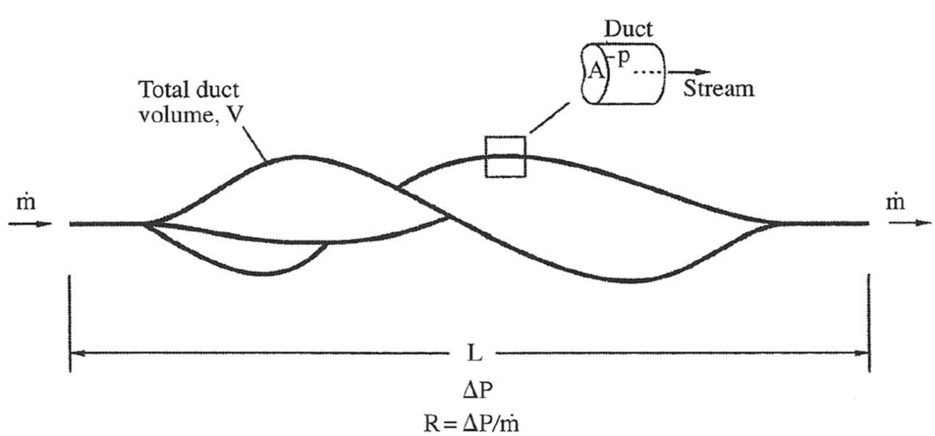

Fig. 2 General architecture for the flow between two points

by elimination is shown in Fig. 3 [7]. The flow resistance between the two points is indicated qualitatively on the abscissa (for details see Ref. [6]). On the ordinate is plotted the number of sides (n) of the regular polygon, such that a larger n value accounts for greater freedom in the mind and hand of the person who contemplates and draws the design.

"Flow access" is defined by the arrow on the abscissa of Fig. 3. The discernible direction of change is indicated with two arrows, to the left for greater flow access, and downward with greater freedom. The world of point-to-point flow designs is divided into two worlds, the possible designs versus the impossible designs. The apparent frontier between the two is populated by the regular polygonal cross sections. These are the most valuable, and on that frontier is located the key milestone, the round cross section ($n = \infty$).

The evolving flow architecture is illustrated on the horizontal line at $n = 6$. Hexagonal shapes become better as they migrate to the left,

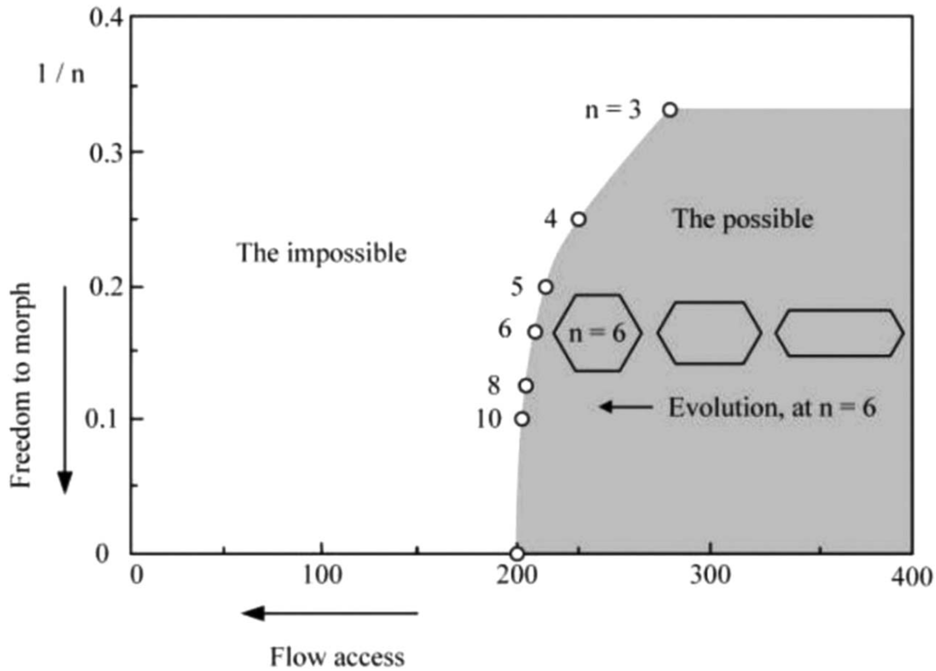

Fig. 3 The evolution of cross-sectional shape toward greater flow access

toward the regular hexagon. The world of possible designs is to the right of the frontier. The effect of the increasing freedom in evolutionary change is to extend the domain of the possible into the domain of the impossible, if possible.

The evidence that the design evolution phenomenon illustrated in Figs. 2 and 3 is universal in nature is massive. Our group is one of several that have been reviewing the evidence, bio, and nonbio [8–21]. If design in nature is this way, then why is it that not all the point-to-point flow designs have evolved to be straight ducts with round cross sections? We questioned and answered this several times, class by class [6]:

(a) River cross sections are lemon-slice shaped, not round. In the simplest, a river cross section has a free surface and two scales, the width (w) of the free surface, and the maximum depth (d). Empirically, it is known that large rivers are wide and deep, and tiny channels are narrow and shallow. Measurements of river channels of all sizes revealed that w/d is essentially a constant in the range of 2–10 [22]. In other words, measurements show that the river cross section has one scale (w or d). We showed that the proportionality between width and depth emerges in the direction of increasing flow access through the cross section (of size $w \times d$) by varying w (or d) subject to fixed cross-sectional area. Four designs are aligned with direction in Fig. 4. Their relative flow resistance is indicated under each drawing (relative to the semicircle, which is represented by 1).

(b) River channels are not straight. They exhibit meanders, which are quasi-sinusoidal shapes with wavelengths (λ) that scale with the stream width (w). The empirical data on the scaling $\lambda \sim w$ were reviewed in Ref. [23]. The average ratio λ/w ranges from 2 to 3 in laboratory studies to 6.5–11 in natural channels with long history. Two explanations for the deviation from the straight path have been proposed, and both account for the fact that the channel flow is relatively inviscid (high Re), and the medium in which it evolves is deformable, erodible. One is the theory of incipient hydrodynamic instability [24–39]. The other is the incipient buckling of the river column in apparent "longitudinal compression" due to the impulse of the stream entering the column and the reaction of the same stream as it exits the column [40–46]. Meanders occur naturally in many other deformable channels (e.g., blood vessels, hoses) and are covered by the same theory [44].

(c) River basins, deltas, lungs, snowflakes, and all vascular systems are dendritic (tree shaped), not straight channels from one point to another point. The reason is that every dendritic flow architecture connects one point with an infinity of points (area or volume), not with another point. Our group has shown on multiple occasions how to predict tree shaped flows (bio, nonbio) by invoking the evolution of the architecture toward greater access point-area and point-volume. This voluminous work is reviewed in books and review articles [13–22].

Figures 1(*a*)–1(*c*) are just three classes of flow architectures from nature. They are not human made. Many more examples of flow architectures from nature owe their evolving design to the same principle: to facilitate flow access by morphing with freedom. For example, our group has shown how to predict temporal design

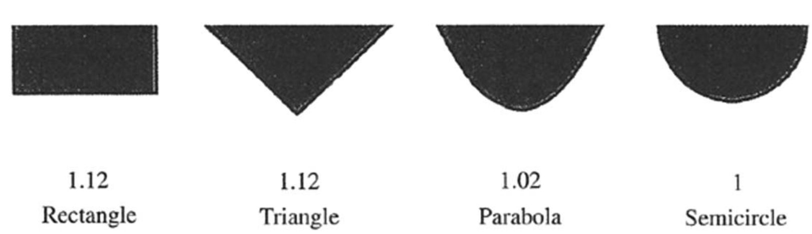

Fig. 4 River channel cross sections for flow access: the proportionality between width and depth

(rhythm) in animal locomotion (swimming, running, flying) [47], and respiration and blood circulation (heart beat) [48]. We showed that even greater opportunities for predicting and fast-forwarding flow design evolution exist in technology: the convergent evolution of the shapes of airplanes [49], helicopters [50], boats with sails [51], fire [52], and wheels [53].

On this diverse background, what is that remains to be elucidated? Three puzzles are identified in the next sections. They seem to go against the tendency toward facilitating flow access via evolutionary design. Cataracts, hydraulic jumps, and ripples seem to "snuf" the flow and contradict the smooth (straight) design that is expected from the textbook.

3 Cataracts Versus Hydraulic Jumps

Cataracts and hydraulic jumps are two of the most common phenomena in nature, Figs. 1(*a*) and 1(*b*). They are different, obviously: the abrupt fall of the stream cannot be confused with the abrupt jump of another stream. Examined together, cataracts and hydraulic jumps have in common the pull of gravity. Cataracts go along gravity, and jumps go against gravity. They also share the impression given to the observer, which is that the abrupt changes "interfere" with the easy flowing of the straight channel expected by the scientist.

Are these differences and impressions correct? Two recent and unconnected publications challenged me to answer this question:

In 2019, Scheingross et al. [54] conducted laboratory experiments with water and debris in open channel flow and showed that cataracts happen naturally. They exhibit pattern, periodicity, and reproducibility. They are not due to pre-existing forms of relief (rocks, seismic faults).

In 2018, Bhagat et al. [55] showed that in a thin liquid film the hydraulic jump does not depend on the orientation of the film with respect to gravity. They showed that at the jump the viscous forces and surface tension balance the momentum change in the flowing film.

Can cataracts and their periodicity be predicted? The most recent study of this phenomenon [54] demonstrated through extensive laboratory visualization and measurements that cataracts form by themselves, and that the mechanism is the removal and deposition of debris [56]. This is in accord with the movement of gravel (Fig. 1(*a*)). The evolutionary design literature reviewed in the preceding section includes the erosion (smoothing) of rolling stones into spheres [57] and the phenomenon of self-lubrication [5], as a more general term for erosion and the tendency of channels to shape themselves to provide greater access to what flows in them. Self-lubrication owes its name to human made systems, such as a rod sliding longitudinally or rotating in its housing, with a film of oil between rod and housing. The rod is free to adjust its position (its center), and this is synonymous to the freedom of the oil film to morph its thickness and shape.

4 Cataracts

To address the question of flow access in the case of cataracts, consider the water flow in a straight river channel that forms the angle β with the horizontal line (Fig. 5). Most rivers flow this way, and β is small, only a few degrees. The geographical distance traveled is L, the river bed length is $L_\beta \cong L$, and the mean water velocity in the inclined channel is V_β.

Figure 5 shows a stepped alternative to the β-inclined path. The long stretch of the river bed is close to the horizontal, i.e., close to being a pool. This is followed by a drop of height z along which the water is in free fall. Along the stepped path ACB the water starts from A and ends at B, just like along the incline.

Which alternative offers greater access to the water flow? In the following analysis I compare the flow time on the path ACB with the straight path AB. This comparison requires assumptions about the Reynolds number regime for the channel flow AB, and the condition of the riverbed. The question about greater flow access should lead to a predictable choice between the two paths, which should shed light on the periodicity of steps when steps provide greater access.

Assume that along the L_β channel the flow is fully turbulent in the fully rough regime. The column of water in the β channel is pulled downstream by the force $(g \sin\beta)\rho AL$, where ρ is the water density and A is the area of the cross section of the stream. This force is balanced by the skin friction integrated over the wetted surface, τpL, where p is the wetted perimeter of the channel cross section A, and τ is the bed shear stress averaged over the entire wetted surface. In the fully turbulent and fully rough regime, the shear stress is [18]

$$\tau = C_f \frac{1}{2}\rho V^2 \tag{1}$$

where the overall skin friction coefficient is independent of Reynolds number and has values in the range of 0.01–0.1 depending on the roughness of the river bed. From the balance of forces on the β-inclined column we determine the water velocity along the channel as follows:

$$V_\beta = \left(\frac{2}{C_f}\frac{A}{p} g \sin\beta\right)^{1/2} \tag{2}$$

The time needed by one water packet to travel from A to B is as follows:

$$t_\beta = \frac{L}{V_\beta} \tag{3}$$

Consider next the stepped route in the limit where the L-long flow is horizontal. In this case, the water travel time is governed by the water that falls freely to the depth z:

$$t_z = \left(\frac{2z}{g}\right)^{1/2} \tag{4}$$

To compare the free fall time with the travel time along the β route, we calculate the ratio as follows:

$$R = \frac{t_z}{t_\beta} \tag{5}$$

Contrary to the impression left by Fig. 5, a travel time from A to C is not added to t_z because the water packets that fall over the dam at C are not the packets that fell to A. What falls over the dam is the pool

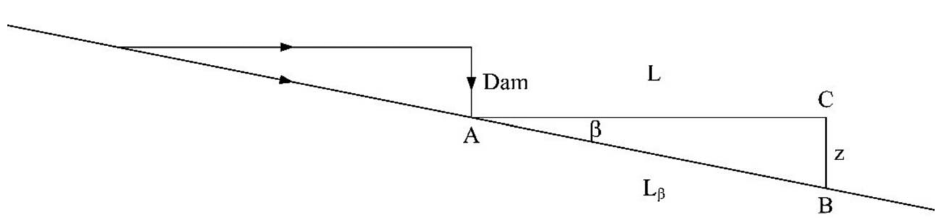

Fig. 5 Flow with intermittent drops along an incline

water. If the dam is tall, the level of the pool would rise approximately uniformly at the rate dictated by the flowrate arriving at A. After using Eqs. (2)–(4) and the small-β approximations ($\sin\beta \cong \beta$ and $z \cong L\beta$), the time ratio becomes

$$R = \left(\frac{4\beta^2 A}{C_f L p}\right)^{1/2} \tag{6}$$

Big rivers are wide and deep, and small rivers are narrow and shallow. The approximate proportionality between width (w) and maximum depth (d) is supported by numerous measurements (Sec. 2(a)) and can be written as follows:

$$\frac{w}{d} = C_{10} \tag{7}$$

The proportionality factor C_{10} is 10 or smaller. As noted earlier, the existence of the proportionality between w and d is in accord with the physics of evolution toward greater flow access in river basins and deltas [21,22]. The shape of the cross section resembles a shallow circular segment, such that $A \geq 1/2\, w\, d$ and $p \geq w$. The ratio A/p is then approximately $1/2\ d$, and in view of Eq. (7),

$$\frac{A}{p} \sim \frac{w}{2C_{10}} \tag{8}$$

from which R takes the form

$$R = \left(\frac{2\beta^2}{C_f C_{10}} \frac{w}{L}\right)^{1/2} \tag{9}$$

If $R < 1$, the flowing river has greater access in the configuration where nearly horizontal stretches are interspaced with waterfalls. The condition for the periodic formation of the stepped path translates into

$$L > \frac{2\beta^2}{C_f C_{10}} w \tag{10}$$

where $2/(C_f C_{10})$ is a constant in the range of 10–100. The qualitative conclusion from Eq. (10) is that when the average inclination (β) of the terrain increases, the distance between successive drops should increase. When the inclination is gentle (small β) the density of steps should be greater. Furthermore, big rivers (large w) should have sparse drops, and rivulets (small w) should have frequent drops. This is in qualitative agreement with observations in nature.

5 Hydraulic Jumps

Figure 6 shows another common observation of natural fluid flow behavior. When the water jet from the faucet impinges on the sink bottom, it first spreads smoothly as a wall jet. Suddenly, the wall jet forms a thicker layer that continues to flow radially, with rolls and bulges that indicate a three-dimensional flow (turbulent flow), not a laminar radial flow. This phenomenon is known as a "jump" based on the established view [58] that at the transition from radial flow to three-dimensional flow the water jumps against gravity and acquires an amount of gravitational potential energy that the original radial jet did not have.

The following analysis goes against the established view: we analyze the radial (r) jet flow in the absence of gravity. The mass flowrate and velocity of the faucet jet ($\dot{m}$, V) are fixed. The analysis proceeds in three steps based on scale analysis:

(i) Consider the initial (smooth, radial) wall jet and determine the scales of its thickness (δ) and radial velocity (u). Model the flow as inertial, i.e., frictionless.
(ii) Model the radial wall jet as viscid, with viscous diffusion having penetrated the δ thickness (as in laminar boundary layer flow), and then determine the corresponding δ and u scales of this flow regime.
(iii) Intersect the two asymptotes determined earlier and deduce the radial distance where the inertial jet ceases to be inviscid.

The liquid jet of mass flowrate $\dot{m}$ and speed V arrives perpendicularly on a plane wall. The liquid spreads radially on the wall as a film (or wall jet) of thickness scale δ and radial velocity scale u. The rate (the flow) of kinetic energy that the jet transfers to the radial film flow is $\dot{m}V^2/2$ and has the scale $\dot{k} \sim \dot{m}V^2$. We distinguish two flow regimes:

(a) *Inertial flow*. Close to the spot impacted by the V jet, the effect of friction (viscous diffusion) posed by the wall is negligible. The flow of kinetic energy, from perpendicular to radial, is conserved

$$\dot{m}\,u^2 \sim \dot{k}, \quad \text{constant} \tag{11}$$

and so is the mass flowrate,

$$\dot{m} \sim \rho\, u\, \delta\, 2\pi r \sim \rho\, u\, \delta\, r \tag{12}$$

Note the omission of the factor 2π, in accord with the rules of scale analysis. Because $\dot{m}$ and $\dot{k}$ are fixed by the size and power of the jet, the radial velocity scale is independent of radial position

$$u \sim \left(\frac{\dot{k}}{\dot{m}}\right)^{1/2} \sim V, \quad \text{constant} \tag{13}$$

From Eqs. (11)–(13) follows the thickness of the radial film

$$\delta \sim \frac{C_1}{r} \quad \text{where } C_1 = \frac{\dot{m}^{3/2}}{\rho \dot{k}^{1/2}} = \frac{\dot{m}}{\rho V} \tag{14}$$

The inertial regime does not spread indefinitely in the radial direction. From the beginning (near $r = 0$), a viscous boundary layer (BL) grows between the radial flow and the solid wall. The thickness of the boundary layer grows in time

$$\delta_{BL} \sim (\nu t)^{1/2} \tag{15}$$

where t is the time scale clocked by the film flow as it travels radially

$$t \sim \frac{r}{V} \tag{16}$$

therefore

$$\delta_{BL} \sim \left(\nu \frac{r}{V}\right)^{1/2} \tag{17}$$

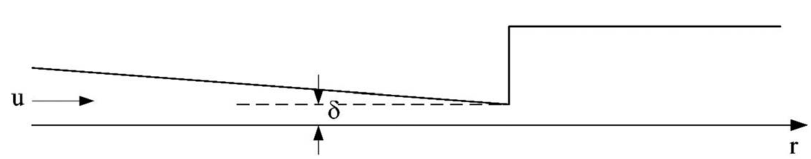

Fig. 6 Flow with sudden thickening along a plane wall

The inertial regime ends when the radial film is penetrated fully by viscous diffusion

$$\delta_{BL} \sim \delta \tag{18}$$

Setting Eqs. (14) and (17) equal, we obtain the scale of the radial distance beyond which the film flow is no longer in the inertial regime:

$$r \sim \left(\frac{\dot{m}^2}{\rho\mu V}\right)^{1/3} \tag{19}$$

(b) *Viscous flow*. Next, make the opposite of the assumption made at (*a*). Assume that from the beginning the radial flow is slowed down by viscous dissipation. We make this assumption in order to contrast regime (*b*) with regime (*a*), so that later we can intersect the asymptotes (*a*) and (*b*) to determine the radial location of the transition from (*a*) to (*b*). The method of intersecting the asymptotes was employed many times in constructal theories that predict flow configurations and transitions [18].

In the viscous regime, the radial velocity changes from u at r, to $u + du$ at $r + dr$. Consider the forces acting on the infinitesimal control volume $\delta\, dr \Delta\theta$, where $\Delta\theta$ is the small angle of the pencil of rays that point from the center ($r = 0$) through the infinitesimal control volume. Oriented radially toward the center is the wall friction force

$$dF \sim \tau\, dr\, 2\pi\, r\, \Delta\theta \sim \tau\, dr\, r\, \Delta\theta \tag{20}$$

where

$$\tau \sim \mu \frac{u}{\delta} \tag{21}$$

In addition, oriented radially toward the center is the reaction F_R due to the flow exiting from the control volume

$$dF_R \sim \rho u\, \delta\, 2\pi\, r\, \Delta\theta\, du \sim \rho u\, \delta\, r\, \Delta\theta\, du \tag{22}$$

In steady state, the balance of forces is between dF and (–dF_R), showing that du must be negative, which means that the flow slows down in the radial direction. From this balance and mass conservation follows

$$-\frac{du}{u^2} \sim \frac{\rho\mu}{\dot{m}^2} r^2\, dr \tag{23}$$

Integrated, this equation yields

$$\frac{1}{u} \sim \frac{\rho\mu r^3}{\dot{m}^2} + C_2 \tag{24}$$

where we set $C_2 = 0$ such that $u \to \infty$ as $r \to 0$. Finally, by substituting u from Eq. (24) into Eq. (12), we obtain

$$\delta \sim \frac{\mu r^2}{\dot{m}} \tag{25}$$

The conclusion is that in the viscous regime the film flow slows down, and the film becomes thicker rather rapidly in the r direction. This trend is the opposite of what happens in the inertial regime. Intersecting Eqs. (14) and (25),

$$\frac{\dot{m}}{\rho V r} \sim \frac{\mu r^2}{\dot{m}} \tag{26}$$

we obtain the radial distance that marks the transition between the assumed regimes, (*a*) and (*b*),

$$r \sim \left(\frac{\dot{m}^2}{\rho\mu V}\right)^{1/3} \tag{27}$$

In what direction should the change occur, from (*a*) to (*b*), or from (*b*) to (*a*)? According to the tendency toward greater flow access (constructal law), the change should be toward greater access for momentum vertically, into the plate, from the high momentum (movement) of the arriving jet to the slow movement of the liquid that comes to rest on the plate. In other words, the flow that changes configuration toward greater access is the momentum transfer across the water film, not the radial flow of the liquid (which, by the way, does not change configuration because it is radial in both configurations, (*a*) and (*b*)).

The answer then is that the change should be from (*a*) to (*b*), because in this direction the liquid film is consistently the thickest. In this direction of change, more of the liquid is slowed down by the wall.

Guided by Fig. 1(*b*), if the impinging jet flow is water with $\dot{m} \sim 0.01$ kg/s, $V \sim 1$ m/s, $\rho = 1000$ kg/m^3, and $\mu = 0.01$ g/(s · cm), then Eq. (27) yields $r \sim 6$ cm, which is comparable with the radial distance to the hydraulic jump observed in Fig. 1(*b*).

Why does the sudden jump happen at the transition between the two regimes? Because of gravity, or because of the constructal law? To find the answer, another approach is to invoke the constructal law, according to which the flow (the wall jet) begins to roll when its local Reynolds number reaches the order of 10^2 [18]

$$\frac{u\,\delta}{\nu} \sim 10^2 \tag{28}$$

For the scales of u and δ we use Eqs. (24) and (25); the r result from Eq. (27) is of order 10 cm, which confirms observations and the previous estimate. In conclusion, the thickening of the radial sheet marks the change from laminar flow to turbulent flow. We explore the wider implications of this conclusion in Sec. 7.

6 Roll Waves

Extensive measurements on roll waves in the laboratory are available in Ref. [59]. Here, the analysis for roll waves is based on the model shown in Fig. 7. We assume that β is small such that $L_\beta \cong L$ and $z \cong \beta L$. The instantaneous volume of water is $zL/2$, or $\beta L^2/2$. The mass of the water in the pool is $\rho\beta L^2/2$, and it is due to the stream $\dot{m}'$ that fed the pool during the time t,

$$\dot{m}'\, t = \frac{1}{2}\rho\beta L^2 \tag{29}$$

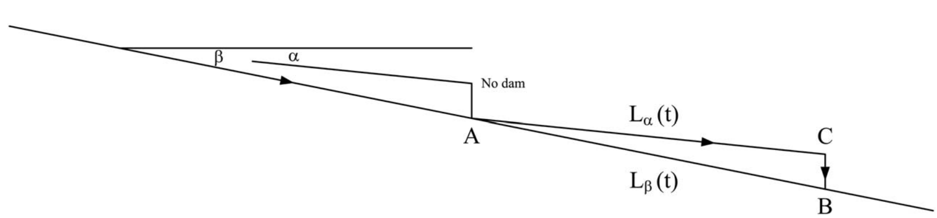

Fig. 7 Roll waves down an incline

From this we find that the dimensions of the pool increase in time as $t^{1/2}$, namely

$$L \cong L_\beta = \left(\frac{2\dot{m}'\,t}{\rho\beta}\right)^{1/2}, \quad z = \left(\beta\frac{2\dot{m}'\,t}{\rho}\right)^{1/2} \tag{30}$$

The time that would be required by water to spill over the edge, from C to B is (cf., Eq. (4)),

$$t_{\rm CB} = \left(\frac{2}{g}\right)^{1/2}\left(\beta\frac{2\dot{m}'\,t}{\rho}\right)^{1/4} \tag{31}$$

In the absence of the pool, the time required to flow down the incline (from A to B) is given by Eqs. (2) and (3)

$$t_{\rm AC} = \frac{L_\beta}{V_\beta} \tag{32}$$

where the bulk velocity (V_β) along the incline is derived by assuming that the flow is a laminar fully developed sheet. The longitudinal force that pulls the liquid column DL_β is balanced by the bottom skin friction τL_β, where the sheet thickness (D) and shear stress (τ) are as follows:

$$D = \frac{\dot{m}'}{\rho V_\beta} \quad \text{and} \quad \tau = 3\mu\frac{V_\beta}{D} \tag{33}$$

The longitudinal force balance yields

$$V_\beta = \left(\frac{g\beta}{3\mu\rho}\right)^{1/3}(\dot{m}')^{2/3} \tag{34}$$

The travel time down the incline follows from Eq. (32) in combination with Eq. (34) and L_β of Eq. (30)

$$t_{\rm AC} = \frac{2^{1/2}3^{1/3}t^{1/2}\mu^{1/3}}{(\dot{m}')^{1/6}\beta^{5/6}g^{1/3}\rho^{1/6}} \tag{35}$$

The time estimates ($t_{\rm CB}$, $t_{\rm AC}$) increase monotonically with the age (t) of the pool, but they increase differently, as $t^{1/4}$ and $t^{1/2}$, respectively. In the beginning, route CB takes longer than route AC. Figure 8 and Eqs. (31) and (35) show that the two curves intersect at the "transition" time

$$t_{\rm TR} = \frac{2}{3^{3/4}}\beta^{13/3}g^{-2/3}\rho^{-1/3}(\dot{m}')^{5/3}\mu^{-4/3} \tag{36}$$

which dictates the transition length scale

$$L_{\rm TR} = \frac{2}{3^{2/3}}(\dot{m}')^{4/3}\beta^{5/3}\rho^{-2/3}g^{-1/3}\mu^{-2/3} \tag{37}$$

The constructal law dictates the direction in which the transition must take place, that is the natural selection between two changes that are available, AC → CB and CB → AC. The natural direction must be from $t_{\rm AC}$ to $t_{\rm CB}$, because along this piecewise continuous curve the flow access down the incline requires *consistently* a shorter time.

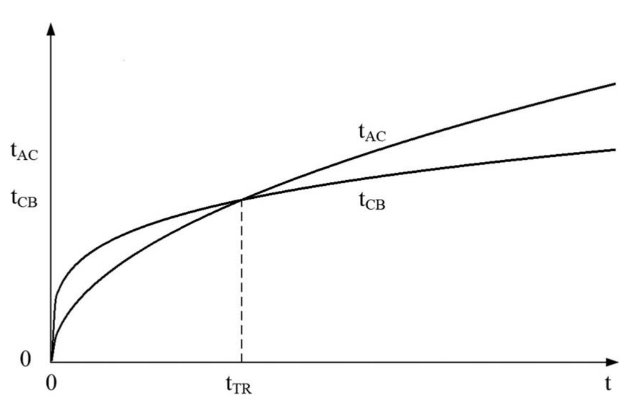

Fig. 8 The intersection of the asymptotes (31) and (35)

Finally, another conclusion is that the distance $L_{\rm TR}$ marks the distance between consecutive ripples. Furthermore, $t_{\rm TR}$ is the time scale of water flow along the distance $L_{\rm TR}$ on the inclined surface, and the ridge at the end of $L_{\rm TR}$ represents the edge (B) shown in Fig. 7.

7 Summary

It may seem contradictory to propose to examine "flow access" in the same sentence with "obstacles," as I did in the title of this article. The image of obstacles suggests lack of access, not access. As we saw in Secs. 3–6, there was no contradiction. More than that, we saw an illustration of what "theory" means [7,60]. It is a purely mental viewing that (if present in the mind already) empowers the mind to imagine other phenomena that impress the observer with "obstacles" when in fact the features that look like obstacles facilitate flow access.

Theory, the mental viewing, means *idea* (from the Greek *idein*, which means seeing with the eyes of the mind). I published this mental image in 1975 [61], Fig. 9, where I showed how to use a cascade of thermal contacts to facilitate the capture of the heat current that leaks along a mechanical support at low temperatures. Lim et al. published a similar image in 1992 [62], Fig. 10, and showed the benefits from apparent "obstacles" in the quest for more power from energy stored by melting a phase-change material. We discovered that the most power than can be produced with a hot stream ($\dot{m}$, T_∞) is accessible in "cascades" of phase-change materials with descending melting points ($T_{\rm m,1}$, $T_{\rm m,2}$, …). The stream temperature should fall in a particular sequence of steps dictated by the recommended selection of melting materials, each expressed in dimensionless terms [$\tau_1 = T_{m,1}/(T_\infty T_0)^{1/2}$, $\tau_2 = T_{m,2}/(T_\infty T_0)^{1/2}$, *etc.*] that were determined numerically so that the total power ($\dot{W}^{(2)}$ in Fig. 10) is maximum. It is useful to predict the cascade (τ_1, τ_2, …) when the need is to increase the access of the flow of power from the hot stream to the environment that includes you, the user of the installation.

The image of the cascade, today and earlier, is important for two reasons. First, it shows the background on which I felt eligible to question the apparent negatives associated with flow obstacles (Secs. 3–6). Second is a trend that threatens the integrity of all sectors of science [63]: new work is being published without citing the published precedents of the same idea. The most recent example is the 2021 paper by Xu et al. [64], which is about the cascade pictured in Fig. 10 and published by Lim et al. [62] in 1992. Numerous infractions of this kind were signaled in the published papers listed in Ref. [7], and reviewed pictorially in Ref. [65].

By questioning flow access in the context of three distinct fluid flow phenomena (Figs. 1(*a*)–1(*c*)), we came to the conclusion that natural flow architectures that look like obstacles owe their occurrence to the tendency toward greater access for what flows: water down an incline (Figs. 1(*a*) and 1(*c*)), and momentum perpendicular to the radial sheet flow (Fig. 1(*b*)), which causes the laminar-turbulent transition.

Agreements between classical notions of turbulence and constructal predictions have happened earlier. In the incipient rolling of slender flow regions (shear layers, jets, and plumes) the natural selection of configuration (called "transition") between laminar flow and turbulent flow is always in favor of the configuration that offers faster access to the transversal transfer of longitudinal momentum to the neighboring layers. Predictions of the onset of turbulence cover a wide range of previously disconnected classical observations and correlations. For their derivation, terminology, and application, the reader is directed to my convective heat transfer textbook [18] as follows:

- All the "critical" numbers of transition to turbulence (forced convection, natural convection, boundary layers, and duct flow).

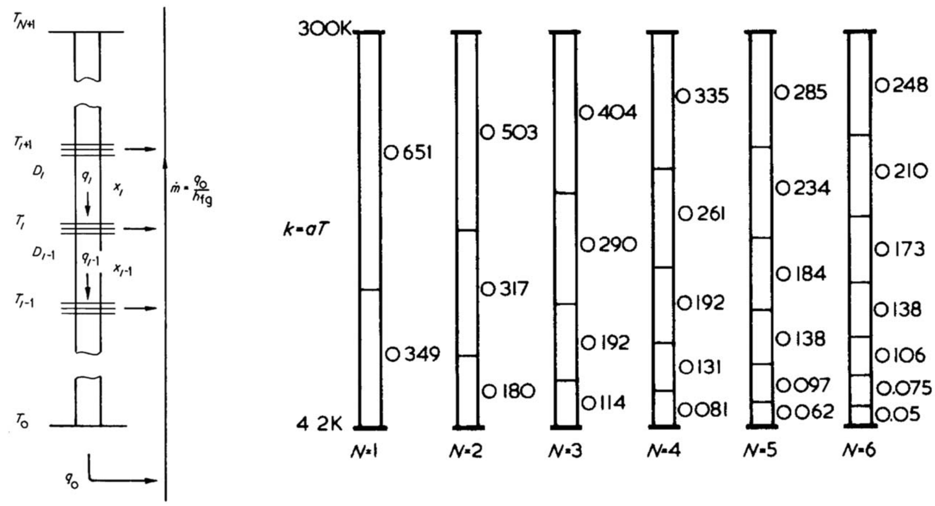

Fig. 9 Cascade of cooling stations along a conducting support for low temperature objects [61]

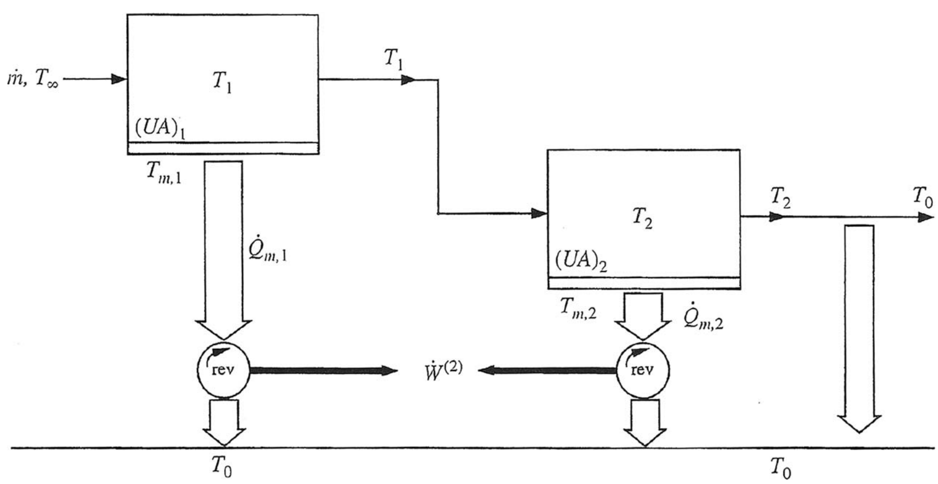

Fig. 10 Cascade flow over phase-change materials for the production of power from a hot stream [62]

- The universal local Reynolds number that is of order 10^2, for example, Eq. (28).
- The frequency of undulating (buckling) turbulent jets, wakes, and plumes (the universal Strouhal number).
- The constant-angle shapes (wedge, cone) of turbulent flow regions with large scale structure.
- The pulsating frequency of billows rising from pool fires.
- The smallest eddy that universally has a Reynolds number of order 10^2.
- The thicknesses of the viscous and conduction sublayers in fully turbulent flows, Fig. 11.
- The analogy between fluid friction and heat transfer.
- The analogy between rolling eddies and rolling stones.

Many other stop & go (or in & out) phenomena with apparent obstacles, such as respiration [48], heat exchanger structure [66], ice making and surface cleaning [21], can be conceptualized, analyzed, and unified as demonstrated in this article.

8 Theory Versus Empiricism

If you know the principle you can predict not only consequences that are observable but you can also make unintended predictions. I made unexpected predictions throughout my career, even before I called them "constructal law" in 1996. In retrospect, the pre-1996 ideas constitute a body of constructal *prefigurations*, such as the predictions of turbulence listed earlier.

If you know the principles, you know where the observed configuration came from. You know its *cause*. That is where biomimetics and reverse engineering are successful: in the minds of those who know the *science* of design, the physics principle. Reverse engineering is the mental process where one uses reasoning in order to deduce how an observed artifact was made (weapon, software, airplane). Artifacts are objects made by humans, and they empower the human and machine species (recall that the word "machine" means artifact, contrivance).

Many more objects happen by themselves, *naturally* (animals, rivers, winds, valuable minerals, celestial bodies). The many are not artifacts. To claim that a person can "reverse engineer nature" is incorrect [65]. Objects that are not made by people are not contrivances. Nature was not made by a person. If you doubt that, consider the fact that nature on earth and elsewhere is much older than the biosphere of the earth. Rivers and winds like ours are visible on planets without biospheres.

One can examine, describe, and catalog objects that happened by themselves, and that is empiricism. On the other hand, if one questions why many of such objects look similar, or why they behave similarly, the idea that pops up in the mind of the curious is that the coincidences are due to a *common cause*. That idea is theory,

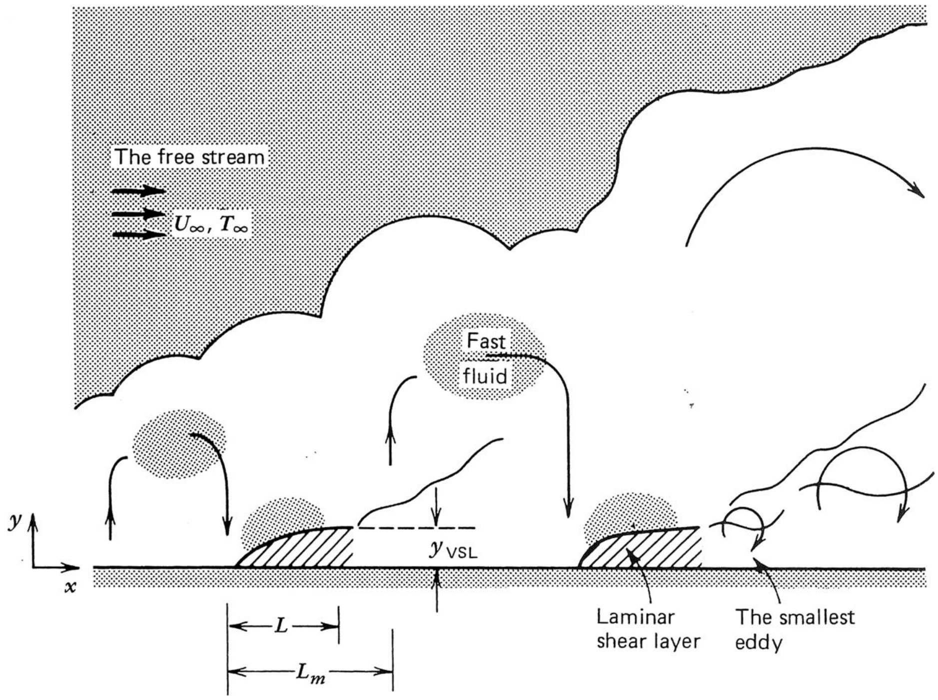

Fig. 11 The formation of the viscous sublayer as the time-averaged superposition of laminar boundary layers with thickness-based Reynolds numbers of order 10^2 [18]

the new physics that empowers future observers not to waste time searching for the new idea.

Theory must not be confused with empiricism. Empiricism is "the facts" established after observation and experiment (measurements) intended to validate or invalidate a theory. For example, the theory presented here for cataracts, jumps, and roll waves can be compared subsequently against data measured in Refs. [54,55,59], respectively. Theory and empiricism are eminently different. For this very reason, they are useful when practiced together.

The broad conclusion from this article is that the constructal law governs the evolution of configuration in flows with apparent obstacles. By analyzing the three phenomena of Fig. 1, we subjected the constructal law successfully to Ellis and Silk's test [67] of a scientific theory, namely "the issue boils down to clarifying one question: what potential observational or experimental evidence is there that would persuade you that the theory is wrong and lead you to abandoning it? If there is none, it is not a scientific theory."

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Conflict of Interest

There are no conflicts of interest. This article does not include research in which human participants were involved. Informed consent not applicable.

Data Availability Statement

No data, models, or code were generated or used for this paper.

Nomenclature

d = depth, m
g = gravitational acceleration, m s^{-2}
$\dot{k}$ = kinetic energy rate, W
$\dot{m}$ = mass flowrate, kg s^{-1}
$\dot{m}'$ = mass flowrate per unit length, kg s^{-1} m^{-1}
p = perimeter, m
r = radial position, m
t = time, s
u = radial velocity, m s^{-1}
w = width, m
z = vertical drop, m
A = cross-sectional area, m^2
D = sheet thickness, m
F = friction force, N
R = time ratio, Eq. (5)
V = jet velocity, m s^{-1}
C_f = skin friction coefficient, Eq. (1)
C_1 = factor, Eq. (14)
C_2 = factor, Eq. (24)
C_{10} = factor of order 10
F_R = reaction force, N
L, L_β = flow lengths, m

Greek Symbols

β = angle
δ = film thickness
θ = angular position
λ = wavelength, m
μ = viscosity, kg s^{-1} m^{-1}

Subscripts

$()_{BL}$ = boundary layer
$()_{TR}$ = transition

References

[1] Klemm, F., 1964, *A History of Western Technology*, MIT Press, Cambridge.
[2] Burstall, A. F., 1965, *A History of Mechanical Engineering*, MIT Press, Cambridge.
[3] Prandtl, L., 1969, *Essentials of Fluid Dynamics*, Blackie & Son, London.

[4] Simiu, E., and Scanlan, R. H., 1986, *Wind Effects on Structures: Modern Structural Design for Wind*, 2nd ed., Wiley, New York.
[5] Bejan, A., and Lorente, S., 2008, *Design With Constructal Theory*, Wiley, Hoboken, NJ.
[6] Bejan, A., and Lorente, S. R., 2004, "The Constructal Law and the Thermodynamics of Flow Systems With Configuration," Int. J. Heat Mass Transfer, **47**(14–16), pp. 3203–3214.
[7] Bejan, A., 2020, *Freedom and Evolution: Hierarchy in Nature, Society and Science*, Springer, New York.
[8] Wainwright, S. A., Biggs, W. D., Currey, J. D., and Gosline, J. M., 1976, *Mechanical Design in Organisms*, Princeton University Press, London.
[9] Schmidt-Nielsen, K., 1984, *Scaling: Why Is Animal Size So Important?*, Cambridge University Press, Cambridge, UK.
[10] Vogel, S., 1988, *Life's Devices: The Physical World of Animals and Plants*, Princeton University Press, Princeton, NJ.
[11] Weibel, E. R., Taylor, C. R., and Bolis, L., 1998, *Principles of Animal Design: The Optimization and Symmorphosis Debate*, Cambridge University Press, Cambridge, UK.
[12] Ahlborn, B. K., 2004, *Zoological Physics: Quantitative Models of Body Design, Actions, and Physical Limitations of Animals*, Springer, Berlin.
[13] Reis, A. H., 2006, "Constructal Theory: From Engineering to Physics, and How Flow Systems Develop Shape and Structure," ASME Appl. Mech. Rev., **59**(5), pp. 269–282.
[14] Chen, L., 2012, "Progress in Study on Constructal Theory and Its Applications," Sci. China Technol. Sci., **55**(3), pp. 802–820.
[15] Chen, L., Feng, H., Xie, Z., and Sun, F., 2019, "Progress of Constructal Theory in China Over the Past Decade," Int. J. Heat Mass Transfer, **130**, pp. 393–419.
[16] Rocha, L., 2009, *Convection in Channels and Porous Media: Analysis, Optimization, and Constructal Design*, VDM Verlag, Saarbrücken.
[17] Miguel, A. F., and Rocha, L. A. O., 2018, *Tree-Shaped Fluid Flow and Heat Transfer*, Springer, Berlin, Heidelberg, New York.
[18] Bejan, A., 2013, *Convection Heat Transfer*, 4th ed., Wiley, New York.
[19] Bejan, A., and Zane, J. P., 2013, *Design in Nature: How the Constructal Law Governs Evolution in Biology, Physics, Technology, and Social Organization*, Doubleday, New York.
[20] Bejan, A., 2016, *The Physics of Life: The Evolution of Everything*, St. Martin's Press, New York.
[21] Bejan, A., 2000, *Shape and Structure, From Engineering to Nature*, Cambridge University Press, Cambridge, UK.
[22] Bejan, A., 1997, *Advanced Engineering Thermodynamics*, 2nd ed., Wiley, Hoboken.
[23] Bejan, A., 1982, "Theoretical Explanation for the Incipient Formation of Meanders in Straight Rivers," Geophys. Res. Lett., **9**(8), pp. 831–834.
[24] Cruickshank, J. O., and Munson, B. R., 1981, "Viscous Fluid Buckling of Plane and Axisymmetric Jets," J. Fluid Mech., **113**(1), pp. 221–239.
[25] Suleiman, S. M., and Munson, B. R., 1981, "Viscous Buckling of Thin Fluid Layers," Phys. Fluids, **24**(1), pp. 1–5.
[26] Cruickshank, J. O., and Munson, B. R., 1982, "An Energy Loss Coefficient in Fluid Buckling," Phys. Fluids, **25**(11), pp. 1935–1937.
[27] Cruickshank, J. O., and Munson, B. R., 1983, "A Theoretical Prediction of the Fluid Buckling Frequency," Phys. Fluids, **26**(4), pp. 928–930.
[28] Schumm, S. A., and Khan, H. R., 1972, "Experimental Study of Channel Patterns," GSA Bull., **83**(6), pp. 1755–1770.
[29] Werner, P. W., 1951, "On the Origin of River Meanders," EOS Trans. Am. Geophys. Union, **32**(6), pp. 898–902.
[30] Callander, R. A., 1969, "Instability and River Channels," J. Fluid Mech., **36**(3), pp. 465–480.
[31] Callander, R. A., 1978, "River Meandering," Annu. Rev. Fluid Mech., **10**(1), pp. 129–158.
[32] Hansen, E., 1967, "The Formation-of Meanders As a Stability Problem," Basic Res. Prog. Rep., **13**, pp. 9–13.
[33] Hayashi, T., 1970, "The Formation of Meanders in Rivers," Proc. Jpn. Soc. Civil Eng., **1970**(180), pp. 61–70.
[34] Leopold, L. B., and Wolman, M. G., 1960, "River Meanders," GSA Bull., **71**(6), pp. 769–793.
[35] Leopold, L. B., and Wolman, M., 1970, "River Channel Patterns," *Rivers and River Terraces*, G. H. Dury, ed., Praeger Publishers, New York.
[36] Leopold, L. B., Wolman, M. G., and Miller, J. P., 1964, *Fluvial Processes in Geomorphology*, W. H. Freeman, San Francisco.
[37] Engelund, F., and Skovgaard, O., 1973, "On the Origin of Meandering and Braiding in Alluvial Streams," J. Fluid Mech., **57**(02), p. 289.
[38] Gorycki, M. A., 1973, "Hydraulic Drag: A Meander-Initiating Mechanism," GSA Bull., **84**(1), pp. 175–186.
[39] Parker, G., 1976, "On the Cause and Characteristic Scales of Meandering and Braiding in Rivers," J. Fluid Mech., **76**(03), p. 457.
[40] Bejan, A., 1981, "On the Buckling Property of Inviscid Jets and the Origin of Turbulence," Lett. Heat Mass Transfer, **8**(3), pp. 187–194.
[41] Bejan, A., 1982, "The Meandering Fall of Paper Ribbons," Phys. Fluids, **25**(5), p. 741.
[42] Stockman, M. G., and Bejan, A., 1982, "The Nonaxisymmetric (Buckling) Flow Regime of Fast Capillary Jets," Phys. Fluids, **25**(9), p. 1506.
[43] Bejan, A., 1982, "Theory of Instantaneous Sinuous Structure in Turbulent Buoyant Plumes," Wärme- Stoffübertrag, **16**(4), pp. 237–242.
[44] Anderson, R., and Bejan, A., 1983, "Buckling of a Turbulent Jet Surrounded by a Highly Flexible Duct," Phys. Fluids, **26**(11), p. 3193.
[45] Kimura, S., and Bejan, A., 1983, "The Buckling of a Vertical Liquid Column," ASME J. Fluids Eng., **105**(4), pp. 469–473.
[46] Anand, A., and Bejan, A., 1986, "Transition to Meandering Rivulet Flow in Vertical Parallel-Plate Channels," ASME J. Fluids Eng., **108**(2), pp. 269–272.
[47] Bejan, A., and Marden, J. H., 2006, "Unifying Constructal Theory for Scale Effects in Running, Swimming and Flying," J. Exp. Biol., **209**(2), pp. 238–248.
[48] Bejan, A., 1997, "Theory of Organization in Nature: Pulsating Physiological Processes," Int. J. Heat Mass Transfer, **40**(9), pp. 2097–2104.
[49] Bejan, A., Charles, J. D., and Lorente, S., 2014, "The Evolution of Airplanes," J. Appl. Phys., **116**(4), p. 044901.
[50] Chen, R., Wen, C. Y., Lorente, S., and Bejan, A., 2016, "The Evolution of Helicopters," J. Appl. Phys., **120**(1), p. 014901.
[51] Bejan, A., Ferber, L., and Lorente, S., 2020, "Convergent Evolution of Boats With Sails," Sci. Rep., **10**(1), p. Art. no. 1.
[52] Bejan, A., 2015, "Why Humans Build Fires Shaped the Same Way," Sci. Rep., **5**(1).
[53] Bejan, A., 2010, "The Constructal-Law Origin of the Wheel, Size, and Skeleton in Animal Design," Am. J. Phys., **78**(7), pp. 692–699.
[54] Scheingross, J. S., Lamb, M. P., and Fuller, B. M., 2019, "Self-Formed Bedrock Waterfalls," Nature, **567**(7747), pp. 229–233.
[55] Bhagat, R. K., Jha, N. K., Linden, P. F., and Wilson, D. I., 2018, "On the Origin of the Circular Hydraulic Jump in a Thin Liquid Film," J. Fluid Mech., **851**.
[56] Lorenzini, G., and Mazza, N., 2004, *Debris Flow Phenomenology and Rheological Modelling*, WIT Press, Southampton, UK.
[57] Bejan, A., 2016, "Rolling Stones and Turbulent Eddies: Why the Bigger Live Longer and Travel Farther," Sci. Rep., **6**(1), p. 21445.
[58] Sabersky, R. H., Acosta, A. J., and Hauptmann, E. G., 1971, *Fluid Flow, A First Course in Fluid Mechanics*, 2nd ed., Macmillan, New York.
[59] Brock, R. R., 1967, Development of Roll Waves in Open Channels, W. M. Keck Laboratory of Hydraulics and Water Resources Division of Engineering and Applied Science California Institute of Technology, Pasadena, CA, KH-R-16.
[60] Bejan, A., 2022, Heat Transfer: Evolution, Design and Performance, Wiley, Hoboken.
[61] Bejan, A., 1975, "Discrete Cooling of Low Heat Leak Supports to 4.2 K," Cryogenics, **15**(5), pp. 290–292.
[62] Lim, J. S., Bejan, A., and Kim, J. H., 1992, "Thermodynamic Optimization of Phase-Change Energy Storage Using Two or More Materials," ASME J. Energy Resour. Technol., **114**(1), pp. 84–90.
[63] National Science Foundation, NSF-CFR-689: "Plagiarism Means the Appropriation of Another Person's Ideas, Processes, Results or Words Without Giving Appropriate Credit."
[64] Xu, B., Lu, S., Jia, W., Du, H., Fan, M., and Wang, R., 2021, "Thermodynamic Study on Charge-Discharge Processes and Cycle of Cascaded Latent Heat Storage in Heating System," Energy Build., **249**, p. 111216.
[65] Bejan, A., 2022, *Time and Beauty: Why Time Flies and Beauty Never Dies*, World Scientific, Singapore, Chap. 6.
[66] Biserni, C., Dalpiaz, F. L., Fagundes, T. M., and Rocha, L. A. O., 2017, "Constructal Design of T-Shaped Morphing Fins Coupled With Trapezoidal Basement: A Numerical Investigation by Means of Exhaustive Search and Genetic Algorithm," Int. J. Heat Mass Transfer, **109**, pp. 73–81.
[67] Ellis, G., and Silk, J., 2014, "Defend the Integrity of Physics," Nature, **516**(7531), pp. 321–323.