We study the stochastic quantization of Non-Abelian Antisymmetric Tensor Fields as Reducible Gauge Theories. Then we prove that the fictitious time divergent longitudinal component reproduces the so-called Faddeev-Popov ghost effects from 1-loop contributions.

§ 1. Introduction

A Stochastic Quantization Method (in short, SQM) was first introduced by Parisi and Wu as the third quantization method of gauge theories. Their statement is as follows: In this scheme there is no need to fix the gauge degrees of freedom, i.e., no need to introduce any artificial auxiliary fields (ghost and multiplier fields). It is the main advantage of the SQM. It is essentially governed by equivalent two descriptions, one is based on the Langevin equation, and the other is on the Fokker-Planck equation. The Langevin equation
\[ \frac{\partial \phi(x, t)}{\partial t} = \dot{\phi}(x, t) = -\frac{\delta S_{\text{ct}}}{\delta \phi(x, t)} + \eta(x, t), \tag{1.1} \]
which describes a hypothetical process of a field \( \phi(x, t) \) with respect to an additional fictitious-time \( t \) is suitable for numerical calculations and perturbative analyses. The expectation value \( \langle \cdot \rangle \) is calculated from the correlations of the random noise \( \eta(x, t) \) with a gaussian distribution,
\[ \langle \eta(x, t) \eta(x', t') \rangle = 2 \delta^{(n)}(x-x') \delta(t-t'). \tag{1.2} \]
The Fokker-Planck equation defines the probability distribution of the field at a given fictitious time \( t \),
\[ \dot{P} = \int d^n x \frac{\delta}{\delta \phi(x, t)} \left[ \frac{\delta S_{\text{ct}}}{\delta \phi(x, t)} + \frac{\delta}{\delta \phi(x, t)} \right] P, \tag{1.3} \]
and also the expectation value of an arbitrary functional is given as
\[ \langle f[\phi(x, t)] \rangle = \int P[\phi, t] f[\phi(x)] d\phi. \tag{1.4} \]
It is well known that an \( n \)-point function of non-gauge theories by the SQM (both based on the Langevin equation and on the Fokker-Planck one) really reproduces that by the Euclidean path integral quantization method in a thermal equilibrium limit \( (t \to \infty) \).
It means that there is a manifest correspondence between the stochastic quantization scheme and the ordinary one in this case.

However, when we quantize the (irreducible) gauge theories by this method, we encounter a $t$-divergent term instead of fixing the gauge degrees of freedom. In fact, when we treat the Yang-Mills (YM) fields, we get a 2-point function as follows:

$$\langle A_{\mu}^a(k, t)A_{\nu}^b(k', t)\rangle = (2\pi)^4 \left[ \frac{T_{\mu \nu}}{k^2} (1 - e^{-2k^2 t}) + L_{\mu \nu} t \right] \delta^{ab} \delta^4(k + k')$$  \hspace{1cm} (1.6)

with

$$T_{\mu \nu} = \delta_{\mu \nu} - \frac{k_{\mu} k_{\nu}}{k^2},$$
$$L_{\mu \nu} = \frac{k_{\mu} k_{\nu}}{k^2}.$$  \hspace{1cm} (1.7)

Here, $T_{\mu \nu}$ and $L_{\mu \nu}$ are projection operators of transverse and longitudinal mode, respectively. The second term linearly diverges in the thermal equilibrium limit. This reflects the gauge invariance of the YM-fields. Though this propagator contains a linearly divergent term, the first term converges into an ordinary Landau gauge propagator in $t$-infinity limit (although the 2-point function is not well-defined, gauge invariant quantity, $\langle F_{\mu \nu}^a(k)F_{\rho \sigma}^b(k') \rangle$ for example, reduces to a finite form, i.e., there is no divergence). Since this propagator is Lorentz-covariant, it includes unphysical variables. Thus the unitarity of this theory may be broken. If this method is self-consistent without ghost fields, in order to ensure the unitarity, the ghost effects which cancel the unphysical variable contributions must be hidden somewhere. It seems that the linearly divergent longitudinal term plays a role of ghost fields. By calculating the expectation value of the gauge invariant quantity $\langle F_{\mu \nu}^a(k)F_{\rho \sigma}^b(k') \rangle$ at 1-loop level, Namiki et al. have shown that the contributions from the longitudinal internal lines automatically sum up to the expected Faddeev-Popov contributions, while the transverse lines render the standard Landau gauge results. It is not until we consider the longitudinal contributions that we can find the manifest correspondence between the stochastic quantization and the ordinary one.

In general, when we covariantly quantize the so-called reducible gauge theories (RGT), we have to introduce the "ghost fields for ghost fields", since the primary ghost fields turn out to be gauge fields. This makes the gauge-fixing somewhat complicated. One of the most familiar examples of the RGT is a non-Abelian (self-interacting) antisymmetric tensor fields (NA-ASTF) theory. It is a 1st-stage reducible gauge theory which needs primary and secondary ghosts to fix the gauge degrees of freedom completely. Several years ago, it was covariantly quantized within the framework of the Batalin-Vilkovisky antifield formalism. Indeed, the covariant canonical quantization of Kugo-Ojima formalism is also available. But until now, it has not been quantized in the stochastic scheme yet (based on the Langevin equation...
and also on the Fokker-Planck one). Therefore, it has not been explored whether the longitudinal component of the NA-ASTF has the same property of the YM-fields. Even if the same property exists, it is a non-trivial problem whether the ghost effects attribute to the primary ghosts or to both the primary and the secondary ones. If the SQM is also applicable to the RGT like the YM-fields (irreducible gauge theories), we never mind the gauge-fixing problem, because the ghost effects are naturally included in this procedure without introducing the ghost fields.

The purpose of this paper is to quantize the 2nd-rank NA-ASTF in the 4-dimensional space-time (1st-stage reducible gauge theory) based on the Langevin equation. Then we can analyse the NA-ASTF perturbatively and compare the results by the SQM with those by an ordinary quantization method. Therefore we can evaluate whether the 1-loop longitudinal contributions for the gauge invariant quantity really reproduce the full ghost effects, or not.

Organization of this paper is as follows. In § 2, we introduce the classical action of the NA-ASTF in the 1st-order form, and show the equivalence of the 1st-order and the 2nd-order action. In § 3, we quantize the NA-ASTF based on the Langevin equation, and show the finite parts (transverse parts) of the solutions of the Langevin equations really give the results corresponding to the usual Feynman rules. In § 4, we prove that the linearly divergent longitudinal term reproduces the Faddeev-Popov ghost (both primary and secondary) effects. Section 5 is devoted to discussion. In the Appendix, we quantize the NA-ASTF in the BRST formalism and give the Euclidean Feynman rules.

§ 2. Non-Abelian antisymmetric tensor fields

In Ref. 8), the 2nd-order Lagrangian of NA-ASTF was given as

\[ L = -\frac{1}{8} G^{\mu a} \tilde{K}_{\mu \nu}^{ab} G_{\nu b} \]  

(2.1)

with

\[
\begin{aligned}
G^{\mu a} & = \bar{\epsilon}^{\mu \nu \sigma} B_{\nu \sigma, a} = \bar{\epsilon}^{\mu \nu \sigma} \delta_{\sigma} B_{\nu \sigma}^a, \\
\tilde{K}_{ab}^{\mu \nu} & = \delta_{\mu}^{\nu} \delta^{ab}, \\
K_{\mu \nu}^{\rho \alpha} & = g^{\mu \rho} \delta_{ab} + g^{\mu \rho \sigma} \bar{\epsilon}^{\nu \sigma \alpha} B_{\alpha \sigma}^a.
\end{aligned}
\]

(2.2)

This is a non-Abelian extension of

\[ L = -\frac{1}{8} G^{\mu a} G_{a} \]  

(2.3)

The Lagrangian (2.1) changes by a total divergence under the following gauge transformation including a vector infinitesimal parameter \( \xi_{\mu}^a \)

\[ \delta B_{\alpha \nu} = (\partial_\mu \xi_{\mu}^a + g f^{abc} A_{\mu}^b \xi_{\mu}^c) - (\mu \leftrightarrow \nu) \]  

(2.4)

with

\[ A_{\mu}^a = \tilde{K}_{\mu \nu}^{ab} G_{\nu b} \]  

(2.5)
However, because (2·1) possesses a non-polynomial interaction term, the Langevin equation of \( B_\mu^a \) takes a non-linear form which cannot be solved explicitly. Also in Ref. 8), the convenient 1st-order Lagrangian of NA-ASTF was presented with a subsidiary field \( A_\mu^a \) so as to linearize the Lagrangian,

\[
L = -\frac{1}{8} \varepsilon^{\mu\nu\rho\sigma} B_\mu^a F_\rho^a + \frac{1}{8} A_\mu^a A_\mu^a \tag{2·6}
\]

with

\[
F_\rho^a = \partial_\rho A_\sigma^a - \partial_\sigma A_\rho^a + g f^{abc} A_\rho^b A_\sigma^c.
\]

When we substitute the solution of the field equation for \( A_\mu^a \)

\[
A_\mu^a = \tilde{\bar{K}}_{\mu \nu} G_\nu^b
\]

into (2·6), we can retain (2·1). This fact represents the classical equivalence of (2·1) and (2·6). The gauge transformation is simply

\[
\delta B_\mu^a = (D_\mu \xi^a) - (D_\xi \xi^a), \tag{2·9a}
\]

\[
\delta A_\mu^a = 0, \tag{2·9b}
\]

where \( D_\mu \) is the covariant derivative with the potential \( A_\mu^a \). Gauge invariance is followed from the Bianchi identity \( \varepsilon^{\mu\nu\rho\sigma}[D_\mu, [D_\nu, D_\sigma]] = 0 \).

Since the gauge transformation parameter is a four-dimensional vector, it seems the number of the gauge degrees of freedom is four. Nevertheless, the true degrees are three. This is just the origin of the so-called “reducibility”. In the following sections, we will use this 1st-order Lagrangian.

\section*{3. Stochastic quantization of non-Abelian antisymmetric tensor fields}

As we consider the Langevin equation, we use an Euclidean action of the NA-ASTF,

\[
S_E = \int dt d^4 x \left( \frac{a}{8} \varepsilon_{\mu\nu\rho\sigma} B_\mu^a F_\rho^a + \frac{a^2}{8} A_\mu^a A_\mu^a \right). \tag{3·1}
\]

For convenience, \( a \) is assumed to be a real positive parameter which has mass dimension (at least, \( a \) is not pure imaginary in order to converge the Langevin equations). This parameter disappears upon integrating out the subsidiary field \( A_\mu^a \), by means of the field equation for \( A_\mu^a \),

\[
A_\mu^a = \frac{1}{a} \tilde{\bar{K}}_{\mu \nu} G_\nu^b. \tag{3·2}
\]

Here we use the following notation from Minkowski space-time to Euclidean space-time, metric tensor signature: \( \eta^M_{\mu \nu} = (+, -, -, -) \), \( \eta^E_{\mu \nu} = (+, +, +, +) \), field component: \( i x_0 = x_i, A_0 = i A_i, B_{0i} = i B_{4i}, \varepsilon_{ijk4} = i \varepsilon_{ijk} \) with the definition \( \varepsilon_{123} = -1, \varepsilon_{124} = i \). Then, we can define Euclidean action as follows:

\[
S_M[\phi] = -S_E[\phi]. \tag{3·3}
\]
From this section, $\mu \nu$ indices denote $1 \sim 4$.

Then, we obtain the Langevin equations from (3·1) and (1·1), in position space,

\[
\begin{aligned}
\dot{B}_{\mu \nu}(x, t) &= -\frac{a}{4} \varepsilon_{\mu \rho \sigma \theta} \partial_\rho A_\sigma(x, t) \\
&\quad - \frac{a}{8} \varepsilon_{\mu \rho \sigma \theta} f^{abc} A_\rho^b(x, t) A_\sigma^c(x, t) + \eta_{\mu \nu}(x, t), \\
\dot{A}_\mu^a(x, t) &= -\frac{a^2}{4} A_\mu^a(x, t) + \frac{a}{4} \varepsilon_{\mu \rho \sigma \theta} B_{\rho \sigma}(x, t) \\
&\quad - \frac{a}{4} \varepsilon_{\mu \rho \sigma \theta} f^{abc} A_\nu^c(x, t) B_{\rho \sigma}(x, t) + \xi_{\mu}^a(x, t),
\end{aligned}
\]  
(3·4)

or in momentum space,

\[
\begin{aligned}
\dot{B}_{\mu \nu}(k, t) &= \frac{a}{4} i \varepsilon_{\mu \rho \sigma \theta} k_\rho A_\sigma(k, t) + I_{\mu \nu}(k, t) + \eta_{\mu \nu}(k, t), \\
\dot{A}_\mu^a(k, t) &= -\frac{a^2}{4} A_\mu^a(k, t) - \frac{a}{4} i \varepsilon_{\mu \rho \sigma \theta} B_{\rho \sigma}(k, t) + I_{\mu}^a(k, t) + \xi_{\mu}^a(k, t)
\end{aligned}
\]  
(3·5)

with the interaction terms,

\[
\begin{aligned}
I_{\mu \nu}(k, t) &= -\frac{a}{8} \varepsilon_{\mu \rho \sigma \theta} f^{abc} \int \frac{d^4p d^4q}{(2\pi)^4} A_\rho^b(p, t) A_\sigma^c(q, t) \delta^4(k - p - q), \\
I_{\mu}^a(k, t) &= \frac{a}{4} \varepsilon_{\mu \rho \sigma \theta} f^{abc} \int \frac{d^4p d^4q}{(2\pi)^4} A_\nu^c(p, t) B_{\rho \sigma}^a(q, t) \delta^4(k - p - q).
\end{aligned}
\]  
(3·6)

In (3·5) $\eta_{\mu \nu}(k, t)$ and $\xi_{\mu}^a(k, t)$ are white noises whose correlations are defined as

\[
\begin{aligned}
&\langle \eta_{\mu \nu}(k, t) \eta_{\rho \sigma}(k', t') \rangle = 2(2\pi)^4 \delta_{\mu \rho} \delta_{\nu \sigma} \delta(t - t'), \\
&\langle \xi_{\mu}^a(k, t) \xi_{\nu}^a(k', t') \rangle = 2(2\pi)^4 \delta_{\mu \nu} \delta(k - k') \delta(t - t') \delta_{\rho \sigma}, \\
&\text{otherwise} = 0
\end{aligned}
\]  
(3·7)

with $I_{\mu \nu, \rho \sigma} \equiv \frac{1}{2}(\delta_{\mu \rho} \delta_{\nu \sigma} - \delta_{\mu \sigma} \delta_{\nu \rho})$.

We must solve the free parts of the Langevin equations (3·5) at first. Interaction parts (3·6) will be taken in perturbatively. Since we treat the free parts, we suppress the group indices for a while. We can combine the tensor and vector Langevin equations into the simple matrix form,

\[
\begin{pmatrix}
\dot{B}_{\mu \nu} \\
\dot{A}_\mu^a
\end{pmatrix}
= -\frac{a}{4}
\begin{pmatrix}
0 & -i \varepsilon_{\mu \nu \lambda \rho} k_\lambda \\
\varepsilon_{\rho \sigma \theta \mu} k_\sigma & a \delta_{\theta \nu}
\end{pmatrix}
\begin{pmatrix}
B_{\rho \sigma} \\
A_\sigma^a
\end{pmatrix}
+ \begin{pmatrix}
\eta_{\mu \nu} \\
\xi_{\mu}^a
\end{pmatrix}
\]  
(3·8)

Here we define this matrix with tensor and vector indices,

\[
M_{\mu \nu, \rho \sigma}^{\mu \nu, \rho \sigma} =
\begin{pmatrix}
0 & -i \varepsilon_{\mu \nu \lambda \rho} k_\lambda \\
\varepsilon_{\rho \sigma \theta \mu} k_\sigma & a \delta_{\theta \nu}
\end{pmatrix}
\]  
(3·9)

Therefore we get the solution of (3·8) formally,

\[
\begin{pmatrix}
B_{\mu \nu} \\
A_\mu^a
\end{pmatrix}
= \int dt [e^{(-a/4)M(t - t')}]_{\mu \nu, \rho \sigma} \begin{pmatrix}
\eta_{\rho \sigma}(t) \\
\xi_{\rho}^a(t)
\end{pmatrix}
\]  
(3·10)
Thus we expand the matrix \([e^{-(\alpha/4)M(t-\tau)}]_{\mu\nu,ab}\) in power series. Here the expansions are all expressed by the following matrices, \(F^{\mu\nu,ab}_{\rho\tau}, P_{\nu,ab}, T_{\rho\tau}\) and \(\delta_{\mu\nu}\) with the definitions

\[
F^{\mu\nu,ab}_{\rho\tau} \equiv \begin{pmatrix}
0 & -i\epsilon_{\mu\nu\lambda\kappa}k_\lambda \\
(i\epsilon_{\alpha\beta\gamma\delta}k_\delta) & 0
\end{pmatrix}
\tag{3.11}
\]

and

\[
P_{\mu,ab} = \frac{1}{2} (T_{\mu a} T_{\nu b} - T_{\nu a} T_{\mu b}).
\tag{3.12}
\]

Thus, we may expect the general solution takes this form,

\[
\sum_i (a_i F + b_i P + c_i T + d_i \delta)e^{-\Lambda_i(t-\tau)}.
\tag{3.13}
\]

Here \(\Lambda_i\) is the eigenvalue of \((\alpha/4)M\). In order to fix the coefficients \(a_i \sim d_i\) of this matrix, we have to calculate the eigenvalues. We obtain the eigenvalues as \(\alpha^2/4, 0\) and \(\Lambda_{1,2} = (\alpha/4)\lambda_{1,2} = (\alpha/4)((\alpha \pm \sqrt{\alpha^2 - 8k^2})/2)\) where 1, 2 indices of \(\Lambda\) and \(\lambda\) express + and − respectively. Unless the \(\alpha\) is pure imaginary, both \(\Re(\alpha^2/4)\) and \(\Re\Lambda_{1,2}\) are positive, which make the Langevin equations converge. The last three eigenvalues are three-fold degenerate. As we mentioned in the last section, three zero-eigenvalues reflect the true gauge degrees of freedom of NA-ASTF and do not make the Langevin equations converge in the equilibrium limit. The expansion is explicitly done as follows:

\[
[e^{-(\alpha/4)M(t-\tau)}]_{\mu\nu,ab} = \frac{1}{\lambda_1 - \lambda_2} \left( F^{\mu\nu,ab}_{\rho\tau} - \lambda_2 P_{\mu,ab} + \lambda_1 T_{\rho\tau} \right) e^{-(\alpha/4)\lambda_1(t-\tau)}
\]

\[
- \frac{1}{\lambda_1 - \lambda_2} \left( F^{\mu\nu,ab}_{\rho\tau} - \lambda_1 P_{\mu,ab} + \lambda_2 T_{\rho\tau} \right) e^{-(\alpha/4)\lambda_2(t-\tau)}
\]

\[+ L_{\rho\tau} e^{-(\alpha^2/4)(t-\tau)} + (1 - P)_{\mu,ab} .
\tag{3.14}
\]

Therefore we gain the general solution of (3.8),

\[
\begin{pmatrix}
B_{\mu\nu} \\
A_\rho
\end{pmatrix} = \int_0^t d\tau \left[ \frac{1}{\lambda_1 - \lambda_2} \begin{pmatrix}
-\lambda_2 P_{\mu,ab} & -i\epsilon_{\mu\nu\lambda\kappa}k_\lambda \\
i\epsilon_{\alpha\beta\gamma\delta}k_\delta & \lambda_1 T_{\rho\tau}
\end{pmatrix} e^{-(\alpha/4)\lambda_1(t-\tau)}
\right]
\]

\[
- \frac{1}{\lambda_1 - \lambda_2} \begin{pmatrix}
-\lambda_1 P_{\mu,ab} & -i\epsilon_{\mu\nu\lambda\kappa}k_\lambda \\
i\epsilon_{\alpha\beta\gamma\delta}k_\delta & \lambda_2 T_{\rho\tau}
\end{pmatrix} e^{-(\alpha/4)\lambda_2(t-\tau)}
\]

\[+ \begin{pmatrix}
0 & 0 \\
0 & L_{\rho\tau}
\end{pmatrix} e^{-(\alpha^2/4)(t-\tau)} + \begin{pmatrix}
1 - P_{\mu,ab} & 0 \\
0 & 0
\end{pmatrix} \begin{pmatrix}
\eta_{ab} \\
\xi_{\gamma}
\end{pmatrix},
\tag{3.15}
\]

or symbolically,
\[
B_{\mu}(k, t) = \int_0^t dt \{ G_{\mu\nu,\rho}^{BB}(k, t - \tau) \eta_{\nu\rho}(k, \tau) + G_{\mu\nu,\tau}^{BA}(k, t - \tau) \xi_{\tau}(k, \tau) \},
\]
\[
A_\rho(k, t) = \int_0^t dt \{ G_{\rho\sigma,\nu}^{AB}(k, t - \tau) \eta_{\nu\sigma}(k, \tau) + G_{\rho\sigma,\tau}^{AA}(k, t - \tau) \xi_{\tau}(k, \tau) \}
\]
with various Green's functions
\[
G_{\mu\nu,\rho}^{BB}(k, t - \tau) = \frac{1}{\lambda_1 - \lambda_2} \{ -\lambda_2 P_{\mu\nu,\rho}^a e^{(-\alpha/4)\lambda_1 (t - \tau)} + \lambda_1 P_{\mu\nu,\rho}^a e^{(-\alpha/4)\lambda_2 (t - \tau)} \}
+ (1 - P)_{\mu\nu,\rho},
\]
\[
G_{\mu\nu,\tau}^{BA}(k, t - \tau) = \frac{-i \epsilon_{\mu\nu\lambda} k_\lambda}{\lambda_1 - \lambda_2} \{ e^{(-\alpha/4)\lambda_1 (t - \tau)} - e^{(-\alpha/4)\lambda_2 (t - \tau)} \}
= -G_{\tau,\rho\nu}^{AB}(k, t - \tau) = G_{\tau,\rho\nu}^{AB}(k, t - \tau),
\]
\[
G_{\rho\sigma,\tau}^{AA}(k, t - \tau) = \frac{1}{\lambda_1 - \lambda_2} \{ \lambda_1 e^{(-\alpha/4)\lambda_1 (t - \tau)} - \lambda_2 e^{(-\alpha/4)\lambda_2 (t - \tau)} \} + L_{\rho\tau} e^{(-\alpha/4)(t - \tau)} .
\]

Here we adopt the initial conditions as \( B_{\mu}^a(k, t=0) = A_{\rho}^a(k, t=0) = 0 \), because the gauge invariant quantities, which will be evaluated in § 4, are independent of the initial conditions.

Taking into account the interaction parts, we finally acquire the exact integral equations,
\[
\frac{d}{dt} \begin{pmatrix} B_{\mu}^a \\ A_{\rho}^a \end{pmatrix} = \int_0^t dt \begin{pmatrix} G_{\mu\nu,\rho}^{BB} & G_{\mu\nu,\tau}^{BA} \\ G_{\rho\sigma,\nu}^{BA} & G_{\rho\sigma,\tau}^{AA} \end{pmatrix} \begin{pmatrix} \eta_{\nu\rho}^a + I_{\nu\rho}^a \\ \xi_{\tau}^a + I_{\tau}^a \end{pmatrix} .
\]

By means of the following definitions of the vertex operators \( V_{\alpha}^{abc,\mu\nu\rho\sigma} \) and \( V_{\alpha}^{abc,\mu\nu\rho\sigma} \)
\[
\langle B_{\mu}^a B_{\nu}^b \rangle = \frac{1}{\lambda_1 - \lambda_2} \{ \lambda_1 e^{(-\alpha/4)\lambda_1 (t - \tau)} - \lambda_2 e^{(-\alpha/4)\lambda_2 (t - \tau)} \} + L_{\rho\tau} e^{(-\alpha/4)(t - \tau)} .
\]

\[
\langle A_{\mu}^a A_{\nu}^b \rangle = \frac{1}{\lambda_1 - \lambda_2} \{ \lambda_1 e^{(-\alpha/4)\lambda_1 (t - \tau)} - \lambda_2 e^{(-\alpha/4)\lambda_2 (t - \tau)} \} + L_{\rho\tau} e^{(-\alpha/4)(t - \tau)} .
\]

\[
\langle B_{\mu}^a A_{\nu}^b \rangle = \frac{1}{\lambda_1 - \lambda_2} \{ \lambda_1 e^{(-\alpha/4)\lambda_1 (t - \tau)} - \lambda_2 e^{(-\alpha/4)\lambda_2 (t - \tau)} \} + L_{\rho\tau} e^{(-\alpha/4)(t - \tau)} .
\]
\[ I^a_{\mu
u}(k, t) = V_{B}^{abc\mu\nu\rho}(k, p, q) A_{\rho}^b(p, t) A_{\sigma}^c(q, t), \]
\[ I^a_{\mu
u}(k, t) = V_{A}^{abc\mu\nu\rho}(k, p, q) A_{\rho}^c(p, t) B_{\sigma}^b(q, t), \]
and the perturbation of (3.18) we can represent \( B^\mu_{\nu} \) and \( A^a_{\sigma} \) symbolically and graphically (Fig. 1),

\[
\begin{align*}
B & = G^{BB} \eta + G^{BA} \xi + G^{BB} V_B (G^{AB} \eta + G^{AA} \xi)^2 \\
& + G^{BA} V_A (G^{AB} \eta + G^{AA} \xi)(G^{BB} \eta + G^{BA} \xi) + \cdots, \\
A & = G^{AB} \eta + G^{AA} \xi + G^{AB} V_B (G^{AB} \eta + G^{AA} \xi)^2 \\
& + G^{AA} V_A (G^{AB} \eta + G^{AA} \xi)(G^{BB} \eta + G^{BA} \xi) + \cdots.
\end{align*}
\]

A spiral line is \( G^{BB} \), a left-spiral and right-solid line is \( G^{BA} \), its inverse line is \( G^{AB} \) and a solid line is \( G^{AA} \). A cross indicates the vector noise \( \xi \) and a box indicates the tensor noise \( \eta \). An encircled \( A \) and \( B \) are \( V_A \) and \( V_B \), respectively. Because the noise correlations of \( \langle \eta \eta \rangle \) and \( \langle \xi \xi \rangle \) are usually done together, we draw the stochastic diagrams like Fig. 1. For example, the diagram that \( G^{AB} \eta \) runs parallel with \( G^{AA} \xi \) indicates \( G^{AB} \eta + G^{AA} \xi \). Hereafter the parallel lines mean the sum of two Green's functions or sum of two propagators. This is done only for simplicity (otherwise, the number of stochastic diagrams would increase).

Consequently, we can calculate stochastic diagrams perturbatively. First, we evaluate the lowest-order contributions to the propagators (Fig. 2) and compare with the results given by the ordinary path integral quantization method (see Appendix (A·3)).

Here we give some useful formulas for calculating the stochastic diagrams. These are formulas for internal lines with finite fictitious-time dependence, so if we want for external lines, we take the limit \( t \to \infty \).

\[
\int_0^t dt' \int_0^{t'} dt'' G_{\rho \sigma, \tau}(k, t-\tau) G_{\rho \tau, \sigma}(k, t'-\tau') + G_{\rho \tau, \sigma}(k, t-\tau) G_{\rho \sigma, \tau}(-k, t'-\tau') \cdot \delta(\tau - \tau')
\]

\[
= \frac{1}{\lambda_1 - \lambda_2} \cdot P_{\nu \mu, \rho \sigma} \frac{2}{a} \left[ -\lambda_2 \{ e^{(-a/4)|t-t'|} - e^{(-a/4)|t+t'|} \} + \lambda_1 \{ e^{(-a/4)|t-t'|} - e^{(-a/4)|t+t'|} \} \right] + (1 - P)_{\nu \mu, \rho \sigma} \times \min(t, t'),
\]

\[
\int_0^t dt' \int_0^{t'} dt'' G_{\rho \sigma, \tau}(k, t-\tau) G_{\rho \tau, \sigma}(k, t'-\tau') + G_{\rho \tau, \sigma}(k, t-\tau) G_{\rho \sigma, \tau}(-k, t'-\tau') \cdot \delta(\tau - \tau')
\]

\[
= \frac{2}{a} L_{\nu \mu} \left[ e^{(-a/4)|t-t'|} - e^{(-a/4)|t-t'|} - e^{(-a/4)|t+t'|} + e^{(-a/4)|t+t'|} \right] + \frac{1}{a} L_{\nu \mu} \left[ e^{(-a/2)|t-t'|} - e^{(-a/2)|t+t'|} \right].
\]

\[
\int_0^t dt' \int_0^{t'} dt'' G_{\rho \sigma, \tau}(k, t-\tau) G_{\rho \tau, \sigma}(k, t'-\tau') + G_{\rho \tau, \sigma}(k, t-\tau) G_{\rho \sigma, \tau}(k, t'-\tau') \cdot \delta(\tau - \tau')
\]

\[
i \frac{a_{\nu \mu} \rho \sigma}{\lambda_1 - \lambda_2} \left[ \frac{1}{\lambda_1} \{ e^{(-a/4)|t-t'|} - e^{(-a/4)|t-t'|} \} - \frac{1}{\lambda_2} \{ e^{(-a/4)|t-t'|} - e^{(-a/4)|t+t'|} \} \right].
\]
Then the propagators are derived in the limit $t = t' \to \infty$. (3·21) gives the propagator

$$\langle B^a_{\mu}(k, t) B^b_{\alpha}(k', t) \rangle = 2(2\pi)^4 \delta^4(k + k') \delta^{ab} \left\{ \frac{P_{\mu \nu, \alpha \beta}}{k^2} + (1 - P)_{\mu \nu, \alpha \beta} \cdot t \right\}. \quad (3·24)$$

It agrees with the result which is gained in Abelian Antisymmetric Tensor Fields (A-ASTF) case with 2nd-order action (2-3). The linearly divergent term corresponds to the gauge degrees of freedom, then the number of true gauge degrees of freedom is three just the trace of $(1 - P)_{\mu \nu, \alpha \beta}$. The transverse part is the Landau gauge propagator $(\beta_i = 0)$ in (A·3). The divergent longitudinal contribution will be evaluated in the next section.

(3·22) also gives the propagator

$$\lim_{t \to \infty} \langle A^a_{\mu}(k, t) A^b_{\alpha}(k', t) \rangle = 2(2\pi)^4 \delta^4(k + k') \delta^{ab} \cdot \frac{2}{a^2} \frac{k \cdot k'}{k^2} , \quad (3·25)$$

which is identical with the ordinary result. It reminds us that the theory of 2nd-rank ASTF is equivalent to scalar fields theory in 4 dimensions (the number of independent component of $n$th-rank antisymmetric tensor $B_{\mu_1 \mu_2 \cdots \mu_n}$ in $D$-dimensions is $d \cdot \mathcal{C}_n$). This is apparent from the fact that the free part in the solution of the field equation for $B^a_{\mu}$ is $A^a_{\mu} = \partial_{\mu} \phi^a$. It makes the ASTF action into scalar fields action.

Equation (3·23) gives us

$$\lim_{t \to \infty} \langle B^a_{\mu}(k, t) A^b_{\alpha}(k', t) \rangle = 2(2\pi)^4 \delta^4(k + k') \delta^{ab} \cdot \frac{ie_{\mu \nu \rho \sigma k_{\rho} k_{\sigma}}}{ak^2} . \quad (3·26)$$

It is identical with the ordinary result.

Second we calculate the lowest-order contributions to the 3-point function, i.e., vertex $\langle B^a_{\mu}(k) A^b_{\rho}(p) A^c_{\sigma}(q) \rangle$. Here we suppress the linearly divergent longitudinal term $G_{BBT} \equiv (1 - P)$ which will be examined in the next section. In order to calculate the vertex, we must compute 36 stochastic diagrams as a whole. By using the notations of (3·20), the symbolized equation for vertex is

$$G_{BBT} \cdot V_B \cdot (G^{AB} G^{BA} + G^{AA} G^{AB})^2$$

$$+ G^{BA} \cdot V_A \cdot (G^{BBT} G^{BA} + G^{BA} G^{AA})(G^{AB} G^{BA} + G^{AA} G^{AB})$$

$$+ G^{AB} \cdot V_B \cdot (G^{BBT} G^{BA} + G^{BA} G^{AA})(G^{AB} G^{BBT} + G^{AA} G^{AB})$$

$$+ G^{AA} \cdot V_A \cdot (G^{BBT} G^{BA} + G^{BA} G^{AA})(G^{AB} G^{BBT} + G^{AA} G^{AB})$$

$$+ G^{AA} \cdot V_A \cdot (G^{BBT} G^{BBT} + G^{BA} G^{AB})(G^{AB} G^{BA} + G^{AA} G^{AA}) , \quad (3·27)$$

where $G_{BBT} \equiv (1/(\lambda_1 - \lambda_2))\{ - \lambda_2 \rho e^{-(a(4) \lambda_1) (t - \tau)} + \lambda_1 \rho e^{-(a(4) \lambda_2) (t - \tau)} \}$, or graphically (Fig. 3). It is apparent from (3·27) that the correlations are all represented as the combination of propagators like $G^{AB} G^{BA} + G^{AA} G^{AA}$. This is just the reason why we have derived the previous formulas and drawn the stochastic diagrams with parallel lines like Figs. 1–4. Using these formulas (3·21)~(3·23), after tedious calculations, we get the result exactly,
This is nothing but the Landau gauge \((\beta_1=0)\) vertex given in the Appendix which is obtained from only one diagram.

Finally we can recognize the transverse component just reproduces the standard Landau gauge results. Therefore we will examine the longitudinal component contributions in the next section.

\section{1-loop calculations for gauge invariant quantity}

From now on, we will calculate the 1-loop diagrams including the divergent longitudinal components and show that the ghost contributions surely exist. Here we evaluate the total Lagrangian,

\[
L = \frac{a}{8} \varepsilon_{\mu\nu\rho\sigma} B^a_{\mu\nu}(x) F^a_{\rho\sigma}(x) + \frac{a^2}{8} A^a_{\mu}(x) A^a_{\mu}(x), \tag{4.1}
\]

as a gauge invariant quantity. We will compute the expectation value of (4.1) up to a \(g^2\)-order, i.e., 1-loop order as follows:

\[
\lim_{y \to x} \left\langle \frac{a}{8} \varepsilon_{\mu\nu\rho\sigma} B^a_{\mu\nu}(x) F^a_{\rho\sigma}(y) + \frac{a^2}{8} A^a_{\mu}(x) A^a_{\mu}(y) \right\rangle \\
= \lim_{y \to x} \int \frac{d^4k d^4k'}{(2\pi)^8} e^{-ikx -ik'y} \left\langle \frac{a}{8} \varepsilon_{\mu\nu\rho\sigma} B^a_{\mu\nu}(k) F^a_{\rho\sigma}(k') + \frac{a^2}{8} A^a_{\mu}(k) A^a_{\mu}(k') \right\rangle. \tag{4.2}
\]

Within the expectation value \(\langle \cdot \rangle\) there are 2-point function and 3-point one. 2-point part is

\[
\frac{a}{8} \cdot \{ 2i \varepsilon_{\mu\rho\sigma} k_\rho \langle B^a_{\mu\nu}(k) A^a_{\sigma}(k') \rangle + \alpha \langle A^a_{\mu}(k) A^a_{\mu}(k') \rangle \}, \tag{4.3}
\]

and 3-point part is

\[
\frac{a}{8} g f^{abc} \varepsilon_{\mu\rho\sigma} \int \frac{d^4p d^4q}{(2\pi)^8} \langle B^a_{\mu\nu}(k) A^b_{\rho}(p) A^c_{\sigma}(q) \rangle \delta^4(k' - p - q). \tag{4.4}
\]

We will explore them separately.
2-Point Function

In the lowest-order of perturbation, by (3·18) the combination of $B_{\mu}(k)$ and $A_{\alpha}(k)$ of (4·3) becomes

$$2i\varepsilon_{\mu\nu\rho\delta}k_{\rho}B_{\mu}(k, t) + aA_{\delta}(k, t)$$

$$= \int_0^t d\tau \bigg[ 2i\varepsilon_{\mu\nu\rho\delta}k_{\rho} \{ G_{\mu\nu, \alpha\beta}(k, t-\tau)\eta_{\alpha\beta}(k, \tau) + G^{BA}_{\mu\nu, \gamma}(k, t-\tau)\xi^\gamma(k, \tau) \}$$

$$+ a \{ G^{AB}_{\alpha\beta}(k, t-\tau)\eta^\alpha(k, \tau) + G_{\gamma}(k, t-\tau)\xi^\gamma(k, \tau) \} \bigg]$$

$$= \int_0^t d\tau \{ i\varepsilon_{\mu\nu\rho\delta}k_{\rho} \{ e^{(-\alpha/4)\lambda_1(t-\tau)} + e^{(-\alpha/4)\lambda_2(t-\tau)} \} \eta_{\alpha\beta}(k, \tau)$$

$$+ [ T_{\gamma}(\lambda_1 e^{(-\alpha/4)\lambda_1(t-\tau)} + \lambda_2 e^{(-\alpha/4)\lambda_2(t-\tau)}) + aL_{\gamma}\tau e^{(-\alpha/4)(t-\tau)} ]\xi^\gamma(k, \tau) \}$$

$$= \int_0^t d\tau \{ D_{\mu, \alpha\beta}(k, t-\tau)\eta^\alpha(k, \tau) + D^\delta_{\gamma}(k, t-\tau)\xi^\gamma(k, \tau) \}. \quad (4·5)$$

Due to the tensorial property, $\varepsilon_{\mu\nu\rho\delta}(1-P)_{\mu\nu, \alpha\beta}=0$, the longitudinal contribution $\varepsilon_{\mu\nu\rho\delta}G^{BB}_{\mu\nu, \alpha\beta}$ does not exist in the external lines. Then we have to take into account the diagrams which have $G^{BB}$ within the internal lines. Here we calculate five kinds of diagrams (Fig. 4). The internal lines are expressed in the brackets [·].

$$D^A V_A \{ (G^{BB}(G^{BB})(G^{AB}G^{BA} + G^{AA}G^{AA})) \} V_A G^{AA}, \quad (4·6a)$$

Fig. 4. 1-loop contributions including t-linearly divergent longitudinal parts.
Here the external lines $D^A$ and $D^B$ are denoted by solid lines with their indices. For calculating above diagrams, we can use the following formula like $(3 \cdot 21) - (3 \cdot 23)$,

\[
\int_0^t d\tau \int_0^{\tau'} d\tau' \{ D^A_{\rho \sigma}(k, t - \tau) G^{AB}_{\rho \sigma}(k, t') + D^B_{\rho \sigma}(k, t - \tau) G^{BA}_{\rho \sigma}(k, t') \} \delta(\tau - \tau')
\]

\[
= \frac{2 \alpha}{\alpha^4} \left[ T_{\rho \sigma}(e^{-(\alpha/4)\Lambda_1(\tau - \tau')} - e^{-(\alpha/4)\Lambda_1(t + \tau')} + e^{-(\alpha/4)\Lambda_2(\tau - \tau')} - e^{-(\alpha/4)\Lambda_2(t + \tau')})
\]

\[
+ L_{\rho \sigma}(e^{-(\alpha/4)\Lambda_1(\tau - \tau')} - e^{-(\alpha/4)(\tau + t')}) \right].
\]

Then $(4 \cdot 3)$ with momentum integrations reduces to

\[
\alpha \int \frac{d^4 k d^4 p d^4 q}{(2\pi)^8} \delta^4(k + p + q) g^2(f_{abc})^2 \epsilon_{\mu \nu \rho \sigma} \epsilon_{\mu' \nu' \rho' \sigma'} L_{\mu \nu}(q)
\]

\[
\times \left[ \frac{4}{\alpha^4} \cdot L_{\rho \sigma}(k) L_{\sigma \rho}(p) \cdot t + \left\{ \frac{4}{\alpha^4} \cdot T_{\rho \sigma}(k) L_{\sigma \rho}(p) - \frac{8}{\alpha^4} \cdot L_{\rho \sigma}(k) L_{\sigma \rho}(p) \right\}
\]

\[
- \frac{\alpha}{8} \cdot \frac{T_{\rho \sigma}(k)}{\Lambda_1(k) - \Lambda_2(k)} \cdot \frac{T_{\sigma \rho}(p)}{\Lambda_1(p) - \Lambda_2(p)}
\]

\[
\times \left\{ -\frac{1}{\Lambda_1(k) + \Lambda_1(p)} + \frac{1}{\Lambda_1(k) + \Lambda_2(p)} + \frac{1}{\Lambda_2(k) + \Lambda_1(p)} - \frac{1}{\Lambda_2(k) + \Lambda_2(p)} \right\}
\]

\[
+ \frac{1}{\alpha} \cdot \frac{L_{\sigma \rho}(p)}{\Lambda_1(k) - \Lambda_2(k)} \cdot \frac{T_{\rho \sigma}(k)}{\Lambda_1(k) + \frac{\alpha^2}{4} - \Lambda_2(k) + \frac{\alpha^2}{4}}
\]

\[
+ \frac{1}{\alpha} \cdot \frac{L_{\rho \sigma}(p)}{\Lambda_1(p) - \Lambda_2(p)} \cdot \frac{T_{\rho \sigma}(k)}{\Lambda_1(p) + \frac{\alpha^2}{4} - \Lambda_2(p) + \frac{\alpha^2}{4}}
\]

\[
+ \frac{4}{\alpha^2} \cdot L_{\rho \sigma}(k) L_{\sigma \rho}(p) \right].
\]

Here, $\Lambda_{1,2}$ was defined in the previous section as $\Lambda_{1,2}=1/4((a^2 - \sqrt{a^2 - 8k^2})/2)$. 1st term linearly diverges, and the remainings are finite.

3-Point Function

The calculations of 3-point function $(4 \cdot 4)$ are much simpler than those of 2-point one. As we consider the longitudinal contributions, we only replace $G^{\rho \sigma}$ by $G^{\rho \sigma}$ in $(3 \cdot 27)$. The diagrams are just the same as Fig. 3.

This result also consists of a $t$-divergent term and finite terms.
Now, owing to the symmetry of \((kPq)\)-integrations and \((\nu\rho\sigma)\)-summations, the sum of (4·8) and (4·9) makes a simple result,

\[
\frac{\alpha}{8} \int \frac{d^4 k d^4 p d^4 q}{(2\pi)^8} \delta^4(k + p + q) g^2(f^{abc})^2 \epsilon_{\mu\nu\rho\sigma} \epsilon_{\mu'\nu'\rho'\sigma'} L_{\mu\nu}'(q) L_{\sigma}(p) 
\times \left[ \frac{4}{\alpha^2} \cdot L_{\mu\nu}'(q) L_{\sigma}(p), t - \frac{\alpha}{8} \cdot \frac{T_{\sigma}(p)}{A_1(p) - A_2(p)} \cdot \frac{T_{\mu\nu}'(q)}{A_1(q) - A_2(q)} \right] 
\times \left\{ -\frac{1}{A_1(p) + A_1(q)} + \frac{1}{A_1(p) + A_2(p) + A_1(q)} - \frac{1}{A_2(p) + A_2(q)} \right\} 
\times \frac{1}{2} \left\{ \frac{1}{A_1(p) + \frac{\alpha^2}{4}} - \frac{1}{A_2(p) + \frac{\alpha^2}{4}} \right\} 
\times \frac{1}{2} \left\{ \frac{1}{A_1(q) + \frac{\alpha^2}{4}} - \frac{1}{A_2(q) + \frac{\alpha^2}{4}} \right\} 
\times \frac{4}{\alpha^3} \cdot L_{\sigma}(p) L_{\mu\nu}'(q). \tag{4·9}
\]

This result explicitly coincides with that of the ordinary field theory which is obtained from (A·3), (A·4) and (A·6). The \(t\)-divergent terms which reflect the gauge-invariance are cancelled each other. Thus, the fact that the gauge-invariant quantities do not have any \(t\)-divergent terms also holds for the RGT. Note that this result is just from both the primary ghosts and the secondary ones. Therefore we can conclude that up to a \(g^2\)-order, the SQ of NA-ASTF automatically leads us to the correct Faddeev-Popov (both primary and secondary) ghost effects without resort to any ghost fields.

§ 5. Discussion

In this paper, we have quantized the NA-ASTF by the stochastic method, and by calculating the expectation value of the gauge invariant quantity up to the 2nd-order perturbation, we have shown that the finite parts of the solutions of Langevin equations reproduce the usual Landau gauge results and the divergent longitudinal parts lead both primary and secondary Faddeev-Popov ghost field contributions. These results indicate that the SQM has the validity for quantization of the RGT. We might conclude that we can use the SQM for any-stage RGT without resorting the artificial fields, because the full (primary, secondary, tertiary· · ·) ghost fields contributions are summed up to the longitudinal component of gauge fields (however, whether the SQM has the validity for topological quantum field theory, for example, topological
YM-theory, is not somewhat clear). This will be proven inductively if we treat the $(D-2)$th-rank NA-ASTF $B_{\rho\sigma}^a$ in $D$-dimensions with a subsidiary field $A_\mu^a$ as a $(D-3)$th-stage reducible gauge theory,

$$L_M = -\frac{\varepsilon^{\mu_1\mu_2\rho_1\rho_2}}{2(D-2)} B_{\rho_1\rho_2}^a F_{\mu_1\mu_2}^a + \frac{1}{2} A_\mu^a A^{\mu a} .$$

(5.1)

Although we have examined the bosonic gauge theories, we may expect this feature of divergent term also can be realized in the fermionic gauge theories (supersymmetric gauge theories, etc.). However, the natural prescription for fermions (with spinorial structures) by the SQM has not been established until now.

In general, the covariant quantization of Green-Schwartz Superstring\textsuperscript{10) and Brink-Schwartz Superparticle\textsuperscript{11) needs infinite ghost fields according to the infinitely reducible gauge invariance, i.e., so-called Siegel $\kappa$-symmetry.\textsuperscript{12) Many attempts have been done,\textsuperscript{13) but until now the covariantly gauge-fixed action has not been realized clearly. Incidentally we may say that, if we can treat fermionic fields by the SQM, because we need not introduce any ghost fields, above complexity may be resolved in another way.

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Appendix

In this appendix, we give the Euclidean Feynman rules of BRST covariantly gauge fixed NA-ASTF.

Following the Kugo-Ojima formalism,\textsuperscript{7) covariantly gauge-fixed Lagrangian in Minkowski space-time is given as

$$L_M = -\frac{\alpha}{8} \varepsilon^{\mu_1\mu_2} F_{\mu_1\mu_2}^a + \frac{\alpha^2}{8} A_\mu^a A^{\mu a} + B_{\mu}^a \partial^\mu B_{\mu}^a$$

\[+ i \bar{C}^{\mu a} \partial^\mu (D_\mu C_{\nu} - D_\nu C_\mu)^a + \frac{1}{2} \beta_1 B^{a\alpha} B^\alpha_a + i \bar{C}^{\mu a} \partial^\mu C_{\mu}^a \]

\[+ \bar{d}^{\alpha} \partial_\mu (D_\mu d)^a + \partial^\mu B^a_{\mu} \cdot B^a + i \partial^\mu \bar{C}^a_{\mu} \cdot C^a + i \beta_2 \bar{C}^a_\alpha C^\alpha . \]

(A·1)

Here, $\beta_1$ and $\beta_2$ are gauge parameters for the primary gauge-fixing and secondary one, respectively, instead of the usual $\alpha$. For primary gauge-fixing $\partial^\mu B^a_{\mu} = 0$, we introduce the primary Faddeev-Popov (FP) ghost $\bar{C}^a_{\mu}$, anti-ghost $C^a_{\mu}$ and the Nakanishi-Lautrup (NL) field $B^a_{\mu}$. There are two secondary gauge-fixings. One is for $\partial^\mu C^a_{\mu} = 0$ by introducing the FP ghost $d^a$, anti-ghost $\bar{d}^a$ and the NL field $\bar{C}^a_{\mu}$. The other is for $\partial^\mu \bar{C}^a_{\mu} = 0$ by introducing the the FP anti-ghost $B^a$ and the NL field $C^a$. After integrating out the auxiliary fields except $A_\mu^a$, we arrive at

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\[+ i \bar{C}^{\mu a} \partial^\mu (D_\mu C_{\nu} - D_\nu C_\mu)^a + \frac{1}{2} \beta_1 B^{a\alpha} B^\alpha_a + i \bar{C}^{\mu a} \partial^\mu C_{\mu}^a \]

\[+ \bar{d}^{\alpha} \partial_\mu (D_\mu d)^a + \partial^\mu B^a_{\mu} \cdot B^a + i \partial^\mu \bar{C}^a_{\mu} \cdot C^a + i \beta_2 \bar{C}^a_\alpha C^\alpha . \]

(A·1)

Here, $\beta_1$ and $\beta_2$ are gauge parameters for the primary gauge-fixing and secondary one, respectively, instead of the usual $\alpha$. For primary gauge-fixing $\partial^\mu B^a_{\mu} = 0$, we introduce the primary Faddeev-Popov (FP) ghost $\bar{C}^a_{\mu}$, anti-ghost $C^a_{\mu}$ and the Nakanishi-Lautrup (NL) field $B^a_{\mu}$. There are two secondary gauge-fixings. One is for $\partial^\mu C^a_{\mu} = 0$ by introducing the FP ghost $d^a$, anti-ghost $\bar{d}^a$ and the NL field $\bar{C}^a_{\mu}$. The other is for $\partial^\mu \bar{C}^a_{\mu} = 0$ by introducing the the FP anti-ghost $B^a$ and the NL field $C^a$. After integrating out the auxiliary fields except $A_\mu^a$, we arrive at
Stochastic Quantization of Non-Abelian Antisymmetric Tensor Fields

\[ L_i = -\frac{\alpha}{8} \varepsilon_{\mu\nu\rho\sigma} B^{\mu}_{\rho} F^{\sigma}_{\nu} + \frac{\alpha^2}{8} A_{\mu} A^{\mu} + i \bar{C} \partial^\nu (D^\nu C_\nu - D_\nu C^\nu) \]

\[ + \bar{d}^\mu \partial_\mu (D^\nu d)^\nu - \frac{1}{2\beta_1} \partial_\mu B^{\nu\mu} \partial^\nu B_\mu + \frac{1}{2\beta_1} \partial_\mu B^\nu \partial^\nu B_\nu - \frac{i}{\beta_2} \partial^\nu C_\nu \partial^\mu C_\mu, \]

or in Euclidean space-time,

\[ L'_e = \frac{\alpha}{8} \varepsilon_{\mu\nu\rho\sigma} B^{\mu}_{\rho} F^{\sigma}_{\nu} + \frac{\alpha^2}{8} A_{\mu} A^{\mu} - i \bar{C} \partial_\mu (D^\nu C_\nu - D_\nu C^\nu) \]

\[ + \bar{d}^\mu \partial_\mu (D^\nu d)^\nu + \frac{1}{2\beta_1} \partial_\mu B^{\nu\mu} \partial^\nu B_\mu + \frac{1}{2\beta_1} \partial_\mu B^\nu \partial^\nu B_\nu - \frac{i}{\beta_2} \partial^\nu C_\nu \partial^\mu C_\mu. \quad (A\cdot2) \]

Here we use the notation introduced in § 3 for Euclidean space-time. Therefore, the necessary propagators and truncated vertices in the previous sections are derived from (A\cdot2) as

\[
\langle B^{\mu}_{\nu}(k) B^{\sigma}_{\rho}(k') \rangle = 2(2\pi)^4 \delta^4(k + k') \delta^{ab} \cdot \frac{1}{k^2} \{ P_{\mu\nu,ab} + \beta_1 (1 - P)_{\mu\nu,ab} \}, \\
\langle A^{\mu}_{\nu}(k) A^{\sigma}_{\rho}(k') \rangle = 2(2\pi)^4 \delta^4(k + k') \delta^{ab} \cdot \frac{2}{\alpha^2} \frac{k_\mu k_\nu}{k^2}, \\
\langle B^{\mu}_{\nu}(k) A^{\sigma}_{\rho}(k') \rangle = 2(2\pi)^4 \delta^4(k + k') \delta^{ab} \cdot \frac{i \varepsilon_{\mu\nu\rho\sigma} k_\sigma}{ak^2}, \\
\langle \bar{C}^{\mu}_{\nu}(k) C^{\sigma}_{\rho}(k') \rangle = (2\pi)^4 \delta^4(k + k') \delta^{ab} \left\{ -\frac{i}{k^2} \left[ \delta^{\mu\nu} - \frac{1}{1 - \beta_2} \frac{k_\mu k_\nu}{k^2} \right] \right\}, \\
\langle \bar{d}^{\mu}(k) d^{\nu}(k') \rangle = (2\pi)^4 \delta^4(k + k') \delta^{ab} \left\{ -\frac{1}{k^2} \right\}, \\
\langle B^{\mu}(k) B^{\nu}(k') \rangle = (2\pi)^4 \delta^4(k + k') \delta^{ab} \cdot \frac{1}{k^2}. \quad (A\cdot3) \]

\[
\langle B^{\mu}_{\nu}(k) A^{b}_{\rho}(p) A^{\sigma}_{\rho}(q) \rangle = -\frac{\alpha}{4} \varepsilon_{\mu\nu\rho\sigma} g^f_{abc}, \\
\langle A^{\mu}_{\nu}(k) C^{b}_{\rho}(p) C^{\sigma}_{\rho}(q) \rangle = (\delta_{\mu\rho} - p_{\mu} \delta_{\rho\sigma}) g^f_{abc}, \\
\langle A^{\mu}_{\nu}(k) d^{b}_{\rho}(p) d^{\sigma}_{\rho}(q) \rangle = -i \delta_{\mu\rho} g^f_{abc}, \quad (A\cdot4) \]

Using the above tools, we can calculate the vertex function \( \langle B^{\mu}_{\nu}(k) A^{b}_{\rho}(p) A^{\sigma}_{\rho}(q) \rangle \) as follows:

\[
-g^f_{abc} \cdot \frac{8}{\alpha^2} \varepsilon_{ab\rho\sigma} \cdot \{ P + \beta_1 (1 - P) \}_{\nu\mu,ab}(k) \cdot \frac{p_{\rho} q_{\sigma} q_{\sigma}}{k^2 p^2 q^2} \cdot (2\pi)^4 \delta^4(k + p + q). \quad (A\cdot5) \]

In the Landau gauge (\( \beta_1 = 0 \)), this is just the vertex obtained by the SQM in § 3 (3\cdot28).

Now, we calculate the 1-loop ghost contributions to the truncated propagator from the primary ghosts and the secondary ones separately. Therefore, the contributions are

\[
\Sigma_1^{\mu\nu}_{\rho\sigma} = -g^f_{abc} f^{abc} f^{abc} \cdot \frac{2 \mu \sigma q + \rho \sigma q}{p^4 q^2}, \quad (A\cdot6) \\
\Sigma_2^{\mu\nu}_{\rho\sigma} = g^f_{abc} f^{abc} \cdot \frac{\mu q + \rho q}{p^2 q^2}. 
\]
where $\Sigma_1^{ad\mu\sigma}$ is from the primary ghosts, and $\Sigma_2^{ad\mu\sigma}$ is from the secondary ones. Here we omit the momentum integrations. Then, after some calculations, the contribution to the gauge-invariant quantity $\langle (a/8) \varepsilon_{\mu\nu\rho\sigma} B^{a\mu}_{\nu\rho}(x) F^{a\nu}_{\rho\sigma}(y) + (a^2/8) A^{a\nu}_{\mu}(x) A^{a\sigma}_{\nu}(y) \rangle$ agrees with (4.10).
References