Inner Region and Nonstatic Effects of Nuclear Forces in the Triplet Odd State

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Nuclear forces at small distances and nonstatic effects are discussed in the triplet odd state by use of the potential model. It is investigated whether singular behavior such as the repulsive core is indispensable in this state. Analysis shows that the hard-core like repulsion is not indispensable and nonsingular potentials reproduce the solutions of the phase shift analysis very well. The suppression of rapid increase of the $^3F_3$-phase shift at high energies is obtained by introducing the quadratic spin-orbit potentials modified so as to be effective in the coupled states. The behavior of the $^3F_3$-phase shift at $E \sim 50$ Mev is also discussed in connection with properties of potentials in the intermediate region.

§ 1. Introduction

The analysis of nuclear forces at low energies has successfully established the one-pion-exchange (OPE) contribution in the region I ($x \geq 1.5$).\(^1\) The contribution of this OPE-potential-tail plays the important role at high energies as well as low energies.

The modified phase shift analysis including the OPE-contribution has achieved the remarkable improvement and established the unique solution.\(^5\) The success of these approach is essentially owing to the Taketani theory\(^4\) to approach from the outside of the nuclear interaction toward the inside, combining our reliable meson-theoretical prediction at large internucleon distances with a phenomenological description at short distances. The determination of the unique solution of the phase shifts in $pp$-scattering have led to the good representation of nucleon-nucleon interaction in terms of the potential model.\(^5,6,7\)

The properties of the intermediate region (the region II, $x = 0.7 \sim 1.5$) inferred from the potential model have shown the importance of the heavy mesons such as $\rho$ and $\omega$, which has led to the one-boson-exchange model (OBEM)\(^9,10,11\) representing the main parts of nuclear forces in terms of the exchange of bosons (the pion and the resonant states of mesons). Essential features of the region II can be understood on the basis of the exchange of one, two and more pions

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\(^2\) $x$ is the internucleon distance in the unit of the pion-Compton wave length.
and of heavy mesons. This understanding makes it possible to clarify properties of the inner most region (the region III, $x \lesssim 0.7$), establishing the region II quantitatively.

What an approach is powerful at present in order to investigate characteristic features such as core in the inner region? In the region where nuclear forces are very strong and energy concerned becomes high, relativistic or at least nonstatic treatment is desirable. There are two such treatments; dispersion-theoretic one and potential model in momentum space. Although the dispersion-theoretic approach is fully relativistic and has been thought very desirable in many aspects, it has needed some serious approximations in actual application. Moreover, some problems yet unsolved are present concerning the low energy behavior of phase shifts with high angular momentum\(^\text{11}\) and convergence of iterative procedure in high partial wave amplitudes.\(^\text{12}\) Momentum space calculations have been performed only for the OPEP,\(^\text{13}\) and usefulness in dealing with the inner region is not clear yet. Although the concept, potential, is inherently nonrelativistic, in order to investigate the inner region the potential approach is most useful at present because of its manageability and feasibility of intuitive understanding. In the application of the potential model, however, higher order nonstatic effects in momentum than the spin-orbit potential must be properly taken into account in terms of potential. It seems an effective and successful way to convert essential features obtained by the potential approach into general and complete treatments on nucleon-nucleon interaction.

The problems left as open questions are: (1) whether the hard-core like repulsion exists in the triplet odd state and (2) on nonstatic effects other than the spin-orbit force. All the potentials so far proposed assume the hard-core in the triplet odd state only for convenience sake. Investigation of the existence of the hard-core in the triplet odd state is important to answer the question whether the hard-core exists in all the two-nucleon states or not.\(^\text{14}\) In this paper, analysis is done with both potentials with the hard-core and without repulsive core.

The rapid increase of the $^3F_t$-phase shift at high energies takes place due to the strong spin-orbit potential introduced to fit the $^3P$-wave phase shifts, and the suppression of the $^3F_t$-phase shift is one of problems which is not satisfactorily answered. Hamada-Johnston's potential has the large $^3F_t$-phase shift, since the introduced quadratic spin-orbit potentials are effective only in the uncoupled states.\(^\text{9}\) The situation is the same for the Yale-potential. Then Breit proposed the spin-orbit potential effective only in the states with $J \leq 2$, where $J$ is the magnitude of the total angular momentum.\(^\text{7}\) In this paper, we adopt the quadratic spin-orbit potentials modified in the regions II and III so as to be effective in the coupled states. With this quadratic spin-orbit potentials, the $^3F_t$-phase shift is reasonably suppressed without destroying the $^3P$-fits. Their strength is of the same order of magnitude with those of the quadratic spin-
orbit potentials introduced in the other states. However, these quadratic spin-orbit potentials should be considered as a phenomenological representation for all the higher order nonstatic effects, because other nonstatic potentials appear at the same time in the derivation of the quadratic spin-orbit potentials.

The behavior of the $^3P_0$-phase shift at intermediate energy region (laboratory energy $E=40 \sim 100$ MeV) is important to determine the quantitative properties of potentials in the region II. The experiments to reduce the ambiguity of the $^3P_0$-phase shift have performed recently at $E \approx 50$ MeV. We discuss also the $^3P_0$-problem.

In § 2, the potential forms used in the analysis is presented. Importance of the quadratic spin-orbit potentials is discussed in the presence of the hard-core in § 3. In § 4, analysis is done without repulsive core, and we show that the hard-core is not indispensable. Discussions concerning $\delta(^3P_0)$ at $E \approx 50$ MeV are done in § 5.

§ 2. Choice of potential parameters

The shape of the potential employed is the same as that used in the previous paper for the cases with the hard-core (denoted by HC) with the radius $x_c$;

$$V(x) = V_0(x) + V_T(x) S_{13} + V_{LS}(x)(L \cdot S) + V_q(x) Q_{13} + V_{LL}(x) L^2, \quad x \geq x_c$$

$$= + \infty, \quad x < x_c.$$

As the shape of the core-less nonsingular potentials, we adopt the square well in the region III (denoted by SQ), with the radius $x_1$;

$$V(x) = V_0(x) + V_T(x) S_{13} + V_{LS}(x)(L \cdot S) + V_q(x) Q_{13} + V_{LL}(x) L^2, \quad x \geq x_1$$

$$= V_0^0 + V_T^0 S_{13} + V_{LS}^0 (L \cdot S) + V_q^0 Q_{13} + V_{LL}^0 L^2, \quad x < x_1,$$

where

$$Q_{13} = - (1/2) [ (\sigma_1 \cdot L) (\sigma_2 \cdot L) + (\sigma_2 \cdot L) (\sigma_1 \cdot L)] = \delta_{2L} L^2 - (L \cdot S)^2,$$

and in the triplet odd state,

$$V_0(x) = \mu c^2 \left( \frac{f^2}{4\pi} \right) \frac{e^{-x}}{x} \left( 1 + a_0 x e^{-x} + b_0 x e^{-2x} \right),$$

$$V_T(x) = \mu c^2 \left( \frac{f^2}{4\pi} \right) \frac{e^{-x}}{x} \left( 1 + 3 \frac{x}{x^2} \right) \left( 1 + a_T x e^{-x} + b_T x e^{-2x} \right),$$

$$V_{LS}(x) = \mu c^2 \left( C_1 e^{-3x} + C_2 e^{-x} \right),$$

$$V_q(x) = - \frac{\mu c^2}{2} \left( \frac{\mu}{M} \right)^2 \left( \frac{f^2}{4\pi} \right) \frac{e^{-x}}{x^3} \left( 1 + \frac{x}{x^2} \right) \left( 1 + a_q x e^{-x} + b_q x e^{-2x} \right),$$

$$V_{LL}(x) = - \frac{\mu c^2}{2} \left( \frac{\mu}{M} \right)^2 \left( \frac{f^2}{4\pi} \right) \frac{e^{-x}}{x^3} \left( 1 + \frac{x}{x^2} \right) \left( 1 + a_{LL} x e^{-x} + b_{LL} x e^{-2x} \right),$$

and in the triplet even state,
with the pion-nucleon coupling constant $\frac{f^2}{4\pi} = 0.08$, the average pion mass $= 139.4\text{ MeV}$ and $\mu/M = 0.1489$ ($M$ being the nucleon mass).\textsuperscript{a) $V_0$, etc., are constants. The $e^{-x^2/x^2}$-dependence in $V_{LS}(x)$ means that the main parts of $^3V_{LS}$ in the intermediate region come from the exchange of the vector bosons, $\rho$ and $\omega$. The parametrization of the other terms are the same as Hamada-Johnston's\textsuperscript{g)} except the reduction factor $\lambda$ in $V_\rho$ and $V_{LL}$, which is a phenomenological description that the effects of $V_\rho$ and $V_{LL}$ cancel with those of the other nonstatic terms in the Born approximation for the OPEP-parts of $V_\rho$ and $V_{LL}$.

For later references we give here the Schrödinger equations when the potentials are given by Eq. (1) or (2):

\begin{align}
L &= J:
\left[ \frac{d^2}{dx^2} + \frac{k^2}{x^2} - \frac{J(J+1)}{x^2} \right] v_J(x) \\
&= [U_0 + 2U_T - U_{LS} + \{1 - J(J+1)\} U_Q + J(J+1) U_{LL}] v_J(x),
\end{align}

\begin{align}
L &= J\pm 1:
\left[ \frac{d^2}{dx^2} + \frac{k^2}{x^2} - \frac{J(J-1)}{x^2} \right] u_J(x) = & [U_0 - \frac{2(J-1)}{(2J+1)} U_T + (J-1) U_{LS} \\
& - (J-1)U_Q + J(J-1) U_{LL}] u_J(x) + \frac{6\sqrt{J+1}}{(2J+1)} U_T u_J(x), \\
\left[ \frac{d^2}{dx^2} + \frac{k^2}{x^2} - \frac{(J+1)(J+2)}{x^2} \right] w_J(x) = & [U_0 - \frac{2(J+2)}{(2J+1)} U_T - (J+2) U_{LS} \\
& - (J+2)U_Q + (J+1)(J+2) U_{LL}] w_J(x) + \frac{6\sqrt{J+1}}{(2J+1)} U_T w_J(x),
\end{align}

\begin{align}
\text{where } k^2 = ME/2\mu^2 \text{ and } U_c = MV_c/\mu^2, \text{ etc. } \text{ The potential in each state aside the tensor coupling term is as follows:}
\begin{align}
^3P_0; V_0 &= V_c - 4V_T - 2V_{LS} - 4V_Q + 2V_{LL}, \\
^3P_1; V_1 &= V_c + 2V_T - V_{LS} + V_Q + 2V_{LL}, \\
^3P_2; V_2 &= V_c - (2/5) V_T + V_{LS} - V_Q + 2V_{LL}, \\
^3F_0; V_0' &= V_c - (8/5) V_T - 4V_{LS} - 16V_Q + 12V_{LL}, \\
^3F_1; V_1' &= V_c + 2V_T - V_{LS} + 11V_Q + 12V_{LL}, \\
^3F_2; V_2 &= V_c - (2/3) V_T + 3V_{LS} - 9V_Q + 12V_{LL}.
\end{align}

\text{The potential parameters are given in Table I and the shapes are shown}

\textsuperscript{a) In the case where the discrimination of the states is necessary, we denote the potential with the spin $S$ and the parity $\Pi$ with $^{3S+1}P_0^\Pi$ etc.}
Fig. 1. Triplet odd potentials. HC means the hard-core potentials and SQ means the core-less nonsingular potentials. The marks of potentials correspond to those in Table I.
Table I. Potential parameters for the hard-core and core-less nonsingular potentials. Notations are given in Eqs. (1)~(3). The following parameters are fixed: \( n=3 \), \( \nu=6.7 \), \( \lambda=0.25 \), \( a_Q=2 \), \( b_Q=-0.01 \), \( a_{LL}=-2.5 \), \( b_{LL}=-0.01 \).

<table>
<thead>
<tr>
<th>Case</th>
<th>Outer parameters</th>
<th>Inner parameters ((V^0\text{ in Mev}))</th>
<th>Comments</th>
</tr>
</thead>
</table>
|      | \( a_C \) \( b_C \) \( a_T \) \( b_T \) \( G_1 \) \( G_2 \) \( x_C \) | \( x_1 \) \( V_c^0 \) \( V_T^0 \) \( V_{LS}^0 \) \( V_Q^0 \) \( V_{LL}^0 \) | Standard for HC
|       | \( G_1 < G_1(\text{HC-1}) \) \( V_Q=V_{LL}=0 \) large \( x_C \) Outer \( V_T \) is weak |
| HC-1 | -12 10 0 \(-1.3\) \(-5.8\) \(-3\) 0.2837 | \( \frac{1}{2} \) \( 75.65 \) \( 6.19 \) \(-182.48\) \(-20\) 26.64 |
| HC-2 | -11 10 0 \(-1.3\) \(-6.2\) \(-3\) 0.2837 | \( \frac{1}{2} \) \( 24.38 \) \(-180.40\) \(-42.17\) 55 |
| HC-3 | -12 10 0 \(-1.3\) \(-5.8\) \(-3\) 0.2837 | \( \frac{1}{2} \) \( 136.11 \) \(-10.07\) \(-184.59\) \(-20\) 22.39 |
| HC-4 | -15 15 \(-1.3\) \(0.55\) \(-5.8\) \(-3\) 0.2837 | \( \frac{1}{2} \) \( 84 \) \(-46.81\) \(-176.25\) \(-20\) 59.38 |
| SQ-1 | -18 12 0 \(-1.3\) \(-5.8\) \(-3\) 0.698 | \( \frac{1}{2} \) \( 75.65 \) \(-6.19\) \(-182.48\) \(-20\) 26.64 |
| SQ-2 | -18 12 0 \(-1.3\) \(-5.8\) \(-3\) 0.698 | \( \frac{1}{2} \) \( 24.38 \) \(-180.40\) \(-42.17\) 55 |
| SQ-3 | -18 12 0 \(-1.3\) \(-5.8\) \(-3\) 0.698 | \( \frac{1}{2} \) \( 136.11 \) \(-10.07\) \(-184.59\) \(-20\) 22.39 |
| SQ-4 | -20 12 \(-1.3\) \(0.55\) \(-5.8\) \(-3\) 0.698 | \( \frac{1}{2} \) \( 84 \) \(-46.81\) \(-176.25\) \(-20\) 59.38 |
| SQ-5 | -20 12 \(-1.3\) \(0.55\) \(-5.8\) \(-3\) 0.698 | \( \frac{1}{2} \) \( 75.65 \) \(-6.19\) \(-182.48\) \(-20\) 26.64 |
| SQ-6 | -20 12 \(-1.3\) \(0.55\) \(-5.8\) \(-3\) 0.698 | \( \frac{1}{2} \) \( 136.11 \) \(-10.07\) \(-184.59\) \(-20\) 22.39 |
| SQ-7 | -18 12 0 \(-1.3\) \(0\) \(0\) \(0.698\) | \( \frac{1}{2} \) \( 92.02 \) \(6.6\) \(-269.55\) \(-30\) 37.6 |

We regard HC-1 as the standard potential for the hard-core potentials and SQ-1 with the same outer parameters (except \( V_c(x) \)) with those of HC-1 as the standard one for the coreless nonsingular potentials. The other potentials are discussed by referring to HC-1 and SQ-1.

§ 3. Importance of quadratic spin-orbit potentials

The rapid increase of \( \delta(^3F_3)^0 \) at \( E \geq 200\) MeV observed in the potential model so far proposed is mainly due to the strong spin-orbit potential necessary to the \( \delta(^3P_2) \)-fit. The quadratic spin-orbit potentials introduced by Hamada and Johnston play no essential role in reducing \( \delta(^3F_3) \), since these potentials are chosen so as to be effective in the uncoupled states (e.g. \( ^1D_1 \) and \( ^3D_2 \)). In this section we introduce \( V_Q \) and \( V_{LL} \) effective in the coupled states and discuss the role of such quadratic spin-orbit potentials for the hard-core potential.

Without \( V_Q \) and \( V_{LL} \), \( \delta(^3F_3) \) becomes large at 310 MeV (e.g. 0.07 in HC-3) and makes the hump in the cross section. The discrepancy with the result of the phase shift analysis becomes serious even qualitatively at 660 MeV (\( \delta(^3F_3) = 0.3 \sim 0.4 \)) compared with the values of the phase shift analysis (0.03 \sim 0.06)\(^{(2)}\).

\(^{(2)}\) \( \delta(^3L_J) \) means the \( ^3L_J \)-phase shift and its numerical values are shown in radians.
The more singular $V_{LS}$ with the shorter range gives the more rapid increase of $\delta \langle \bar{F}_0 \rangle$: For an example HC–3 with $x_0 = 0.3465$ and $V_{LS}(x) = -\mu c^2 \left(4.0 \exp \left(-5.5x\right)/x^5 + 4.5 \exp \left(-6.7x\right)/x^4\right)$ gives $\delta \langle \bar{F}_0 \rangle \cong 0.08$ at $310 \text{ MeV}$ and $\cong 1.2$ at $660 \text{ MeV}$.

Therefore, besides $V_{LS}$ we need some additional nonstatic terms or some $L$-dependence of potentials. The latter possibility was adopted by the Yale group, cutting off the contribution of $V_{LS}$ to the states with $J \geq 3$. Generally, as shown by Okubo and Marshak,\textsuperscript{16} $L^2$-dependence of potential appears in all the terms as nonstatic effects. In spite of this general consideration, it is not clear why only $V_{LS}$ needs $L$-dependence. In this paper, we adopt the former possibility, that is, the suppression of $\delta \langle \bar{F}_0 \rangle$ comes from some additional nonstatic effects. The reasons are: (1) the meson theory predicts the terms such as $V_Q$ and $V_{LL}$;\textsuperscript{19} (2) the nonstatic effect, $V_{LS}$, appears even at rather low energies, and therefore the higher order nonstatic effects in momentum also appear at high energies ($E \gtrsim 200 \text{ MeV}$); (3) in fact, the evidence is found in the $^3D_2$- and $^3F_2$-states; (4) in the OBEC calculation, where all nonstatic effects are automatically included, no rapid increase of $\delta \langle \bar{F}_0 \rangle$ arises even at $660 \text{ MeV}$\textsuperscript{20}. The

![Fig. 2. Effective potentials in the $^3P_J$ and $^3F_J$-states, for both cases with and without $V_Q$ and $V_{LL}$. The singular attraction of $V_4$ disappears with $V_Q < 0$ and $V_{LL} > 0$.](https://academic.oup.com/ptp/article-abstract/33/1/55/1921293/Inner-Region-and-Nonstatic-Effects-of-Nuclear)
nonstatic potentials, \( V_q \) and \( V_{L.\ell} \), in Eq. (3) are used in analysis. This form is a phenomenological extension of \( V_q \) and \( V_{L.\ell} \) of the OPEP with the pseudovector.

Fig. 3. Phase shifts (in radians) calculated from the hard-core potentials. The marks correspond to those in Table I. The HC-2 phase shifts are almost same with the HC-1's.
coupling,\textsuperscript{*1} and should be interpreted as a substitution of all other nonstatic effects than $V_{LS,LL}$\textsuperscript{*1,2}. In order to reduce $\delta(F_3)$ by $V_Q$ and $V_{LL}$ without making the $^3P_J$-fit worse, it is evident from Eq. (5) that we need
\begin{equation}
V_Q < 0 \quad \text{and} \quad V_{LL} > 0,
\end{equation}
consequently $a_Q > 0$ and $a_{LL} < 0$ are demanded ($b_Q$ and $b_{LL}$ are not essential due to the large impact parameter of $F$-wave). In the effective potential $V_4$ for $^3F_4$, the attraction $3V_{LS}$ can be well cancelled by $-9V_Q + 12V_{LL} \approx 21|V_Q|$ because of the large kinematical factors in spite of the weakness of $V_Q$ and $V_{LL}$ of $O(\mu/M)^5$ (HC–1, HC–2). On the other hand, in the absence of the $V_Q$ and $V_{LL}$, $V_4$ overwhelms the centrifugal potential. This is illustrated in Fig. 2. Owing to $V_Q < 0$ and $V_{LL} > 0$, $\delta(F_3)$ is reasonably suppressed; $\langle \delta(F_3) \rangle \approx 0.045$ at 310 MeV and $\delta(F_3) \approx 0.10$ at 660 MeV), but the other phase shifts scarcely change (compare HC–1 and HC–3 in Fig. 3). The reasons are that the $^3P_2$ state is sensitive to the region III, because the $^3P_2$-wave is pulled in by $V_{LS} < 0$. The $V_Q$ and $V_{LL}$ make $\delta(F_3)$ lower, and to retain the $^3P_2$-fit the hard-core radius should be smaller. $\delta(F_0)$ and $\delta(F_1)$ are quite insensitive to this change of $x_c$, since these waves are pushed out by $V_{LS}$. Thus HC–1 and HC–2 reproduce the solutions of the phase shift analysis (Fig. 3 and Table II). HC–2 has the slightly stronger $V_{LS}$ in the region II and gives the larger polarizations (better at $E=100–200$ MeV but worse at 310 MeV) than HC–1.

\section*{4. Core-less nonsingular cases}

In order to investigate whether the hard or soft core is necessary in the triplet odd state, analysis using the potential (2) is made in this section. We use the potential in $\mathcal{A}_{x>x_c}$ except $V_C$ slightly modified, and the square-well depths ($V_{C0}^0, V_{C0}^r, V_{LS}^0, V_Q^0$ and $V_{LL}^0$) are varied so as to obtain the best-fit. Solving Eq. (5) with respect to $V_C^0$, etc., we obtain\textsuperscript{***1)
\begin{equation}
\begin{align*}
V_C^g &= (-1.6V_C^0 + 16V_s^0 + 1.71V_z^0 - 4.11V_t^g)/12, \\
V_C^r &= (-3.5V_C^0 + 3.75V_s^0 + 1.6V_z^0 - 1.6V_t^r)/12, \\
V_{LS}^0 &= (-2V_C^0 - 3V_s^0 + 5V_z^g)/12, \\
V_Q^0 &= (2.36V_s^0 + 3V_t^0 - 5.5V_z^0 - 1.93V_t^o + 1.93V_z^0)/12, \\
V_{LL}^0 &= (1.94V_s^0 + 3V_t^0 - 6.5V_z^0 - 1.5V_t^o + 2.5V_z^0)/12.
\end{align*}
\end{equation}
\textsuperscript{*1)} This does not necessarily mean that the pseudoscalar coupling is excluded, since the effect of the $V_Q$ and $V_{LL}$ of the OPEP is small in the triplet odd state, and the $V_Q$ and $V_{LL}$ of Eq. (3) with the opposite sign of $\lambda$ will give no essentially different results.
\textsuperscript{**1)} Although $V_{LL}$ is derived in the same way as $V_Q$, $V_{LL}$ is also regarded as the $L$-dependence of $V_C$.
\textsuperscript{***1)} In Eq. (5), the fourth equation for $V_3$ can be dropped, because this equation is nearly equivalent to the second equation for $V_1$ in the case ($V_Q \approx -V_{LL}$) in which we are interested.
Although the effects of the tensor coupling should be included in $V_1^0$, $V_1^0$ and $V_1^0$, these effects are small. Then Eq. (7) is a good approximate expression, which we use to understand qualitative features of the region III. $V_2^0$ is determined through only the $^3P_2$-state potentials and free from ambiguities of the $^5F_2$-state potentials. This may be the consequence that $(L \cdot S)$ is the only term linear in $\sigma_1$ and $\sigma_2$ and kinematically independent of the other terms. As the first approximation, we consider the cases of $V_0^0 = V_2^0 = 0$ (SQ-3). Then to fit the "experimental" $\delta(^3P_2)$, we have

$$
\begin{align*}
V_0^0 &= 520 \sim 570 \text{ MeV}, \\
V_1^0 &= 280 \sim 320 \text{ MeV}, \\
V_2^0 &= -20 \sim -70 \text{ MeV}, \\
V_3^0 &= 140 \sim 131 \text{ MeV}, \\
V_4^0 &= -12.5 \sim -7.6 \text{ MeV}, \\
V_5^0 &= -165 \sim -204 \text{ MeV}.
\end{align*}
$$

(8)

$\delta(^3P_2)$ is insensitive to the region III up to $\sim 310 \text{ MeV}$, but turns to be sensitive at $660 \text{ MeV}$. In the following, discussions concerning $V_0^0$ and $V_2^0$ are parallel with those in § 3. If we use $V_0^0$, $V_1^0$ and $V_2^0$ in Eq. (8), $\delta(^3P_2) = 0.06 \sim 0.065$

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![Fig. 4](https://example.com/fig4.png)  
**Fig. 4.** Correlation between $\delta(^3P_2)$ and $V_0^0$, the effective potential depth in the $^3F_2$-state for SQ-case. Four cases of the outside central and tensor potentials are shown by $a_0 = -20, a_T = 0, b_T = -1.3$  
$a_0 = -18, a_T = 0, b_T = -1.3$  
$a_0 = -20, a_T = -1.3, b_T = 0.55$  
$a_0 = -18, a_T = -1.3, b_T = 0.55$.

![Fig. 5](https://example.com/fig5.png)  
**Fig. 5.** Correlation among $V_0^0, V_2^0, V_2^0$ as a function of $V_4^0$, when $V_0^0 = 0$. The experimental $\delta(^3P_2)$ determine $V_4^0$ (square well depth) as $V_4^0 = -30 \sim -40 \text{ MeV}$. The solid lines correspond to $V_2^0 = -30 \text{ MeV}$ and the dotted lines to $V_2^0 = -40 \text{ MeV}.$
Fig. 6. Phase shifts (in radians) calculated from the core-less nonsingular potentials. The marks correspond to those in Table I. The phase shifts absent in the figure are as follows: SQ-2 and SQ-3 are almost same with SQ-1 and SQ-5 and SQ-6 are almost same with SQ-4. ($\delta(3F_2)$ of SQ-4 are almost same with that of SQ-1.)
at 310 MeV and 0.24~0.26 at 660 MeV (SQ-3, SQ-6) due to \( V_0^\circ = -400 \sim -550 \text{ MeV} \) through Eqs. (5) and (8), as shown in Fig. 4. In order to reduce \( \delta((^3P_0)) \), \( V_1^\circ \sim 0 \) is necessary, which is obtained by \( V_0^\circ \approx -20 \text{ MeV} \) and \( V_1^\circ \approx 20 \text{ MeV} \) because of \((-9V_0^\circ + 12V_2^\circ) = 21V_1^\circ \approx 420 \text{ MeV} \). In these cases (SQ-1, SQ-4), \( V_0^\circ \sim 100 \text{ MeV} \) is necessary to keep the above cited values of \( V_0^\circ, V_1^\circ \) and \( V_2^\circ \).

Thus the nonsingular repulsive central potential (\( V_0^\circ \sim 100 \text{ MeV} \)) appears when \( V_0^\circ \) and \( V_2^\circ \) are as weak as possible. Since \( V_0 < 0 \) in the region II, the discontinuity of \( V_c \) at \( x_i \) is rather abrupt. This discontinuity may grow to the repulsive core, when \( x_i \) becomes smaller (\( x_i \leq 0.5 \)). However, if we use a somewhat stronger \( V_0^\circ \) and \( V_2^\circ \), we eliminate the repulsive feature in \( V_c \). Figure 5 shows the correlation among \( V_0^\circ, V_2^\circ \) and \( V_2^\circ \) as a function of \( V_0^\circ \), when \( V_0 = 0 \). For example, we obtain \( V_0^\circ \approx 0, V_0^\circ \approx -40 \sim -50 \text{ MeV} \) and \( V_2^\circ \approx 50 \sim 60 \text{ MeV} \) for \( V_0^\circ \approx 600 \text{ MeV} \) (SQ-2, SQ-5). The phase shifts given by such nonsingular potentials are shown in Fig. 6. Thus, in the triplet odd state, the repulsive core is not indispensable, in contrast with the singlet even state. The strong correlation between \( V_0^\circ \) and \( V_2^\circ \), indicates that a part of functions of the hard-core commonly used is to represent some non-static effects in the triplet odd state, since \( (V_c + V_{LL})^3 \) is repulsive in the region III in any case.

Both the hard-core and the nonsingular core-less potentials explain the phase shifts up to 310 MeV. In order to determine which is correct, the analysis should be extended to the higher energies. Although the nonrelativistic potential model is not valid quantitatively, the phase shifts calculated at 660 MeV serve to see whether qualitative differences between two types of potentials appear or not. Table II shows that even at 660 MeV it is difficult to distinguish these two possibilities qualitatively, if corrections entering at high energies are similar for both cases.*

§ 5. \( \delta((^3P_0)) \) at intermediate energies and potential in the region II

The attempts to reduce the ambiguity of \( \delta((^3P_0)) \) at \( E \leq 100 \text{ MeV} \) have recently been made by several experimental groups.\(^{21,22,23}\) The phase shift analyses at 52 MeV using \( I_0(\theta), P(45^\circ), D(70^\circ), C_{KP}(90^\circ) \) and \( C_{nn}(90^\circ) \) give \( \delta((^3P_0)) = 0.25 \sim 0.30,^{19,24} \) which are consistent with the results of the analyses at \( E = 50 \sim 100 \text{ MeV} \).\(^{25}\) The analyses including \( A(\theta) \) show \( \delta((^3P_0)) = 0.20 \sim 0.23,^{26} \) which joins

\(^{21}\) In the core-less potential, the central potential depth in region II is more attractive than that of the potential with hard core. This attractiveness makes \( \delta_{\text{ann}} \) at low energy (\( E \leq 10 \text{ MeV} \)) positive, where \( \delta_{\text{ann}} \) is defined as
\[ \delta_{\text{ann}} = (1/9) (\delta((^3P_0)) + 3\delta((^3P_1)) + 5\delta((^3P_2))) \]
and seems to be negative from the analysis of experimental data.\(^{27}\) This problem may be discussed in detail elsewhere.
to the values at $E \geq 140$ MeV. Here the information about $\delta(\beta)$ inferred by using both potential shapes in §§ 3 and 4 is discussed.

The small $\delta(\beta)$ at $E \leq 100$ MeV is the general consequence of the potentials adjusted to the high energy side; at 52 MeV $\delta(\beta) < 0.25$. When $V_r$ in the region II is weak, $\delta(\beta)$ becomes very small; $\delta(\beta) = 0.20$ at 52 MeV (HC–4 and SQ–4–6). In these cases, $\delta(\beta)$ at $E = 100 \sim 200$ MeV becomes slightly high and outside the standard deviation. The reason may be due to the short-tail of $V_{LS}$ (Fig. 1). If $\delta(\beta) > 0.20$ is confirmed, the fit of $\delta(\beta)$ at $E = 100 \sim 200$ MeV probably demands the stronger tail of $V_{LS}$ than those in Table I and the weak $V_r$ in the region II such as those in HC–4 and SQ–4–6. This feature is the same with Hamada-Johnston's.

In order to increase $\delta(\beta)$ at $E < 100$ MeV without destroying the high energy fit, the $V_r$ being moderately strong in the region II and negative or weak in the region III is favourable; $\delta(\beta) = 0.23 \sim 0.24$ at 52 MeV (HC–1, 2 and SQ–1–3). The large $C_{np}(=0.3$ at $\theta = 90^\circ$) obtained by these potentials are outside the experimental errors. The experimental value $^{30} C_{np}(90^\circ) = 0.13 \pm 0.11$ requires large $\delta(\beta)$ ($\geq 0.25$). $\delta(\beta)$ larger than 0.25 at 52 MeV is given by a weak spin-orbit tail. For example, SQ–7 is the case of SQ–1 without outer $V_{LS}$.

If $\delta(\beta)$ is really large ($\delta(\beta) > 0.25$ at 52 MeV), it gives rise to the following questions: Whether the large $\delta(\beta)$ at $E < 100$ MeV can be reconciled with $\delta(\beta) \approx 0.20$ at 310 MeV, and more strictly with the small $\delta(\beta)$ such as YLAN3M's at $E \sim 140$ MeV. For the hard-core potential of Eq. (1), a pos-

### Table II

(A) Nuclear bar phase shifts (in radians) calculated from the hard-core and the core-less nonsingular potentials, whose parameters are given in Table I.

(B) The singlet even phase shifts are also shown, which reproduce the experimental quantities, combined with the triplet odd phase shifts.

<table>
<thead>
<tr>
<th>$E$(MeV)</th>
<th>Case</th>
<th>$\alpha_0$</th>
<th>$\beta_0$</th>
<th>$\alpha_0$</th>
<th>$\beta_0$</th>
<th>$\alpha_0$</th>
<th>$\beta_0$</th>
<th>$\alpha_0$</th>
<th>$\beta_0$</th>
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</thead>
<tbody>
<tr>
<td>19.8</td>
<td>HC–1</td>
<td>0.1449–0.0756</td>
<td>0.0306–0.101</td>
<td>0.0008–0.0024</td>
<td>0.0002–0.0004</td>
<td>0.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>HC–2</td>
<td>0.1421–0.0765</td>
<td>0.0308–0.103</td>
<td>0.0010–0.0024</td>
<td>0.0002–0.0004</td>
<td>0.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>HC–3</td>
<td>0.1505–0.0738</td>
<td>0.0316–0.110</td>
<td>0.0011–0.0024</td>
<td>0.0002–0.0004</td>
<td>0.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>SQ–1</td>
<td>0.1474–0.0740</td>
<td>0.0348–0.111</td>
<td>0.0012–0.0024</td>
<td>0.0002–0.0004</td>
<td>0.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>SQ–2</td>
<td>0.1475–0.0739</td>
<td>0.0348–0.114</td>
<td>0.0013–0.0024</td>
<td>0.0002–0.0004</td>
<td>0.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>SQ–3</td>
<td>0.1474–0.0740</td>
<td>0.0351–0.111</td>
<td>0.0012–0.0024</td>
<td>0.0002–0.0004</td>
<td>0.0000</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>SQ–4</td>
<td>0.1323–0.0684</td>
<td>0.0353–0.096</td>
<td>0.0009–0.0024</td>
<td>0.0002–0.0004</td>
<td>0.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>52</td>
<td>HC–1</td>
<td>0.2345–0.1564</td>
<td>0.1013–0.0345</td>
<td>0.0068–0.0129</td>
<td>0.0021–0.0036</td>
<td>0.0005</td>
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<td></td>
<td>HC–2</td>
<td>0.2273–0.1593</td>
<td>0.1016–0.0340</td>
<td>0.0066–0.0130</td>
<td>0.0022–0.0036</td>
<td>0.0003</td>
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<tr>
<td></td>
<td>HC–3</td>
<td>0.2500–0.1569</td>
<td>0.1051–0.0338</td>
<td>0.0066–0.0130</td>
<td>0.0022–0.0037</td>
<td>0.0004</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>SQ–1</td>
<td>0.2360–0.1542</td>
<td>0.1123–0.0336</td>
<td>0.0068–0.0127</td>
<td>0.0024–0.0037</td>
<td>0.0005</td>
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</tr>
<tr>
<td></td>
<td>SQ–2</td>
<td>0.2362–0.1541</td>
<td>0.1150–0.0342</td>
<td>0.0069–0.0127</td>
<td>0.0025–0.0035</td>
<td>0.0003</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>SQ–3</td>
<td>0.2359–0.1542</td>
<td>0.1127–0.0334</td>
<td>0.0067–0.0127</td>
<td>0.0024–0.0037</td>
<td>0.0005</td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>SQ–4</td>
<td>0.2065–0.1392</td>
<td>0.1132–0.0308</td>
<td>0.0067–0.0122</td>
<td>0.0024–0.0035</td>
<td>0.0004</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The repulsive hump of $V_{LS}$ in the region II is undesirable in making $\delta(^3P_0)$ too low at $E \leq 100$ MeV. Even by the tensor modifications, we cannot reconcile the large $\delta(^3P_0)$ at $E < 100$ MeV with $\delta(^3P_0)$ at 310 MeV without destroying the fits of the other phase shifts. While the core-less potential succeeds in reproducing the energy dependence of $\delta(^3P_0)$ above mentioned (SQ-7), it is probably due to the
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The singlet even phase shifts in (B) are calculated from
\[ V^*_1 = \frac{\mu^2}{\hbar^2} \left( \frac{f^2}{4x} \right) \left[ -e^{x}/x (1 + 9e^{-x}/x) \right. \]
\[ + 11e^{-2x}/x^2 \left. + 2L(L+1) \left( \frac{\mu}{M} \right)^2 \times (e^{-x}/2x) (1 + 3|x|/x^2) \right. \]
\[ \times (0.25 + 1.75e^{-x}/x) \]
which gives the low energy parameters
\[
\begin{align*}
\lambda &= -17.84 \times 10^{-13} \text{ cm}, \\
\lambda &= 2.852 \times 10^{-13} \text{ cm}, \\
\lambda &= 0.0143.
\end{align*}
\]

stronger energy variation of the wave penetration into the core region than that for the hard-core potential. In the core-less potentials, however, we cannot obtain the large \( \delta (^1P_0) \) at 52 MeV and the small \( \delta (^1P_0) \) at \( E \sim 150 \text{ MeV} \).

Since the small \( \delta (^1P_0) \) at \( E \sim 150 \text{ MeV} \) is required by the polarization, if \( \delta (^1P_0) \) at \( E < 100 \text{ MeV} \) should be large, there is some drastic energy dependence in potentials which cannot be represented by the quadratic spin-orbit effects and some \( L \)-dependence of potentials. Therefore at \( E \sim 150 \text{ MeV} \), \( \delta (^1P_0) \approx 0.25 \) or \( \approx 0.20 \) is critical and further experiments to reduce the ambiguity of \( \delta (^1P_0) \) below 100 MeV are wanted, since inconsistencies may be present in data available at present.

The phase shifts calculated by the potentials in Table I are shown in Table II. The singlet even phase shifts obtained by the hard-core potential are also cited. These phase shifts together represent the two nucleon data below 310 MeV faithfully.

§ 6. Summary

Analysing the triplet odd phase shifts by the use of both the hard-core and the nonsingular core-less potentials, we obtain the following conclusions:

1. In the triplet odd state, the available data allow the nonsingular potentials without the hard-core like repulsion, in contrast with the singlet even state, where the singular repulsive core is indispensable no matter how it is hard\(^{26}\) or soft.\(^{29}\)

2. The quadratic spin-orbit potentials modified so as to be effective in the coupled states for \( x \leq 1.0 \) play an important role in explaining \( \delta (^3P_0) \) without destroying the fit of the other phase shifts. The strength of \( V_Q \) and \( V_{LL} \) is of the same order of magnitude with those in the singlet even and the triplet even states. Therefore, this \( V_Q \) and \( V_{LL} \) is a good description of nonstatic effects, although phenomenological in essence.

3. If the maximum value of \( \delta (^3P_0) \) appearing at intermediate energies is confirmed as larger than 0.25, some energy-dependence of potentials is required at \( E \leq 150 \text{ MeV} \).
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