The Polarization of Deuteron

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The polarization in the elastic scattering of deuteron from spin zero target is treated on the basis of diffraction theory by using the polarization formalism of Wolfenstein, Lakin and Stapp. The $D$-state contribution of deuteron is small, especially for small angle, in the result of cross section and polarization at 94 Mev deuteron elastic scattering.

§ 1. Introduction

Recently, the polarization effects in the elastic scattering of deuteron by carbon and various other nuclei were measured\(^1\), \(^2\), \(^3\) and were interpreted on the basis of the impulse approximation. Especially at 400 Mev deuteron scattering,\(^2\) the tensor polarization, in addition to the vector polarization, was measured, and was found to be appreciable. The general theory of polarization of the particle with any spin was discussed by Wolfenstein.\(^4\) The general polarization formalism of deuteron has been presented by Lakin.\(^5\) He constructs a complete set of nine $3 \times 3$ matrixes as follows:

\[
T_{11} = -\frac{1}{2} \sqrt{3} (s_x + is_y),
\]

\[
T_{10} = \left( \frac{3}{2} \right)^{1/2} s_z,
\]

\[
T_{20} = \left( \frac{1}{2} \right)^{1/2} (3s_y^2 - 2),
\]

\[
T_{21} = -\frac{1}{2} \sqrt{3} \left[ (s_x + is_y) s_x + s_z (s_x + is_y) \right],
\]

\[
T_{22} = \frac{1}{2} \sqrt{3} (s_x + is_y)^2,
\]

\[
T_{J,M} = (-1)^M T_{J,-M}.
\]

The secondly scattered intensity is given by

\[
I(\theta, \varphi) = I_0(\theta, \varphi) \left[ 1 + \langle T_{20}\rangle \langle T_{20}\rangle + 2 \langle iT_{11}\rangle \langle iT_{11}\rangle - \langle T_{31}\rangle \langle T_{31}\rangle \cos \varphi \right. \\
+ \left. 2 \langle T_{22}\rangle \langle T_{22}\rangle \cos 2\varphi \right],
\]
where \( \langle T_{ij} \rangle_1 \) and \( \langle T_{ij} \rangle_2 \) are the quantum mechanical expectation value of \( T_{ij} \) averaged over the spin directions of beam, at the first and the second scattering. The other general formalism of deuteron polarization has been given by Stapp,\(^6\) taking the \( D \)-state of deuteron into consideration. He pointed out the important effect of simultaneous scattering of both particles of deuteron, especially for large scattering angles.

On the other hand, the diffraction theory of neutron inelastic scattering has been given by Inopin,\(^7\) and this method has been extended for the inelastic scattering of \( \alpha \) particle, deuteron and proton,\(^8\) in which the surface vibration of nucleus is induced. The experimental results are fairly well explained by this model. An experimental verification of this model, which has been pointed out by Blair, is that the oscillation of inelastic scattering angular distribution curve, where the excitation involves no change of parity, is just out of phase with those of elastic angular distribution curve. In the stripping process of deuteron, it has been shown that a significant role was played by the dissociation process arising from diffractive effect.\(^9\) For elastic scattering, the dissociation of deuteron and the polarization of nucleon stripped from deuteron at high energy has been treated by Akhieser and Sitenko\(^{10,11}\) on the assumption of diffraction theory, taking the spin orbit interaction into consideration. In the present paper, the polarization of deuteron elastic scattering is treated by the diffraction theory on the assumption of completely black nucleus, in which account is taken of spin orbit interaction. Although the \( D \)-state contribution of deuteron to both the cross section and polarization is small for small angle and low energy. The component of tensor polarization \( \langle T_{13} \rangle \) is exactly equal to zero in the case that deuteron constitutes only of \( S \)-state.

The calculations of elastic and inelastic scattering of deuteron have been done by many authors.\(^{12,13,14}\) It is shown that the electric break-up of deuteron\(^14\) plays an important role in the elastic scattering of deuteron, but we neglect this process in the present paper.

\section*{§ 2. Calculation}

The free motion of deuteron in a plane perpendicular to the direction of incident deuteron (the \( Z \) axis) is described by the wave function

\[
\psi_D(\rho, r) = \frac{1}{L} \exp[i \mathbf{K} \cdot \mathbf{r}] \phi_0(r),
\]

where \( L \) is the normalization length, \( |i \mathbf{K}| = K \sin \theta \approx K \theta \) and \( \rho \) are the projection of the wave vector \( \mathbf{K} \) and radius vector of the centre of mass of deuteron, and \( r \) is the relative coordinate between neutron and proton. \( \phi_0(r) \) is the wave function of deuteron

\[
\phi_0(r) = [s(r)]^2 + 8^{-1/2} S_{12} d(r),
\]
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$s(r)$ is the $S$-state wave function and $d(r)$ is the $D$-state wave function. For triplet state, the tensor operator\(^{15}\) $S_{12}$ is given by

$$S_{12} = 6 \left( s \cdot \frac{r}{r^2} \right)^2 - 4$$

(5)

where $s$ is the total spin of deuteron $s=1/2(\sigma_u+\sigma_p)$. For the incident wave function, $\Delta K$ of (3) is zero, and so $\psi_d$ is simply $(1/L)\phi_d(r)$. Owing to the presence of an absorbing nucleus, incident wave of deuteron is diffracted and the wave function can be written in the form

$$\psi_0(r, \theta) = \sum_{J,K} \hat{a}_{J,K} \psi_{J,K}(r, \theta).$$

(6)

The scattered amplitude $M(\theta, \varphi)$ is given by

$$M(\theta, \varphi) = -i\left( L^2 K/2\pi \right) \hat{a}_{J,K}.$$  

(7)

We shall consider the effect of spin orbit coupling in the case of diffractional scattering, so $\hat{a}_{J,K}$ is calculated in the following way:\(^{11}\)

$$\hat{a}_{J,K} = \int \int \psi^*(\rho, r) \left[ \tilde{\omega}(r_\rho) + \tilde{\omega}(r_p) - \tilde{\omega}(r_n) \tilde{\omega}(r_p) \right] \psi_0(\rho, r) \, d\rho \, dr$$

where

$$\tilde{\omega}(r) = \omega(r) \left[ 1 + \frac{\hbar^2}{M c^2} \left( \frac{\hbar}{M c} \right)^2 \right] \frac{\sigma t}{2} \left[ k \times \frac{\partial}{\partial r} \right].$$

(8)

where $\sigma$ and $\sigma'$ are phase shifts specifying the complex parts of the central and spin orbit coupling interaction, $\tilde{\gamma}$ is the spin orbit coupling constant. $\sigma$ is the spin matrix of nucleon.

$$\hat{a}_{J,K} = a_{J,K} + i \frac{\tilde{\gamma}}{2} \left( \frac{\hbar}{M c^2} \right)^2 \int e^{i\frac{\hbar}{\Delta K} \hat{\omega}(\rho)} \, d\rho \int e^{-i\frac{\hbar}{\Delta K} \hat{\omega}_d^*(r)} \, dr$$

(9)

where $K = k_n + k_p$ and

$$\omega(\rho) = \begin{cases} 1 & \rho < R \\ 0 & \rho > R \end{cases}$$

(10)

$R$ is radius of nucleus, and

$$a_{J,K} = \int \int \psi^*(\rho, r) \left[ \omega(\rho_\rho) + \omega(\rho_p) - \omega(\rho_n) \omega(\rho_p) \right] \psi_0(\rho, r) \, d\rho \, dr.$$
where
\[ n = \frac{K \times \Delta K}{|K \times \Delta K|} \]
and \( e \) is the component of unit vector \( e \) with direction of \( \Delta K \). The unit vector \( e \) is approximately equal to the unit vector \( E = \frac{K_{out} - K_{in}}{|K_{out} - K_{in}|} \) defined by Stapp.

We neglected the effect of simultaneous scattering \( \omega_o \omega_p \) in the calculation of the scattering amplitude. We shall discuss the term with \( \omega_o \omega_p \) in § 3. \( s_i \) is the component of the total spin \( s \) of deuteron and \( s_{ij} \) is the symmetrical tensor,
\[ s_{ij} = \frac{1}{2} (s_i s_j + s_j s_i) - \frac{2}{3} \delta_{ij} I. \]

The quantum mechanical expectation values of \( s_i \) and \( s_{ij} \) are given as follows.
\[
I_0(s_i) = \text{Tr} \left[ \frac{1}{3} M(\theta, \varphi) M(\theta, \varphi) s_i \right] = \frac{2}{3} \text{Re} \left[ b (2a - \frac{c}{3})^* \right],
\]
\[
I_0(s_{ij}) = \text{Tr} \left[ \frac{1}{3} M(\theta, \varphi) M(\theta, \varphi) s_{ij} \right] = \frac{1}{3} \left[ hh^* (n_i n_j - \frac{1}{3} \delta_{ij}) \right.
+ \left. \left( 2 \text{Re}c^* - \frac{1}{3} cc^* \right) (e_i e_j - \frac{1}{3} \delta_{ij}) \right].
\]

and
\[
I_0 = a^2 + \frac{2}{3} b^2 + \frac{2}{9} c^2.
\]

The differential cross section after the second scattering, under the assumption that the vector \( e \) is approximately equal to the unit vector \( E \), is given by
\[
I(\theta, \varphi) = I_0(\theta, \varphi) \left[ 1 + \frac{3}{2} \langle s_i \rangle \langle s_i \rangle_2 + 3 \langle s_{ij} \rangle \langle s_{ij} \rangle_2 \right].
\]

We can calculate the vector polarization \( \langle iT_{11} \rangle \) and tensor polarization \( \langle T_{20} \rangle \), etc., from Eq. (14) in comparison with Eq. (2).

§ 3. Results and discussion

We calculate the elastic scattering cross section and polarization of deuteron by diffraction theory, and the comparison with experimental results at 94 Mev deuteron energy is shown in Figs. 1 and 2. We employ the wave function of deuteron for the potential without core, introduced by Sugawara and Hulthen,\(^{17}\) which has 4% of \( D \)-state probability. Spin orbit coupling constant \( \gamma = 1/2 \), if spin orbit interaction is caused by Thomas precession.\(^{18}\) However, in the shell
model and recent study of polarization of nucleon, $\gamma$ is very large compared with $1/2$. We employ $\gamma = 15$; and further $\delta = 34.3^\circ$ and $\delta' = 0$ given by Fermi. Haffner analyzed the data of polarization of proton employing the Wood Saxon potential and chose $\gamma = 22$, $\delta = 68.5^\circ$ $\delta' = 0$. The vector polarization $\langle iT_{11} \rangle$ is very large compared with experimental result if we employ the parameter given by Haffner. The $D$-state contribution is very small for both cross section and polarization in small angle scattering. The $D$-state contribution for $\langle iT_{11} \rangle$ is about 5% of the value of $\langle iT_{11} \rangle$. The component of tensor polarization $\langle T_{21} \rangle$ is exactly equal to zero without $D$-state. It is less than 1% even if we take account of $D$-state and even at energy as high as 94 Mev. This is much smaller at lower energies and the statement of Hird, etc., that the negative coefficient of $\cos \varphi'$ in (28) is caused by the effect of $\langle T_{31} \rangle$, seems doubtful. $\langle T_{31} \rangle$ is less than 2% and $\langle T_{30} \rangle$ is less than 4% at 94 Mev.

In §2 and §3, we neglected the term with $\omega_{d}\omega_{p}$. This approximation corresponds to the impulse approximation which neglects the simultaneous scattering of the particles composing the deuteron by nucleus. Stapp pointed out that the simultaneous scattering may play an important role for large angle scattering. For small angles, which we have treated in the present paper the effect of simultaneous scattering may be small. The term with $\omega_{d}\omega_{p}$ has been evaluated by Akhieser and Sitenko, assuming that the wave function of deuteron
is \( rs(r) = Ne^{-\alpha r} \) and that the nuclear radius \( R \) is greater than the radius of deuteron \( R_d = 1/2a \).

The results with the correction of \( \omega_n, \omega_p \) are shown in Fig. 1, with broken lines. We take the radius of carbon \( R = 4.5 \times 10^{-13} \) cm, which Haffner employ in the analysis of deuteron inelastic scattering.\(^{16}\) We cannot explain the fact that the negative sign of vector polarization in smaller angles less than 10° appears at 94 Mev and does not appear at 125 Mev.

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