Preon models with quasi-simple gauge groups of the form $G = G_{hc} \otimes G_{hf}$ are studied, where $G_{hc}(G_{hf})$ denotes the hypercolor (hyperflavor) subgroup of $G$. They are classified to self-dual and dual models according to the behavior of their preon multiplets under the exchange of hypercolor and hyperflavor. Dual preon models are characterized by having no preon mass term, and allow two possibilities. One has $G = SU(7) \otimes SU(7)$ with a dual pair of preons transforming as $\phi_{(21,7)}$ and $\chi_{(7^*,21^*)}$. The other has $G = SO(10) \otimes SO(10)$ with preons given by $\phi_{(10,16)}$ and $\chi_{(16,10)}$.

§ 1. Introduction

There is no experimental evidence for the conjecture that quarks and leptons are composed of more fundamental objects. At present the necessity for it comes solely from theoretical desirability. We want to dispense with many free parameters, in particular with fine tuning of the Higgs boson masses. We also wish to understand the origin of generations observed in light quarks and leptons.

Fundamental constituents, called preons in this work, are usually assumed to be confined by gauge interactions of “hypercolor”. In addition they have “hyperflavor” gauge interactions, which include the ordinary color and flavor ones. The whole gauge group $G$ is then of the form

$$G = G_{hc} \otimes G_{hf}, \tag{1.1}$$

where $G_{hc}(G_{hf})$ stands for the hypercolor (hyperflavor) gauge group. $G_{hf}$ should contain the gauge group $SU_c(3) \otimes SU_l(2) \otimes U_y(1)$ of the Weinberg-Salam theory. We make the following three requirements on the preon model. I) It has gauge interactions only; II) It is anomaly-free with respect to $G$; III) It is asymptotically free with respect to $G_{hc}$. The first requirement means that there should be no elementary Higgs scalars. The second one is necessary to assure the renormalizability of the model, and the third is desirable for the preon confinement.

There is one unsatisfactory point on the gauge group given by Eq. (1.1). Even if $G_{hc}$ and $G_{hf}$ are both simple, the model is not a unified one, because it has
two independent gauge coupling constants, $g_{nc}$ and $g_{hf}$. An orthodox way to unify it is to look for a simple group containing $G_{nc} \otimes G_{hf}$. Another way of unification is possible when $G_{nc}$ and $G_{hf}$ are isomorphic. The $G$ given by Eq. (1·1) becomes quasi-simple by imposing a discrete symmetry connecting $G_{nc}$ and $G_{hf}$. The aim of this work is to study such quasi-simple preon models.

The discrete symmetry in any quasi-simple model should break down spontaneously at an energy scale larger than that for the preon confinement. At this breaking we assume that $G_{nc}$ is kept unbroken but $G_{hf}$ is broken to a smaller subgroup. It has been pointed out that spontaneous breaking of a discrete symmetry may cause a domain wall problem.\(^5\) We note, however, that according to a new scenario of the inflationary universe for the standard model of hot big-bang cosmology\(^6\) there will be no domain walls in the observable part of our universe.

We examine quasi-simple models in which $G_{nc}$ and $G_{hf}$ are simple groups having complex representations. Such simple groups are given by\(^7\): i) $SU(n)$ for $n \geq 5$; ii) $SO(4m + 2)$ for $m \geq 2$; and ii') $E_6$. We have excluded $n = 3$ and 4 in i) because $G_{hf}$ should contain the gauge group with rank four of Weinberg and Salam. All the representations of ii) and ii') have the vanishing anomaly number, while this is not the case for i).

We represent preons by left-handed Weyl spinors. A preon multiplet transforming as $\psi_L(R', R)$ under $G_{nc} \otimes G_{hf}$ is called self-dual if $R' = R$. Preons in such a multiplet are transformed to themselves by the exchange of hypercolor and hyperflavor. Quasi-simple models containing only self-dual multiplets will be called "self-dual preon models". We shall see that those models which are not self-dual should contain a pair of preon multiplets transforming either as $\psi_L(R', R)$ and $\chi_L(R^*, R'^*)$ or as $\psi_L(R', R)$ and $\chi_L(R, R')$, where $R' \neq R$. We shall call them "dual preon models".

In building preon models we must be careful not to introduce exact global symmetries. Otherwise we shall be plagued by unwanted massless Goldstone bosons. In particular, preon multiplets should not be repeated.

We have studied self-dual preon models and dual ones. Preon mass terms are absent only in the latter, which lead to two possibilities. One is given by $G = SU(7) \otimes SU(7)$ with preons transforming as $\psi_L(21, 7)$ and $\chi_L(7^*, 21^*)$. This is a model investigated recently by one of us,\(^8\) which motivated the present work. The other has $G = SO(10) \otimes SO(10)$ with preons given by $\psi_L(10, 16)$ and $\chi_L(16, 10)$.

The character of the first model is unique in that it is neither invariant under the exchange of left and right nor of hypercolor and hyperflavor. It is invariant only if the two exchanges are simultaneously made.
§ 2. $SU(n) \otimes SU(n)$ models

A. Self-dual preon models

The self-dual preon multiplets consist of $\phi_L(R, R)$ and $\phi^c_L(R^*, R^*)$. They are related to each other and to the right-handed Weyl spinors $\phi_R(R, R)$ and $\phi^c_R(R^*, R^*)$ by

$$
\begin{array}{c}
\phi_L(R, R) \xrightarrow{P} \phi_R(R, R) \\
\phi^c_L(R^*, R^*) \xrightarrow{P} \phi^c_R(R^*, R^*)
\end{array}
$$

where $P$ stands for space reflection and $C$ for charge conjugation. Evidently the model is left-right symmetric and is anomaly-free with respect to $G$.

Expanding the Callan-Symanzik function $\beta(g)$ as

$$
\beta(g) = b_0 g^3 + b_1 g^5 + \cdots,
$$

we find that

$$
B = 24 \pi^2 b_0 = \frac{1}{2} \sum_i C(R_i) - \frac{11}{2} C_2(G_{nc}),
$$

(2.1)

where $R_i$'s denote $G_{nc}$ representations of the left-handed spinors. $C(R)$ is defined by

$$
\text{Tr}(T_a(R) T_a(R)) = \frac{1}{2} C(R) \delta_{as}
$$

(2.2)

with the property that $C(R^*) = C(R)$, and

$$
C_2(G) = \frac{1}{2} C(R_{\text{adj}}(G)),
$$

(2.3)

which is equal to $n$ for $G = SU(n)$.

Asymptotic freedom for $G_{nc}$ is realized when $B$ is negative. This is the case in the present model only if $R$ is the fundamental representation of $SU(n)$. We then obtain

$$
B = \frac{1}{2} (C(n) d(n) + C(n^*) d(n^*)) - \frac{11}{2} C_2(SU(n))
$$

$$
= n - \frac{11}{2} n = -\frac{9}{2} n < 0,
$$

where $C(n)$ and $C(n^*)$ are the dimensions of the fundamental and antifundamental representations of $SU(n)$, respectively.
where $d(R)$ denotes the dimension of $R$ and $C(n)=1$. Though it is possible to repeat the preon multiplets without violating asymptotic freedom, we reject the introduction of "preon generations" to avoid unwanted Goldstone bosons.

We see that $n$ should be odd integers in order to have hypercolor-singlet fermions. We also note that the model has an exactly conserved vector charge

$$Q = \frac{1}{n}(N_e - N_{\nu}).$$

It is normalized so that it takes any integral value for hypercolor-singlet states. Since $SU(n)$ contains $SU(3) \otimes SU(2) \otimes U(1)$ and it is left-right symmetric, it should include $SU(2)$ as well. We thus find that $n \geq 7$.

We have obtained left-right symmetric models having $G = SU(n) \otimes SU(n)$ for $n=7, 9, \ldots$, with preons given by $\psi_L(n, n)$ and $\phi^c_L(n^*, n^*)$. The $SU(7) \otimes SU(7)$ model considered by Montvay\textsuperscript{10} belongs to this category. These models allow a $G_{hc}$-invariant preon mass term of the form, $\bar{\psi}^c_L \psi_L + h.c.$

B. **Dual preon models**

The preons are given by a dual pair of $\psi_L(R', R)$ and $\chi_L(R^*, R'^*)$ with $R' \neq R$. Under discrete operations they behave as

$$
\begin{align*}
\psi_L(R', R) &\leftrightarrow \psi_R(R, R') \\
\chi_L(R^*, R'^*) &\leftrightarrow \chi_R(R'^*, R^*)
\end{align*}
$$

where $S$ represents the exchange of hypercolor and hyperflavor.

It will be interesting to compare the above behavior of the preons with that of the quarks in $n$-flavor QCD,

$$
\begin{align*}
\psi_L(n, 1) &\leftrightarrow \psi_R(1, n) \\
\phi^c_L(1, n^*) &\leftrightarrow \phi^c_R(n^*, 1)
\end{align*}
$$

Only the part of chiral flavor symmetry is shown here. We see that 1 corresponds to $R$ and $n$ to $R'$. Furthermore, $P$ and $C$ are replaced by $PS$ and $CS$, respectively. $P$, $C$ and $S$ are not definable individually in $SU(n) \otimes SU(n)$ dual preon models. Their Lagrangians are invariant only under $PS$, $CS$ and $PC$.

The condition for $G_{hc}$ to be asymptotically free is given by
Quasi-Simple Preon Models

\[ B = \frac{1}{2} \{ C(R')d(R') + C(R^*)d(R'^*) \} - \frac{11}{2} C_a < 0. \]  

(2.5)

The anomaly number of \( R \) is defined by

\[ \text{Tr}[[T_a(R), T_b(R)]T_c(R)] = K(R) \text{Tr}[[t_a, t_b]t_c], \]  

(2.6)

where \( t_a \) denotes \( T_a(\square) \), with \( \square \) standing for the fundamental representation. We note that \( K(R^*) = - K(R) \). The requirement that the model is anomaly-free with respect to \( G \) is given by

\[ A_{hc} = K(R')d(R) + K(R^*)d(R'^*) = 0. \]  

(2.7)

This is sufficient because we have \( A_{hc} = - A_{bc} = 0 \).

Defining the anomaly congruence number \( \bar{R}(R) \) for an irreducible representation \( R \) of \( SU(n) \) by

\[ \bar{R}(R) = nK(R)/d(R), \]  

(2.8)

we obtain from Eq. (2.7)

\[ \bar{R}(R') = \bar{R}(R). \]  

(2.9)

We have seen that \( R \) and \( R' \) should satisfy the inequality (2.5) and Eq. (2.9). This is possible only if \( R = \square \) and \( R' = \square^* \) (or \( R = \square^* \) and \( R' = \square \)). Using \( K(\square^*) = 1 \) and \( K(\square) = n-4, \) we get

\[ 2^{n-4}/n-1 = 1, \]  

(2.10)

which gives us a unique solution of \( n = 7 \).

We have obtained a model with \( G = SU(7) \otimes SU(7) \), in which the preons transform as \( \phi_L(21, 7) \) and \( \chi_L(7^*, 21^*) \). This model is invariant under PS and PC, and it allows no preon mass term.

§ 3. \( SO(n) \otimes SO(n) \) and \( E_6 \otimes E_6 \) models

These models are automatically anomaly-free.

A. Self-dual models

When \( G \) is given by \( SO(n) \otimes SO(n) \) with \( n = 4m + 2 \) for \( m \geq 2 \), we need two self-dual multiplets \( \phi_L(R, R') \) and \( \chi_L(R', R') \), where \( R \) and \( R' \) should be opposite...
in the even-oddness of their quartalities.\footnote{If we take either one of them, we cannot make hypercolor-singlet fermions. Under the discrete symmetries P, C and PC they behave as}

\[
\begin{align*}
\phi_\ell(R, R) \xrightarrow{PC \ P} \phi_h(R^*, R^*) \\
\chi_\ell(R', R') &\xrightarrow{PC \ C} \chi_h(R'^*, R'^*) \\
\end{align*}
\]

The two multiplets are unrelated by any discrete symmetry.

In order to assure asymptotic freedom of hypercolor we have to take \( R \) to be the spinor representation \( 4^m \) and \( R' \) to be the vector one \( n \). We check the inequality.

\[
B = \frac{1}{2} \{ C(4^m) \cdot 4^m + C(n) \cdot n \} - \frac{11}{2} C_2
= 2(16^m - 1) - 18m < 0,
\]

where we have used \( C(4^m) = 4^{m-1}, C(n) = 2 \) and \( C_2 = 4m \). The above inequality is satisfied only when \( m = 2 \).

We thus obtain a model having \( G = \text{SO}(10) \otimes \text{SO}(10) \), with the preons given by \( \phi_\ell(16, 16) \) and \( \chi_\ell(10, 10) \). It is left-right symmetric and allows a preon mass term of the form, \( \chi_\ell \chi_\ell + \text{h.c.} \). The model also has an exactly conserved charge given by

\[
Q = -\frac{5}{2} N_\theta + 8 N_\et.
\]

An unwanted Goldstone boson will emerge if \( Q \) is to break down spontaneously.

The self-dual multiplet for \( G = E_6 \otimes E_6 \) is given by \( \phi_\ell(R, R) \), which behaves under P, C and PC as

\[
\begin{align*}
\phi_\ell(R, R) \xrightarrow{PC \ P} \phi_h(R^*, R^*) \\
\end{align*}
\]

Since \( E_6 \) has triality for the congruence classes,\footnote{\( \eta \)} hypercolor-singlet fermions can be formed. Moreover, there is no preon mass term for \( R \) with a non-zero triality. But the model is not asymptotically free even for \( R = 27 \), because

\[
B = \frac{1}{2} C(27) \cdot 27 - \frac{11}{2} C_2
= 81 - 66 > 0,
\]
where we have used $C(27) = 6$ and $C_2(E_8) = 12$. The situation becomes worse for representations of larger dimension.

B. Dual preon models

For $G$ given by $SO(n) \otimes SO(n)$ with $n = 4m + 2$, $m \geq 2$, we have a dual pair of multiplets $\phi_l(R', R)$ and $\chi_l(R, R')$ with $R' \neq R$. They behave under discrete symmetries as

\[
\begin{array}{ccc}
\phi_l(R', R) & \xrightarrow{C} & \phi_{\bar{l}}(R^*, R^*) \\
\xrightarrow{P, PC} & & \xrightarrow{C} \\
S & \xrightarrow{PS, PCS} & S \\
\chi_l(R, R') & \xrightarrow{C} & \chi_{\bar{l}}(R^*, R'^*) \\
\xrightarrow{P, PC} & & \xrightarrow{C}
\end{array}
\]

The model is left-right symmetric and it allows no preon mass term.

Asymptotic freedom of $G_{\text{hy}}$ is obtained only if we choose $R = 4^m$ and $R' = n$. We then have

\[
B = \frac{1}{2} \left[ C(n) \cdot 4^m + C(4^m) \cdot n \right] - \frac{11}{2} C_2 
\]

\[
= 4^{m-1} \cdot (2m+5) - 22m < 0. \tag{3.4}
\]

This inequality is satisfied only when $m = 2$. We get the second dual preon model, which has $G = SO(10) \otimes SO(10)$ with the preons given by $\phi_l(10, 16)$ and $\chi_l(16, 10)$.

The situation for $E_6 \otimes E_8$ is essentially the same as the one for $SO(n) \otimes SO(n)$, except for the fact that there is no asymptotically free model in the former. Recalling our result for the self-dual case, we may discard $E_6 \otimes E_8$ from our consideration of quasi-simple models.

§ 4. Discussion

We have classified the quasi-simple preon models to self-dual and dual ones. Dual preon models are characterized by having no preon mass term. We have only two models of this type, $SU(7) \otimes SU(7)$ and $SO(10) \otimes SO(10)$. They may be regarded as possible candidates for a realistic preon model.

Though the two models share several features in common, there are important differences between them. The first one is that the $SU(7) \otimes SU(7)$ model is not left-right symmetric while the $SO(10) \otimes SO(10)$ is. As the second difference we remark that $SU_{\text{hy}}(7)$ contains new degrees of freedom in addition to those of
the $SU(5)$ GUT, \cite{11} which may be related to the generation of quarks and leptons. On the other hand, $SO_{16}(10)$ cannot contain any degrees of freedom other than those of the $SO(10)$ GUT. \cite{12} The generation should appear as dynamical degeneracy in those composite fermions which remain massless down to the energy scale of Weinberg and Salam.

We wish to express our hearty thanks to members of our Particle Physics Group for many valuable discussions.

References

1) References on various preon models can be found in:


8) M. Ida, Kobe Preprint, KOBE-82-03 (1982).


