On the Electron-Neutron Interaction

Masami Yamada

Department of Physics University of Tokyo

December 1, 1954

According to Foldy,¹ the electron-neutron (e−N) interaction can be divided into two parts, one...
related to the anomalous magnetic moment (a.m.m.), and the other independent of it. Usually we measure the strength of the $e-N$ interaction by the depth of the equivalent square well whose radius is $e^2/m_e^2$, the classical radius of electron. Then the contribution from the a.m.m. is (attractive)

$$V_{1N} = -4.0 \text{ kev}.$$ 

The other part $V_{2N}$, called the "intrinsic $e-N$ interaction" by Foldy, is the $\gamma \mu \beta \alpha \mu$ type interaction between nucleon and electromagnetic field. ($\gamma_\mu$: Dirac matrix, $\beta$: 4-momentum of photon, $\alpha \mu$: electromagnetic 4-potential.) On the other hand, the experiment by Hughes et al. shows that the total $e-N$ interaction is

$$V_N = V_{1N} + V_{2N} = -3.86 \pm 0.37 \text{ kev},$$

and then

$$V_{2N} = 0.14 \pm 0.37 \text{ kev}.$$ 

Meson theoretical calculations of $V_{2N}$ have been done in the lowest-order perturbation theory (first part of Fig. 1 (a) and Fig. 1 (b)). In the symmetrical pseudoscalar meson theory with pseudoscalar coupling with nucleon, $V_{2N}$ can be written as (for the later necessity written for proton, too)

$$V_{2N} = 1.35 \text{ kev} \frac{f^2}{4\pi} \left( \frac{3 - \tau_s}{2} \right) \times \int \frac{du}{u^3} \left[ \frac{1}{6A_0^2} + \frac{u^2}{12A_0^4} \right]$$

$$+ \tau_s \int \frac{du}{u} (1-u)^2 \left[ \frac{1}{6A_0^2} - \frac{u^2}{3A_0^4} \right],$$

(1)

where

$$A_0^2 = u^2 + \left( \frac{m_n^2}{M^2} \right) (1-u).$$

The suffix $\tau$ indicates that (1) comes from the ordinary lowest order Feynman graph. $\tau_s=1$ for proton, and $m_n$ and $M$ are the masses of pion and nucleon respectively, whose ratio is taken as $m_n/M = 0.15$ in the following calculation. If we take the coupling constant $f^2/4\pi = 15$, the value often used recently (although, in the same approximation, this coupling constant gives too large a.m.m. for neutron, and too small a.m.m. for proton in magnitude), (1) is for neutron

$$V_{2N} = -2.58 \text{ kev},$$

which is too attractive. Foldy suggested that a suitable mixture of heavy pseudoscalar meson can fit the theoretical value to the experimental one. Here, however, we shall point out that there exist other contributions in the $\pi$-meson theory which reduce $|V_{2N}|$, or even make $V_{2N}$ positive.

When a.m.m. and $e-N$ interaction are calculated in the lowest-order perturbation theory, the virtual nucleon interacts with the photon only through its charge, and its a.m.m. (here, to avoid double counting, a.m.m. of proton is taken as $1.79 \cdots$ nuclear magneton which is obtained by subtracting one nuclear magneton from the observed value of the magnetic moment), intrinsic $e-N$ interaction and corresponding intrinsic $e-P$ interaction (we call them "intrinsic electron-nucleon interaction" (i.e.n.i.) together) are neglected. We can, however, expect that the calculation including their effect may be closed. Actually, the values of the a.m.m. and the i.e.n.i. of the virtual nucleon will be different from those of the real nucleon, but it is so difficult to evaluate these differences that we use the values of the real nucleon here. The Feynman diagrams of this calculation are shown in Fig. 1. Such a method of calculation has been already applied to the problem of a.m.m. by Miyazawa, and it has been pointed out that the graphs of Fig. 1 may be regarded as
the approximation of the sum of the graphs of Fig. 2.

When the electromagnetic field is expressed by $A_\mu$ only, the interaction of the virtual nucleon with the electromagnetic field through the a.m.m. is linear in the photon momentum $p$, and contributes to both the a.m.m. and the i.e.n.i. of the real nucleon. The i.e.n.i. of the virtual nucleon is quadratic in $p$, and contributes only to the i.e.n.i. of the real nucleon. Interactions of the virtual nucleon higher than quadratic in $p$ contribute nothing to the a.m.m. and the i.e.n.i. of the real nucleon.

The contribution to the i.e.n.i. from the a.m.m. of the virtual nucleon is (second part of Fig. 1 (a))

$$V_{2\alpha} = 1.35 \text{ kev} \left( \frac{3 - \tau_3}{2} \mu_P + \frac{3 + \tau_3}{2} \mu_N \right)$$

$$\times \frac{f^2}{4\pi} \int_0^1 du \frac{u^n}{4A_0^2},$$

where $\mu_P$ and $\mu_N$ are the a.m.m.'s of proton and neutron respectively in the unit of nuclear magneton, and $\mu_P \approx 1.79, \mu_N \approx -1.91$. If we put $f^2/4\pi = 15$ as before,

$$V_{2N\alpha} = 4.04 \text{ kev},$$

and then

$$V_{2NT} + V_{2N\alpha} = 1.46 \text{ kev},$$

which is too repulsive.

Next, we calculate the contribution from the i.e.n.i. of the virtual nucleon (third part of Fig. 1 (a)). As $p^2$ can be treated as a constant, the calculation is the same as that of Dyson's renormalization constant $Z_1$, and the result diverges. $Z_1$ can be replaced by $Z_2$ according to the Ward identity, and the calculated contribution to the i.e.n.i. can be written as

$$V_{2\alpha} = \left( \frac{3 - \tau_3}{6} V_{2P} + \frac{3 + \tau_3}{6} V_{2N} \right) (1 - Z_2).$$

The physical meaning of (3) is clear if we notice the following two facts. Namely, $1 - Z_2$ is the dissociation probability of a nucleon, and $(3 - \tau_3)/6$ and $(3 + \tau_3)/6$ are the probabilities that the virtual nucleon is in proton state and neutron state respectively.

The total i.e.n.i. is

$$V_\alpha = V_{2T} + V_{2\alpha} + V_{2\alpha}. \quad (4)$$

We insert (1), (2) and (3) into (4), and then simultaneous equations for $V_{2P}$ and $V_{2N}$ are obtained.

$$V_{2P} = 4.92 \text{ kev} + [(1/3)V_{2P} + (2/3)V_{2N}] (1 - Z_2)$$

$$V_{2N} = 1.46 \text{ kev} + [(2/3)V_{2P} + (1/3)V_{2N}] (1 - Z_2).$$

If we restrict the value of $Z_2$ to $0 \leq Z_2 \leq 1$ taking into account its physical meaning, $V_{2N}$ is always positive ($1.46 \text{ kev} \leq V_{2N} \leq \infty$) contrary to $V_{2NT}$.

Until now, we have focussed our attention upon the $e-N$ interaction. But we should pay attention to the a.m.m. too, because both of them arise from the same Feynman diagrams. The above sort of calculation leads the lowest order values of them to the good direction. However, these corrections are too large for $e-N$ interaction and too small for a.m.m. To fit them better to the experimental values simultaneously, we must invoke other method of calculation; e.g. more exact treatment of higher order terms, or mixing of other type of meson. As for the latter, neutral scalar meson seems hopeful.

The author wishes to express his sincere thanks to Professors T. Yamanouchi and S. Nakamura for their continual guidance and encouragement.

1) L. I. Foldy, Phys. Rev. 83 (1951), 688; Phys. Rev. 87 (1952), 693.
2) D. J. Hughes, J. A. Harvey, M. D. Goldberg and M. J. Stafne, Phys. Rev. 90 (1953), 497.
3) e.g. B. D. Fried, Phys. Rev. 88 (1952), 142.
4) L. L. Foldy, Phys. Rev. 87 (1952), 675.
5) H. Miyazawa, Soryusiron Kenkyu (mimeographed circular in Japanese) 2 (1950), No. 4, 37.
6) F. J. Dyson, Phys. Rev. 75 (1949), 1736.