Can Primordial Black Holes Solve the Overproduction Problem of Monopoles?

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It has been pointed out that magnetic monopoles are overproduced in the early universe. As an approach to solve this overproduction problem, we investigate the evolution of the universe including primordial black holes for arbitrary mass and arbitrary initial abundance of monopoles. We investigate whether the generations of entropy and baryon asymmetry due to black hole evaporation can dilute the monopole abundance sufficiently without contradiction to present baryon/entropy ratio and primordial nucleosynthesis. It is shown that the introduction of primordial black holes enables us to construct a consistent cosmic scenario for wider range of mass and initial abundance of monopoles. However, it is also shown that for monopoles with standard mass, $10^6$ GeV, we cannot elude the overproduction of monopoles by primordial black holes unless the initial abundance of monopoles is sufficiently small.

§ 1. Introduction

It was shown that GUTs (Grand Unified Theories), which unify strong and electroweak interactions, allow the existence of magnetic monopoles under very general conditions. Moreover, it has been argued that current GUTs, in combination with the standard Big Bang cosmology, predict that monopoles are overproduced by many orders of magnitude in the early universe compared with the upper limit of observations. As pointed out by Preskill and investigated in detail by the present authors, the monopole/entropy ratio, $n_M/S$, can never be reduced to less than $10^{-10}(m/10^{16}\text{GeV})$ by pair annihilations of monopoles and antimonopoles, where $m$ is the monopole mass. On the other hand, the ratio must be smaller than $10^{-24}(m/10^{16}\text{GeV})^{-1}$, from the constraint that the mass density due to monopoles must not exceed the limit on the mass density of the universe imposed by the observed Hubble constant and deceleration parameter. This means that there is a gap of order $10^{10}$ between the prediction of current theories and the upper limit imposed by observations, for monopoles with standard mass of order $10^{16}\text{GeV}$.

In order to avoid this overproduction, various ideas such as cosmic models with the first-order phase transition (especially, one-bubble models), monopole confinement similar to quarks in a hadron, and the change of breaking pattern of gauge symmetry have been proposed. Moreover, it was shown that a consistent scenario cannot be constructed by monopole annihilation due to gravitational clumping.

One of the reasons why some of the above approaches do not work well is that sufficient dilution of monopoles necessarily dilutes the baryon/entropy ratio greatly by entropy production. As for the origin of the cosmic baryon asymmetry, a mechanism by the evaporation of mini-black holes is proposed instead of the usual mechanism in the standard hot universe. Namely, the baryon asymmetry is generated through asymmetric decays of superheavy bosons which are produced by the black hole evaporation. In respect of the problem of monopoles, black holes are useful because the entropy produced at their evaporation reduces the monopole/entropy ratio. These properties of
black holes prompt us to investigate whether the introduction of primordial black holes may solve the overproduction problem of monopoles. With a view of explaining the origin of the cosmic baryon asymmetry, Lindley\textsuperscript{11} investigated the evolution of the universe including primordial black holes in detail. He also estimated the abundance of monopoles emitted by the black hole evaporation and found the condition that monopoles are not produced so sufficiently to contradict the observations. However, he neglected the contribution of monopoles which have existed before the evaporation, supposing implicitly that such "primordial" monopoles are sufficiently diluted by the entropy produced at the evaporation. This supposition is the case only in the cold universe model in which the gauge symmetry has been always broken because of low temperature. In the present paper, we investigate the evolution of the "hot" universe including primordial black holes. At present, we have no direct evidences indicating the existence of primordial black holes. However, non-uniform structure such as galaxies and clusters of galaxies which we observe in the present universe indicates that there may exist (large-scale) fluctuations in the early stage of the universe, and at the Planck time ($t \sim 10^{-43}$ sec.), the universe may be chaotic due to large-amplitude fluctuations of the space-time by the effect of quantum gravity. Therefore, it is not strange that there existed primordial black holes in the early universe as the result of such fluctuations. Moreover, the existence of primordial black holes never contradicts the present observations of the universe, if they had evaporated away in the early stage.

In the present work, we assume that monopole mass is arbitrary for the following reasons. The mass predicted by standard GUTs, $\sim 10^{16}$ GeV, is not proved experimentally. Note that a simple model, minimal $SU(5)$, seems inconsistent with experiments of proton decay.\textsuperscript{13} Since monopole mass is related to the energy scale at which a unified group is broken to a subgroup including a $U(1)$ factor, different models of the unification predict different masses. For example, monopoles become heavier according to various kinds of supersymmetric GUTs,\textsuperscript{14} but much lighter ($\sim 10^{6}$ GeV) according to the Pati-Salam model.\textsuperscript{15} Moreover, there are a sort of models\textsuperscript{16} which distinguish the energy scale when $U(1)$ appears from the grand unification scale by the multiple breaking pattern of gauge symmetry. In the present work, therefore, we take monopole mass, $m$, as a free parameter.

As employed in the previous paper\textsuperscript{3} (hereafter Paper I), we assume that monopoles are produced when the cosmic temperature drops to $m$, and also take the initial monopole /entropy ratio, $\epsilon_i = (n_M/S)_i$, as a free parameter, because the precise estimate of the number density of monopoles created by the phase transition is unknown. It is obvious that the estimate imposed from the particle horizon gives a lower limit simply, and monopoles are created more abundantly, in practice.

As for primordial black holes, we assume that black holes with mass $M$ (monochromatic mass spectrum) are produced when the cosmic temperature is equal to the Planck mass, $m_p(\sim 10^{19}$ GeV), and also take $M$ and the initial energy density ratio of black holes to radiation, $\rho_0 = (\rho_{BH}/\rho_r)_{T=m_p}$, as free parameters.

Consequently, we regard the four values, $m$, $\epsilon_i$, $M$ and $\rho_0$ as free parameters and search for the region of the parameter space where we can construct a consistent scenario. In §2, basic equations necessary to describe the evolution of the universe before the black hole evaporation are shown and their solutions are given. In §3, the evolution after the evaporation is described and constraints imposed by observations are given. In §4,
the results are shown and in §5, some remarks are given.

§ 2. Basic equations and the solution before the evaporation of primordial black holes

Before monopoles are produced (i.e., \( m < T < m_p \)), the time-evolutions of the cosmic scale factor, \( R \), the energy density of radiation, \( \rho_r \), and that of black holes, \( \rho_{BH} \), are described as

\[
\begin{align*}
\frac{1}{R} \frac{dR}{dt} &= \sqrt{\frac{8\pi \rho}{3m_p^2}} = \left( \frac{4\pi^3 N}{45} \right)^{1/2} \frac{T^2}{m_p} \sqrt{1 + h(T)}, \\
\rho &= \rho_r + \rho_{BH}, \\
\frac{d\rho_r}{dt} &= -\frac{4}{R} \frac{dR}{dt} \rho_r, \\
\frac{d\rho_{BH}}{dt} &= -\frac{3}{R} \frac{dR}{dt} \rho_{BH},
\end{align*}
\]

(1)

where \( N \) denotes the statistical weight of particles. The evolution of black hole/radiation energy density ratio, \( h \), and the cosmic time-temperature relation are obtained as

\[
h(T) = (m_p/T)h_0, \\
t(T) \approx \begin{cases} 
\left( \frac{45}{16\pi^3 N m_p^2} \right)^{1/2} \left( \frac{T}{m_p} \right)^{-2}, & h < 1, \text{i.e., } T > h_0 m_p, \\
\left( \frac{5}{\pi^3 N m_p^2} \right)^{1/2} \left( \frac{T}{m_p} \right)^{-3/2} h_0^{-1/2}, & h > 1, \text{i.e., } T < h_0 m_p,
\end{cases}
\]

(3a, 3b)

where we assume that the universe is radiation-dominant initially (i.e., at \( T = m_p \), \( h_0 < 1 \)).

When the cosmic temperature drops to \( m \), monopoles are produced. The evolution of the universe after this time is given by

\[
\begin{align*}
\frac{dn_M}{dt} &= -D n_M^2 - \frac{3}{R} \frac{dR}{dt} n_M, \\
\frac{d\rho_r}{dt} &= D n_M^2 - \frac{4}{R} \frac{dR}{dt} \rho_r, \\
\frac{d\rho_{BH}}{dt} &= -\frac{3}{R} \frac{dR}{dt} \rho_{BH}, \\
\frac{1}{R} \frac{dR}{dt} &= \sqrt{\frac{8\pi \rho}{3m_p^2}}, \\
\rho &= \rho_r + \rho_{BH} + mn_M,
\end{align*}
\]

(4)

where \( D \) is the coefficient which characterizes the annihilation process. In these equations, entropy production due to monopole annihilations is included. As employed in Paper I, we assume that \( D \) is given by

\[
D \approx \begin{cases} 
dT^{-2}, & T_d < T < m, \\
0; & T < T_d,
\end{cases}
\]

(5a, 5b)

where \( d \approx \frac{4\pi^3}{5} \xi(3)\alpha N_c \approx 1 \times 10^4 (N_c/10^2)^{-1} \), and \( N_c \) denotes the statistical weight of charged particles defined as \( N_c = \sum_i (q_i/e)^2 \). The critical temperature \( T_d \) is given by

\( * \) We employ units \( c = \hbar = k_B = 1 \) and the Planck mass \( m_p = G^{-1/2} \approx 10^{19} \text{ GeV} \).
Primordial Black Holes and Monopole Problem

\[ T_d = \left( \frac{16\pi^2 a}{\xi(3)} \right)^2 n^{-8} N_c^{-2} m \approx 9 \times 10^{-5} \left( \frac{N_c}{10^2} \right)^{-2} n^{-8} m, \]

where \( n \) denotes the monopole charge in the unit of a Dirac charge \( g_D = (2\sqrt{a})^{-1} \approx 5.9 \), i.e., \( n = g/g_D \). Equation (5) describes that for \( T > T_d \), monopoles annihilate by the diffusion process while for \( T < T_d \), they do not annihilate effectively, as discussed previously.

For convenience, we introduce a new variable, \( x \), defined as

\[ x = \epsilon / T = n_m(T) / T \cdot S(T), \]

where \( \epsilon \) is defined as the monopole/entropy ratio, \( n_m/S \). Then, Eq. (4) is rewritten as

\[
\begin{align*}
T \frac{dx}{dT} &= \frac{4}{3} Bmx^2 + Bx - \left( 1 + h + \frac{4}{3} xm \right)^{1/2}x, \\
T \frac{dh}{dT} &= -\left( 1 + h + \frac{4}{3} xm \right)^{1/2} \frac{4}{3} Bmx^2 h, \\
\frac{1}{T} \frac{dT}{dt} &= -\left( \frac{4\pi^3 N}{45} \right)^{1/2} \left( \frac{T^2}{m_p} \right) \left( \frac{1 + h + \frac{4}{3} xm}{45} \right)^{1/2} \frac{1}{3} Bmx^2
\end{align*}
\]

for \( T_d < T < m \),

where

\[ B = \left( \frac{\pi N}{45} \right)^{1/2} dm_p \approx 4 \times 10^2 \left( \frac{N}{10^2} \right)^{1/2} \left( \frac{N_c}{10^2} \right)^{-1} m_p. \]

The equations for \( T < T_d \) are obtained by eliminating the terms which include \( B \) in Eq. (8). Note that, in the model without primordial black holes (i.e., \( h = 0 \)), \( x \) is constant and equal to \( B^{-1} \) when monopole annihilation proceeds. This is because the energy release due to the annihilation just compensates the dilution of thermal energy by the cosmic expansion.

Equation (8) cannot be solved analytically, but it is not difficult to obtain the characteristic properties of its solutions. We gain two types of the solutions with respect to the initial condition, that is, the values of \( h \) and \( x \) at \( T = m \), \( h_i \) and \( x_i = \epsilon_i / m \).

If initially \( x_i \) is larger than a critical value, which depends on the ratio \( h \), \( x_\omega(h) \), then \( x \) approaches \( x_\omega \) almost instantaneously (i.e., the necessary time to approach \( x_\omega \) is much shorter than the cosmic expansion time-scale at \( T = m \)) while \( h \) and \( T(\approx m) \) remain almost constant. The critical value, \( x_\omega \), is given by

\[
\begin{align*}
x_\omega &\approx \left\{ \begin{array}{ll}
\frac{h}{2B} (1 + h)^{-1/2} ; & 1 \ll h \ll \left( \frac{B}{m} \right)^2, \\
B^{-1} ; & h \ll 1.
\end{array} \right.
\end{align*}
\]

During this stage, monopoles annihilate very rapidly, but the entropy production is negligible because the abundance of monopoles produced at \( T = m \) is smaller than that of radiation, \( \epsilon_i < 1 \). Subsequently, \( x \) evolves as \( x \propto x_\omega(h) \). During this stage, the decrease of monopole abundance is "quasi-static" in the sense that \( dx/dT = 0 \). On the other hand, if \( x_i \) is smaller than \( x_\omega(h) \), \( x \) evolves as \( x \propto h \), and eventually reaches \( x_\omega(h) \). During this
Fig. 1. Schematic diagram of the evolutions of the monopole fraction before the evaporation of primordial black holes. The evolutions are described by two quantities, $x = \epsilon/T = n_m/T_S$ and $h = \rho_{BH}/\rho_r$. Their relation to the temperature is obtained from the relation, $h \propto T^{-1}$. The numerical values indicated in this figure and also in the subsequent figures are evaluated by putting $n = 1$, $N = N_0 = 10^5$, $N_B = 10$, $m_p = 10^{-8} m_p$, $\epsilon_{CP} = 10^{-2}$ and $p = 10^{-4}$. Various initial ($T = m$) points are shown by full circles, and the subsequent $(T_d < T < m)$ evolutions are described for different initial cases ($\odot$ to $\ominus$). Within these cases, cases $\odot$ and $\oslash$ correspond to the models without primordial black holes. The line, $x = x_\infty(h)$, is a critical line, where monopole annihilation is "quasi-static" in the sense that $dx/dT = 0$. Above this line, monopoles annihilate very rapidly and $x$ reaches $x_\infty$ almost instantaneously. Below the line, the abundance of monopoles is so small that they do not annihilate in practice. Some dashed lines denote the evolutions after $T = T_d$ for different situations at $T = T_d$. For more details, see text.

stage, the annihilation of monopoles is negligible because the initial abundance is sufficiently small. Subsequently, $x$ evolves as $x \approx x_\infty(h)$ in the same way as in the previous case.

The evolution of $h$ is given by a simple relation, $h \propto T^{-1}$, during the stage that monopoles annihilate by diffusion process ($T_d < T < m$). This is because during this stage, the energy density of radiation overcomes that of monopoles, $\rho_r > \rho_m$, independent upon whether the universe is dominated by black holes or by radiation. (Note that this is also the case in the model without primordial black holes, i.e., $h(T) = 0$.)

Figure 1 shows the solutions during the stage that monopoles annihilate by diffusion process ($T_d < T < m$) for different initial values. The subsequent (i.e., $T < T_d$) evolutions are also shown for some cases. In Fig. 1, the evolutions of $x(\equiv \epsilon/T)$ are described as a function of $h(\equiv \rho_{BH}/\rho_r)$ and the directions of the evolutions are shown by arrows. The temperature is obtained from the relation, $h \propto T^{-1}$. The evolutions are shown for various initial values (the values when monopoles are produced at $T = m$) depicted by full circles. The cases $\odot$ and $\oslash$ correspond to the situation including no black holes ($h(T) = 0$). In these cases, $x$ reaches $B^{-1}$ instantaneously ($\odot$) or at $T = B_{\epsilon_1}$ ($\oslash$, if $B_{\epsilon_1} > T_d$). As shown in Paper I, at $T = T_d$ the ratio, $\epsilon$, reaches "Preskill limit" $\epsilon_p$ given by

$$
\epsilon_p = \frac{64}{\xi(3)} \left( \frac{45\pi}{N} \right)^{1/2} a^{3n^{8}N_{c}^{-1}m/m_{p}}
$$
unless the initial value $\epsilon_i$ is smaller than $\epsilon_p$. Subsequently $x$ increases as $x \propto T^{-1}$.

Since the annihilation of monopoles becomes negligible when the temperature drops to $T_d$, the universe evolves adiabatically when $T < T_d$. Therefore, $x$ takes off from $x_\infty$ and evolves as $x \propto h$, because both $x$ and $h$ evolve as $\propto T^{-1}$. Dashed lines in Fig. 1 describe the evolutions at this stage ($T < T_d$) for different values of $x$ at $T = T_d$. For example, in the case (5), where initial value of $x$ is very small, the temperature has dropped to $T_d$ until $x$ reaches $x_\infty$. Therefore $x$ does not evolve as $x = x_\infty(h)$ but as $x \propto h$ even after $T$ becomes less than $T_d$.

The calculations in this section are not dependent on the mass of the black holes, $M$, because the present result depends only upon their energy density, $h$.

§ 3. The evaporation of primordial black holes and the constraints imposed by observations

First, we assume that black holes evaporate and that the evaporation temperature and the lifetime of a black hole with mass $M$ are given by

$$T_{BH}(M) = \frac{m_p^2}{8\pi M}$$

and

$$t_{BH}(M) = \left(\frac{8\pi}{3}\right) \frac{\pi^2}{10} \frac{M^3}{N m_p^4}.\quad (12)$$

If the black holes evaporate at the radiation-dominant era (i.e., at $h < 1$), the entropy production due to their evaporation is negligible. In order to affect the evolution of the universe, the black holes must evaporate after the universe becomes dominated by black holes. The cosmic temperature when the primordial black holes with mass $M$ evaporate is obtained for Eqs. (3b) and (12) as

$$T_b = T(t_{ev}(M)) = \frac{135}{128\pi^5} \left(\frac{N}{6}\right)^{1/3} h_0^{-1/3} \left(\frac{M}{m_p}\right)^{-2} m_p.\quad (13)$$

We assume that the energy density of black holes is instantaneously transformed into that of radiation just when the cosmic temperature drops to $T_b$, i.e.,

$$\rho_r(T_a) = \rho_{BH}(T_b) + \rho_r(T_b) \approx \rho_{BH}(T_b), \quad (14a)$$

$$\rho_m(T_a) = \rho_m(T_b). \quad (14b)$$

Therefore, the cosmic temperature and the monopole/entropy ratio just after the evaporation are given by

$$T_a = h_b^{1/4} T_b = h_0^{1/4} (T_b/m_p)^{3/4} m_p$$

$$\approx 3 \times 10^{-2} \left(\frac{N}{10^2}\right)^{1/4} \left(\frac{M}{m_p}\right)^{-3/2} m_p\quad (15)$$

and

$$\epsilon_a = 3 T_a \rho_m(T_a) = \left(\frac{h_b m_p}{T_b}\right)^{-3/4} \epsilon_b$$

$$\approx 3 \times 10^{-2} \left(\frac{N}{10^2}\right)^{1/4} \left(\frac{M}{m_p}\right)^{-3/2} h_0^{-1} \epsilon_b, \quad (16)$$
respectively, where $\epsilon_b$ is defined as the monopole fraction before the evaporation, $\epsilon_b \equiv \epsilon(T_b)$. Note that the temperature just after the evaporation, $T_a$, is not dependent on $h_0$ but only on $M$. This is because the black holes evaporate at the black hole-dominated era and therefore their energy density when they evaporate, $\rho_B(T_b)$, is determined only by their mass, $M$.

Now, we investigate the situation of the universe after the evaporation for various values of four parameters, $\epsilon_i, m, h_0$ and $M$. First, we estimate $T_b$ (as a function of $M$ and $h_0$) from Eq. (13). Second, $x_b$, i.e., the value of $x$ at $T = T_b$ is evaluated by using the result of §2 for various cases of $\epsilon_i, m$ and $T_b$ (see Fig. 1). Then, from Eqs. (15) and (16) and noting the relation $\epsilon_b = T_b x_b$ we obtain the values of $T_a$ and $\epsilon_a$. Thus we can obtain the values after the evaporation of primordial black holes, $\epsilon_a$ and $T_a$.

However, we must impose the following six conditions in order that the primordial black holes play an important role in the early universe without conflict with observations.

(a) The temperature after the evaporation should be lower than $m$, $T_a < m$, because otherwise GUT symmetry is restored and monopoles are reproduced when the temperature drops to $m$.

(b) The primordial black holes should evaporate away after the universe becomes dominated by them, $T_b < T(h=1)$, and after monopoles are produced, $T_b < m$.

Note that this condition guarantees that primordial black holes evaporate after they enter the horizon. Namely, condition (b) gives the constraint to the parameters as follows:

\[
\begin{cases}
M \gtrsim 10^{-1} \left( \frac{N}{10^2} \right)^{1/6} h_0^{-1/4} \left( \frac{m}{m_p} \right)^{-1/2} m_p, & h_0 > \frac{m}{m_p}, \\
M \gtrsim 10^{-1} \left( \frac{N}{10^2} \right)^{1/6} h_0^{-2/3} m_p, & h_0 < \frac{m}{m_p}.
\end{cases}
\]  

(17)

On the other hand, the condition that they evaporate after entering the horizon is obtained as

\[
M \gtrsim 5 \left( \frac{N}{10^2} \right)^{1/2} h_0^{1/4} m_p,
\]

and automatically satisfied from Eq. (17) (except for the case that both $h_0$ and $m/m_p$ are nearly equal to unity).

(c) The annihilations of monopoles after the evaporation should be negligible, because otherwise the monopole/entropy ratio, $\epsilon$, becomes equal to Preskill limit, $\epsilon_p$, just as in the model without primordial black holes. This condition is described as

\[
\begin{cases}
T_a < T_d \\
or \\
T_d > T_d \quad \text{and} \quad \epsilon_a < \epsilon_p.
\end{cases}
\]

(d) Monopole abundance should be reduced to the value less than the upper limit imposed by observation, i.e., $\epsilon_a \cdot m/m_p < 10^{-27}$.

(e) Baryon asymmetry should be sufficiently generated. Namely, $n_B/S \gtrsim 10^{-10}$. Here, we suppose two scenarios.
**Scenario 1:** Baryon asymmetry is generated through asymmetric decays of the relatively light Higgs bosons (or X-bosons) produced at the black hole evaporation. According to this scenario, for a black hole heavier than \( m_{\text{BH}} > \frac{\rho_{\text{BH}}}{24\pi N} \), the baryon number density

\[
n_B(T_a) = \frac{\varepsilon_{CP} N_H}{24\pi N} \left( \frac{m_H}{m_p} \right)^{-2} \rho_{\text{BH}}(T_b) \frac{N_H}{M},
\]

is generated, where \( m_H \) and \( N_H \) denote the mass and the statistical weight of Higgs bosons, and \( \varepsilon_{CP} \) is the magnitude of CP violation. Since \( S(T_a) = \frac{2\pi^2}{45} NT_a^3 \), the baryon/entropy ratio just after the evaporation is easily obtained from Eqs. (15) and (18) as

\[
\frac{n_B}{S}(T_a) \approx \frac{1}{32\pi} \frac{N_H}{N} \left( \frac{m_H}{m_p} \right)^{-2} \left( \frac{M}{m_p} \right)^{-1} \left( \frac{T_b}{m_p} \right)^{3/4} h_0^{1/4} \varepsilon_{CP} \frac{50}{\pi} \frac{N_H}{N^{3/4}} \left( \frac{m_H}{m_p} \right)^{-2} \left( \frac{M}{m_p} \right)^{-5/2}.
\]

Comparing Eq. (19) with the lower limit imposed by observations, we obtain the following constraint:

\[
M < 4 \times 10^6 \left( \frac{N_H}{10^3} \right)^{2/15} \left( \frac{N}{10^2} \right)^{3/10} \left( \frac{m_H}{10^8 m_p} \right)^{-4/5} \varepsilon_{CP}^{1/5} m_p.
\]

**Scenario 2:** The sufficient baryon asymmetry remains after the evaporation in spite of the entropy production. If we suppose that the baryon/entropy ratio before the evaporation is order \( 10^{-2} \), then the ratio should remain more than \( 10^{-10} \) after the evaporation is described as

\[
\Delta = \frac{n_B}{S(T_a)} = \frac{n_B}{S(T_b)} = 10^{-8}.
\]

From Eq. (16), the “dilution factor”, \( \Delta \), is given by

\[
\Delta \approx 3 \times 10^{-2} \left( \frac{N}{10^2} \right)^{1/4} h_0^{-1/4} \left( \frac{M}{m_p} \right)^{-3/2},
\]

so the condition is obtained as

\[
M < 2 \times 10^4 \left( \frac{N}{10^2} \right)^{1/4} h_0^{-2/3} m_p.
\]

(f) Primordial nucleosynthesis should proceed as in the standard model. Namely, \( T_a > 10^{-21} m_p \sim 10^{-21} \text{MeV} \). If we employ Scenario 1 as the origin of baryon asymmetry, this condition is automatically satisfied because the universe must be reheated sufficiently. On the other hand, in Scenario 2, this is not the case. From Eq. (15) and the condition, \( T_a > 10 \text{MeV} \), we obtain the following constraint on the mass, \( M \),

\[
M < 9 \times 10^{12} \left( \frac{N}{10^2} \right)^{1/6} m_p.
\]

We investigate where these conditions (a)~(f) are satisfied in the four-dimensional...
space of parameters, \((M, h_0, m, \varepsilon_i)\). As shown in Paper I, condition (d) that monopole abundance is less than \(10^{-27}(m/m_p)^{-1}\) in the model without primordial black holes is given by \(\min[\varepsilon_i, \varepsilon_p] \leq 10^{-27}(m/m_p)^{-1}\). From Eq. (11), we know that this condition is equivalent to the following condition:

\[
\begin{align*}
\text{or} & \\
m \leq & 6 \times 10^{-11} m_p , \\
m & \geq 6 \times 10^{-11} m_p \quad \text{and} \quad \varepsilon_i < 10^{-27} \left( \frac{m}{m_p} \right)^{-1}.
\end{align*}
\] (25)

In other words, if

\[
m \geq 6 \times 10^{-11} m_p \quad \text{and} \quad \varepsilon_i > 10^{-27} \left( \frac{m}{m_p} \right)^{-1},
\] (26)

the overproduction problem remains in the model without primordial black holes. Therefore, we limit our investigation to the "overproduction region", Eq. (26), chiefly.

§ 4. Results

The primordial black holes evaporate at different stages of the cosmic evolution.

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**Fig. 2.** Schematic diagram of the allowed region of the parameters relating to monopoles, \((m, \varepsilon_i)\) if we employ appropriate values as the parameters relating to primordial black holes, \((M, h_0)\), in Scenario 1 of baryon asymmetry. The region where we must construct a model with \(h_t > 1\) (the "cold" universe model) is also shown. The dotted region denotes where the overproduction problem is solved without primordial black holes. The line, \(\varepsilon_i = \varepsilon_h(h_t < 1)\), corresponds to the lower limit of initial monopole abundance imposed from the particle horizon ("horizon limit"). For details, see text.

**Fig. 3.** The same schematic diagram as Fig. 2 but in Scenario 2. Here, the condition that \(h_t > 1\) is always unnecessary and we obtain the same result even if we admit such a condition. The horizon limit, \(\varepsilon_h\), is not shown, but it is the same as in Fig. 2.
described in Fig. 1, for different values of the four parameters. For example, they evaporate after the temperature drops to \( T_d \) for some sets of the parameters, but evaporate before \( x \) reaches \( x_\infty \) for others. Therefore, different formulae are necessary to describe the evolution of the universe after the evaporation of primordial black holes for different sets of the four parameters. It is much complicated to describe all the cases, so we describe here some typical cases only, and show the summary of the result.

First, we pursue the region \((m, \epsilon_i)\) where we can construct a consistent scenario by employing appropriate values for \( M \) and \( h_0 \). In Figs. 2 and 3, the allowed region of \((m, \epsilon_i)\) is described for the cases that Scenario 1 and Scenario 2 are taken as the origin of baryon asymmetry, respectively. In Fig. 2, the region which is allowed if we assume that the value of \( h \) at \( T = m \) is greater than unity, \( h_i > 1 \), is also shown. The condition that \( h_i > 1 \) means that the universe has already become dominated by black holes when monopoles are produced. We call such a scenario of the universe "cold" universe model in the sense that radiation is not a dominant component of the universe at the monopole production. On the other hand, the condition, \( h_i > 1 \), is not necessary anywhere in the allowed region in Fig. 3. Even if we admit the case that \( h_i > 1 \), the allowed region is not enlarged for Scenario 2.

In both Scenarios 1 and 2, the smaller mass sides of the allowed regions (i.e., \( m < 10^{-9} m_p \) or \( 10^{-6} m_p \) for Scenario 1; \( m < 10^{-6} m_p \) for Scenario 2) are obtained by the effect of black holes which evaporate after (or during) the annihilations for monopoles, \( T_b < T_d \). Because the difference between the Preskill limit, \( \epsilon_p \), and the value imposed by observations, \( 10^{-27}(m/m_p)^{-1} \), is not so large in these regions, the difference is easily compensated by the entropy production due to the evaporation of a small amount of black holes. On the other hand, the sides of the smaller initial abundances of the allowed regions (i.e., \( \epsilon_i < 10^{-14} \) or \( 10^{-14}(m/m_p)^{-1} \) for Scenario 1; \( \epsilon_i < 10^{-19}(m/m_p)^{-1} \) for Scenario 2) are obtained independent of the annihilations of monopoles because of their small abundance. In these regions, monopole abundance is small originally, and therefore it is possible that the difference between the initial abundance and the value imposed by observations is compensated by a small amount of black holes.

These boundaries of the allowed regions are easily obtained as follows. First, let us deal with the case in which monopoles are scarce and do not annihilate. Since the fraction at the black hole evaporation, \( \epsilon_b \), is equal to the initial one, \( \epsilon_i \), Eq. (16) is rewritten as \( \epsilon_b \sim 3 \times 10^{-2}(M/m_p)^{-3/2}h_0^{-1}\epsilon_i \). (We use numerical values given in the caption of Fig. 1.) Therefore, the condition of monopole dilution (d) is reduced to

\[
(M/m_p)^{-3/2} h_0^{-1} m/m_p \epsilon_i < 3 \times 10^{-26}.
\]

For Scenario 1, this constraint implies that the largest allowed region in the \((m, \epsilon_i)\)-plane is attained when \( M \sim 6 \times 10^7 m_p \) (the condition of baryon asymmetry (e), see Eq. (20)) and \( h_0 = m/m_p \) (or \( h_0 = 1 \)). Accordingly, we obtain the condition \( \epsilon_i < 10^{-14} \) (or \( \epsilon_i < 10^{-14}(m/m_p)^{-1} \)). For Scenario 2, from the two conditions of baryon asymmetry (e) and nucleosynthesis (f), the optimal values of \( M \) and \( h_0 \) are \( M \sim 10^{13} m_p \) and \( h_0 \sim 10^{-13} \) (see Eqs. (23) and (24), and see Fig. 5 also), for which we obtain the condition \( \epsilon_i < 10^{-19}(m/m_p)^{-1} \). Second, let us deal with the case in which the black holes evaporate after the era of monopole annihilation, i.e., \( T_b < T_d \). Since the fraction \( \epsilon \) remains constant for \( T < T_d \), \( \epsilon_b \) is obtained as \( \epsilon_b = \epsilon(T_d) = x_w(h(T_d)) \cdot T_d / (T_d/B) \text{Max}[1, \sqrt{h(T_d)}] \sim 3 \times 10^{-7}(m/m_p) \).
Max[1, 10^2 h_0^{1/2}(m/m_p)^{-1/2}] (see Eqs. (6), (9) and (10)). Therefore, from Eq. (16), the condition of monopole dilution (d) is reduced to

$$\text{Max}[(M/m_p)^{-3/2}h_0^{-1}(m/m_p)^2, 10^2(M/m_p)^{-3/2}h_0^{-1/2}(m/m_p)^{3/2}] < 10^{-19}.$$ 

For Scenario 1, this constraint is optimized when $M \sim 6 \times 10^7 m_p$ and $h_0 = m/m_p$ (or $h_0 = 1$), the same as the case of no monopole burning. Since Max $[m/m_p, 10^2 m/m_p] < 10^{-7}$ (or Max $[(m/m_p)^2, 10^4(m/m_p)^{3/2}] < 10^{-7}$), we obtain the condition $m/m_p < 10^{-9}$ (or $m/m_p < 10^{-6}$). For Scenario 2, the optimal values are also $M \sim 10^{13} m_p$ and $h_0 \sim 10^{-13}$. Since Max $[(m/m_p)^2, 10^{-4}(m/m_p)^{3/2}] < 10^{-12}$, we obtain the condition $m/m_p < 10^{-5}$.

In Fig. 2, the initial abundance imposed from the particle horizon (in the case that $h_0 < m/m_p$), $\epsilon_H$, is also described. Since a monopole is a topological defect of vacuum (Higgs fields), the number density of monopoles produced at the phase transition is constrained by the size of the particle horizon at their production, $l_H$,

$$n_M(T=m) > p/l_H^3,$$ (27)

where $p(\sim 10^{-1})$ is the probability that the topological defect is created. Therefore, the fraction of monopoles created actually is larger than a value (“horizon limit”), which is given as a function of $h_0$,

$$\epsilon_H(h_0) = \frac{p/l_H^3(m/m_p)^3}{S(m)}$$

$$\begin{cases} 
80\left(\frac{p}{10^{-1}}\right)\left(\frac{N}{10^2}\right)^{1/2}\left(\frac{m}{m_p}\right)^3; & h_0 < \frac{m}{m_p}, \\
40\left(\frac{p}{10^{-1}}\right)\left(\frac{N}{10^2}\right)^{1/2}h_0^{3/2}\left(\frac{m}{m_p}\right)^{3/2}; & h_0 > \frac{m}{m_p}. 
\end{cases}$$ (28)

If the fraction of monopoles produced actually at $T=m$ is the lower limit, $\epsilon_H$, the introduction of primordial black holes allows us to construct a consistent model for monopoles with mass $(10^{-7} < m/m_p < 10^{-5}$ for both Scenarios 1 and 2. On the other hand, without primordial black holes, we can construct a consistent model for $m/m_p < 10^{-7}$ only.

In Figs. 4 and 5, the parameter regions relating to black holes $(M, h_0)$ where we can construct a consistent scenario for monopoles with mass about $10^{-6} m_p$ and for $\epsilon = \epsilon_H$ are described, for Scenarios 1 and 2, respectively. The numerical values indicated show that...
the allowed regions of the parameters are not wide, but the parameters must be fine-tuned. Of these allowed regions, the upper right region in Fig. 4 \((4 \times 10^{-7} (m/m_p)^{-1} < h_0 < 1)\) corresponds to the case in which the black holes evaporate after the era of monopole annihilation. The other allowed regions correspond to the cases in which monopoles do not annihilate because of their small abundance. In both Scenarios, the lower limits to \(M\) are imposed by the condition of monopole dilution (d). The upper limit to \(M\) is imposed by the condition of baryon asymmetry (e) (see Eq. (20)) in Scenario 1. On the other hand, in Scenario 2, it is imposed by the two conditions of baryon asymmetry (e) and nucleosynthesis (f) (see Eq. (23)).

In the above investigation, we have not taken into account the effect of gravitational clumping of monopoles (i.e., the formation of black holes and monopole stars, and the subsequent burning of the monopole stars or the evaporation of the black holes) after the evaporation of the primordial black holes. As shown in Paper I, gravitational clumping enables us to construct no consistent scenario if the monopole fraction \(\epsilon\) is equal to or less than \(\epsilon_p\) at the monopole-dominated era. In the cosmic model without primordial black holes, the fraction \(\epsilon\) cannot be larger than \(\epsilon_p\) because of the annihilation by diffusion process. However, in the cosmic models including primordial black holes, it may be possible that the monopole fraction after the evaporation, \(\epsilon_a\), is larger than \(\epsilon_p\) and that gravitational clumping offers a consistent scenario. Then, the universe after the evaporation must be cooler than \(T_d\), \(T_a < T_d\), because otherwise the fraction is reduced to \(\epsilon_p\) by annihilations.

We investigate the scenario that secondary black holes are produced as results of gravitational clumping of monopoles and evaporate away eventually. Note that the burning of monopole stars is incomplete and slower than the evaporation of the black holes. We must check up on whether this scenario is consistent with observations. According to Paper I, the baryon/entropy ratio and the cosmic temperature just after the evaporation of the secondary black holes are obtained as

\[
\frac{n_B}{S} \sim 7 \times 10^{11} \left(\frac{N_H}{10}\right) \left(\frac{N}{10^2}\right)^{1/2} \left(\frac{m_H}{10^{-8} m_p}\right)^2 \epsilon_{CP} \left(\frac{m}{m_p}\right)^{11} \epsilon_a \tag{29}
\]

and

\[
T \sim 5 \times 10^{-2} \left(\frac{N}{10^2}\right)^2 \left(\frac{m}{m_p}\right)^3 \epsilon_a \epsilon_a m_p, \quad \tag{30}
\]

respectively, where \(\epsilon_a\) is the monopole/entropy ratio just after the evaporation of the primordial black holes and is given by Eq. (16). Here, two conditions, (e) \(n_B/S \gtrsim 10^{-10}\)
and (f) \( T > 10^{-21}m_p \), should be satisfied. Moreover, the conditions (a) and (b) in §3 should be satisfied. In this scenario, the condition (c) is regarded as follows,

\[ T_a < T_d \quad \text{and} \quad \epsilon_a > \epsilon_p. \]

The condition (d) is, of course, not necessary because all monopoles form black holes and the black holes evaporate away. However, these conditions cannot be satisfied by any values of \( \epsilon_a \) obtained after the evaporation of the primordial black holes. As the result, the effect of gravitational clumping is not useful to solve the overproduction problem of monopoles also in the cosmic model including primordial black holes.

§ 5. Concluding remarks

In the present work, it was shown that indeed the introduction of primordial black holes enables us to enlarge the region of parameters relating to monopoles, \((m, \epsilon_i)\), where we can construct a consistent scenario. But it was shown that for monopoles with standard mass, \( \sim 10^{16}\text{GeV} \), it does not enable us to construct a consistent scenario, unless the initial monopole/entropy ratio, \( \epsilon_i \), is smaller than \( 10^{-14} \) (\( 10^{-11} \) if we admit the case that \( h_i > 1 \), the “cold” universe model). However, such a small abundance is inconsistent with the limit imposed by the size of the particle horizon (i.e., \( \epsilon_H \sim 10^{-7} \) for \( m \sim 10^{16} \text{ GeV} \)). Namely, as long as we regard the “horizon limit” as a lower limit, the introduction of primordial black holes is helpless for monopoles heavier than \( 10^{-5}m_p \).

In Paper I and the present work, we have neglected the annihilation of monopoles when \( T < T_d \). Precisely speaking, however, it is not the case. As pointed out by Dicus, Page and Teplitz,\(^9\) the three-body annihilation process becomes to affect the evolution of the universe eventually after the universe becomes dominated by monopoles. Then, the monopole fraction, \( \epsilon \), evolves as \( \epsilon \propto T^{2/5} \). At the later stage, monopoles become cooler than radiation (“decoupling”) and the fraction evolves \( \epsilon \propto T_{\text{red}}^{13/19} \).\(^*\) It is true that the monopole fraction, \( \epsilon \), decreases at such later stages, but the dilution of baryon asymmetry is the same as that of monopoles, i.e., \( n_B/S \propto n_M/S \), because large entropy is produced by a small amount of monopole annihilation. Therefore, it is clear that the annihilations at the later stage are not useful to solve the overproduction problem. Even if we take into account the annihilations occurring when \( T < T_d \), the qualitative conclusion that primordial black holes are helpless for heavier monopoles will not be changed although some numerical values such as the allowed region may be slightly changed. This is because the later annihilations do not last for a long period, which depends on \( m \) and \( \epsilon_i \), though. Note that the later annihilation stops at \( T \sim 1\text{MeV} \) when relativistic charged particles almost vanish.

In the previous paper\(^9\) and the present paper, we investigated whether the overproduction problem of monopoles is solved by pair annihilations, gravitational clumping and the evaporation of primordial black holes or not. As the result, it becomes clear that the overproduction problem is solved if monopole mass is small. But, none of such cosmological methods are effective for standard monopole mass, \( \sim 10^{16}\text{GeV} \). This result suggests that the monopole problem cannot be solved without drastic changes of the standard cosmological scenario and/or unified theories. In this respect, some inflationary scenarios are very attractive, because there exist no monopoles essentially

\(^*\) In this respect, the details will be showed in other publication.
within the horizon. However, we have not understood yet whether, in these scenarios, the reheating occurs so sufficiently after the inflation that baryon asymmetry is sufficiently generated. This problem is very important also in the respect of the problem of monopoles, and its investigation is expected.

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References

4) See, for example,
5) See, for example,
13) See, for example,
14) See, for example,
16) See, for example,