THE HYDROGEN CONTENT OF WHITE DWARF STARS
IN RELATION TO STELLAR EVOLUTION

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1. Introduction.—The physical theory of the state of matter in white
dwarf stars was first treated by R. H. Fowler in 1926. His investigation
rested on a new departure in wave mechanics which had been made by
Fermi and Dirac a few months previously. Fowler obtained the equation
of state \( P = K\rho^3 \), applying in conditions approximately realised in the
interior of a white dwarf. In 1929 E. C. Stoner and W. Anderson independently
put forward a modified equation of state supposed to take account of the
change of mass with velocity of the electrons, and astronomical discussions
have generally been based on their theory. The modification is, however,
fallacious; and I think there can be no doubt that the original formula
\( P = K\rho^3 \) is accurate.

The untenability of the Stoner-Anderson formula has been pointed out
in earlier papers.* In reference to a more recent attempt to defend it by
Chandrasekhar †, it seems sufficient to say that the wave function treated
by him represents a freely dispersing system of electrons moving under no
forces, and has no relation to the problem in hand.

Observational data for white dwarfs, although meagre, provide sufficient
material for an instructive comparison with theory. There is evidently need
for a re-survey of this subject, which had become confused by the distractions
of the Stoner-Anderson formula. Almost as important as the elimination
of actual error is the restoration of simplicity to a subject which had become
highly and, as it now appears, unnecessarily complicated. The formulæ
derived from the Fowler equation of state are very simple, and this makes it
easy to assess the effect of uncertainties of the observational data.

The most definite line of investigation is a determination of the hydrogen
content of these stars. White dwarf conditions are exceptionally favourable
for a determination of hydrogen content. The procedure is entirely different
from that employed for ordinary stars. It is an impressive illustration
of the applicability of physical theory to the stars that the determinations of
hydrogen content of Sirius and its companion are in near agreement, that of
the companion being perhaps rather the higher; the methods used for the
two stars have scarcely anything in common, and diverge through different
branches of atomic physics.

Since the theory of maintenance of a star’s heat is intimately connected
with the supply of hydrogen, we are led on to discuss the special difficulty of
including white dwarfs in the evolutionary scheme. In particular, the

* M.N., 95, 194; 96, 20, 1935; Relativity Theory of Protons and Electrons,
§ 13.5, etc., 1936.
existence of white dwarfs is found to have an important bearing on recent suggestions as to the mechanism of liberation of subatomic energy.

2. The Principal Formulae.—The following formulae relate to stars at zero temperature (black dwarfs). It is explained in § 6 that the corrections needed to take account of the actual internal temperatures of white dwarfs are likely to be inappreciable.

The physical theory of degenerate matter gives

\[ P = \frac{1}{\frac{8}{3}} \left( \frac{3}{8\pi} \right) \frac{h^2}{m} \sigma^\frac{3}{4}, \]  

(2.1)

where \( \sigma \) is the number of free electrons per unit volume, \( h \) Planck’s constant, and \( m \) the mass of an electron. Let \( \mu_1 \) be the number of units of molecular weight per free electron, so that

\[ \rho = \frac{\sigma}{\mu_1}, \]  

(2.2)

where \( H \) is the mass of a hydrogen atom. By (2.1) and (2.2)

\[ P = K \rho^\frac{3}{4}, \]  

(2.3)

where

\[ K = \frac{1}{\frac{8}{3}} \left( \frac{3}{8\pi} \right) \frac{h^2}{m} \left( \frac{1}{\mu_1} \right)^\frac{3}{4} \approx [12.9955] \mu_1^{-\frac{3}{4}}, \]  

(2.4)

the square bracket indicating the decimal logarithm of the coefficient.

The relation (2.3) gives a polytrope with index \( n = 1.5 \), and the corresponding relation between the mass and radius is (Internal Constitution of the Stars, equation (57.3))

\[ \left( \frac{GM'}{M'} \right)^\frac{1}{n} \left( \frac{R}{R'} \right)^\frac{3}{n} = \left( \frac{\frac{5}{2} K}{4\pi G} \right)^\frac{1}{n}, \]

with the values (from Emden’s tables) \( M' = 2.718, R' = 3.657 \). Hence

\[ MR^3 = \frac{M'R'^3}{16\pi^2G^3} \left( \frac{\frac{5}{2} K}{4\pi G} \right)^3 = [61.6349] \mu_1^{-5}. \]  

(2.5)

For the Sun \( MR^3 = [65.8241] \). We shall henceforth measure \( M \) and \( R \) in terms of the Sun’s mass and radius. In these units (2.5) becomes

\[ MR^3 = [5.8108] \mu_1^{-5}. \]  

(2.6)

An important observational datum is the Einstein shift of the spectral lines, which is proportional to \( M/R \). We denote its value (expressed as a Doppler shift in kilometres per second) by \( a \). Since \( a = 0.634 \) on the Sun, the general value is

\[ a = 0.634 M/R. \]

Eliminating \( R \) by (2.6) we have

\[ a = 15.8 M^{\frac{3}{2}} \mu_1^{\frac{3}{4}}. \]  

(2.7)

Or, eliminating \( M \),

\[ a = [5.6131] R^{-4} \mu_1^{-5}. \]  

(2.8)

Equations (2.6), (2.7), (2.8) determine \( \mu_1 \), and hence the hydrogen content, when any two of the three quantities \( M, R, a \) are known by observation.
The value of the central density (which in the \( n = 1.5 \) model is just 6 times the mean density) in a white dwarf of given mass or radius or Einstein shift is also of interest. Using (2.6) we find

\[
\rho_c = [5.1168] M^2 \mu_1^5 = [4.7384] R^{-5} \mu_1^{-5} = [3.3191] a_8 \mu_1^{\frac{5}{8}}. \tag{2.9}
\]

3. The Hydrogen Content.—The "modified" molecular weight \( \mu_1 \) differs from the ordinary molecular weight \( \mu \), being the weight per free electron instead of the weight per free particle. The difference is important for hydrogen and helium, which give \( \mu_1 = 1.0, 2.0 \) as compared with \( \mu = 0.5, 1.33 \). It is especially noteworthy that the value of \( \mu_1 \) for helium is precisely the same as for the other common light elements \( C, N, O, Mg, Si, Ca \). Determinations of hydrogen abundance in white dwarfs are therefore entirely independent of the abundance of helium—unlike the determinations for ordinary stars which depend on \( \mu \). For \( Ti, Fe \) and elements up to atomic number 40, \( \mu_1 \) ranges from 2.1 to 2.3; and it increases up to 2.5 for the heaviest elements. We adopt \( \mu_1 = 2.1 \) for the non-hydrogen content; the error is scarcely likely to exceed \( \pm 0.1 \). The following table then gives the percentage by weight of hydrogen \( X \) corresponding to different values of \( \mu_1 \):

<table>
<thead>
<tr>
<th>( \mu_1 )</th>
<th>( X )</th>
<th>( \mu_1 )</th>
<th>( X )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>0</td>
<td>1.5</td>
<td>36.4</td>
</tr>
<tr>
<td>2.0</td>
<td>4.6</td>
<td>1.4</td>
<td>45.5</td>
</tr>
<tr>
<td>1.9</td>
<td>9.6</td>
<td>1.3</td>
<td>56.0</td>
</tr>
<tr>
<td>1.8</td>
<td>15.2</td>
<td>1.2</td>
<td>68.2</td>
</tr>
<tr>
<td>1.7</td>
<td>21.4</td>
<td>1.1</td>
<td>82.7</td>
</tr>
<tr>
<td>1.6</td>
<td>28.4</td>
<td>1.0</td>
<td>100</td>
</tr>
</tbody>
</table>

We shall in §§ 4, 5 determine \( X \) in three white dwarfs for which more or less satisfactory observational data are available. Apart from the actual value of such determinations, we may notice that it is a striking confirmation of the general validity of the physical theory of degenerate matter that the values of \( \mu_1 \) which satisfy the observational data lie in the narrow permitted range 1.0 to 2.1.

A determination of \( X \), whether for an ordinary star or a white dwarf, is necessarily a venturesome undertaking; for it assumes that all sources of deviation from the adopted model have been exhaustively studied, so that the hydrogen content remains the only unknown when we come to the fine adjustment of theory and observation. On the whole, the determination for a white dwarf seems less precarious than the determination for an ordinary star; in the latter the highly complicated calculation of the outflow of radiation involves many factors for which we can only give probable estimates. At present, however, the theoretical advantage of the white dwarf determinations is largely counterbalanced by the meagerness of the observational data.

4. The Companion of Sirius.—For this star \( M = 0.95 \), so that by (2.7)

\[
a = 14.8 \mu_1^{\frac{5}{8}}.
\]

Thus the extreme values of \( a \), corresponding to "all hydrogen" and "no
hydrogen”, are 15 and 51 km. per sec. The observed value, found both by Adams and Moore, is 20 km. per sec. This is much nearer to the “all hydrogen” value, and it is unlikely that observational uncertainties can be so large as to upset the conclusion that the companion of Sirius contains a rather large proportion of hydrogen.

The value $a=20$ gives $\mu = 1.20$, $X = 68$. To illustrate the effect of observational uncertainty, the values $M = 0.85$, $a = 25$ (which may perhaps be regarded as extreme limits) give $X = 36$.

For Sirius itself B. Strömgren found $X = 40$ on the assumption of no helium content, with larger values if helium is abundant. It would seem therefore that both components have a roughly equal abundance of hydrogen, that of the companion being perhaps rather the greater. This agrees with our usual view of the evolution of a double star. The mass before fission would have rapid rotation, and the material would be fairly well stirred by the convection currents due to rotation. In so far as the mixing was imperfect, hydrogen would preponderate in the outer part—both on account of diffusion and because the elimination of hydrogen by transmutation occurs in the hot central parts. The companion, being formed from the outer material, would therefore start with rather more hydrogen than Sirius; and the excess would slowly increase owing to the more rapid transmutation of hydrogen in the brighter component.

On the other hand our result negatives the view, often suggested, that the reason why a star contracts into a white dwarf is that it has used up its supply of hydrogen.

Instead of using $M$ and $a$, we might have used $M$ and $R$, adopting the value of $R$ obtained from the absolute magnitude and the effective temperature corresponding to the observed spectral type. This, however, is much more precarious, because there is great uncertainty in extrapolating the usual temperature-type relation to a white dwarf. Since the temperature corresponding to a given spectral type is considered to be higher in a dwarf than in a giant, we should be tempted to adopt a still higher value in a white dwarf; but actually, to agree with the observed $a$, the temperature must be lower. From $M = 0.95$, $a = 20$, we derive $R = 0.030$. From the visual magnitude 8m.53 (Kuiper, 1935) and parallax $11^m.373$, we obtain the absolute visual magnitude $11^m.39$. Combining this with $R$, the effective temperature $T_e$ is 7020°, which is unexpectedly low for a star classed as A5. Presumably therefore we should adopt effective temperatures lower than usual in treating other white dwarfs.

5. Van Maanen No. 2.—According to Kuiper, the radius calculated from the absolute magnitude and spectral type is $R = 0.0089$. The observed radial velocity is 240 km. per sec., which must be largely Einstein shift. If we assume that $a$ is between 200 and 280 km. per sec., and apply (2.8), we obtain $\mu = 2.0$ to 1.88, giving $X = 5$ to 11 per cent. The corresponding mass is 2.8 to 3.9 times that of the Sun.

It would appear that in this star the hydrogen is nearly exhausted. It is difficult to judge whether the accuracy of the observational data is sufficient to render this conclusion trustworthy. If Kuiper in deriving $R$ employed
the ordinary temperature-type relation instead of the specially low temperature scale appropriate to white dwarfs (§ 4), his $R$ would be too small and the resulting $\mu_1$ would be too large.

$\alpha^2$ Eridani.—The combined mass of the double star is 0.65. The mass of the fainter component, which is an ordinary red dwarf, may be estimated from its absolute magnitude as 0.23, giving $M = 0.42$ for the white dwarf. The absolute visual magnitude is $11^m.3$.

Using (2.6) to determine $R$, we can calculate the effective temperature corresponding to any assumed hydrogen content. We have, for example,

<table>
<thead>
<tr>
<th>$X$</th>
<th>$\mu_1$</th>
<th>$T_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.1</td>
<td>10,700°</td>
</tr>
<tr>
<td>28</td>
<td>1.6</td>
<td>8,300°</td>
</tr>
<tr>
<td>45</td>
<td>1.4</td>
<td>7,200°</td>
</tr>
</tbody>
</table>

The star is classed as B9, so that it seems scarcely possible to assign to it an effective temperature much below 10,000°. It would appear therefore that $\alpha^2$ Eridani, like Van Maanen No. 2, contains comparatively little hydrogen.

A.C. 70° 8247.—This star, discovered by Kuiper, is remarkable as having the smallest radius known. We adopt $R = 0.005$ as a rough estimate. The point of interest is that even this small radius is consistent with a reasonable value of the mass. For zero hydrogen content (2.6) gives $M = 12.7$; for 28 per cent. hydrogen $M = 49$. For higher hydrogen content $M$ increases to values which we should deem unlikely.

6. Corrections for Temperature.—It is believed that the temperature in the interior of the companion of Sirius is not greater than that of a main series star. The argument may be put as follows. Taking the usual opacity law $k \propto \rho / g T^4$, and assuming first the same temperature in both stars, the luminosity will be proportional to $g / \rho$. The actual difference of luminosity (7 magnitudes) roughly corresponds to the difference of $p$; more precisely, the guillotine factor $g$ is required to be about 10 times greater in the white dwarf. It is argued that the high proportion of occupation of the energy states in degenerate matter will increase the guillotine factor much more than 10-fold, so that the assumption of equal temperature makes the white dwarf much too bright. The temperature must therefore be lower than on the main series.

The argument is not convincing, because the white dwarf state is far beyond the range to which Kramers' theory of absorption is intended to apply, and the energy-transitions contemplated are of an entirely different nature. At present there is not even a rudimentary theory of absorption in degenerate matter. Rather vague considerations incline me to accept the current view that the opacity in degenerate matter is abnormally low; but there is very little to guide us.

If the temperature is not zero, the usual gas pressure $R \rho T / \mu$ (as well as the radiation pressure, if appreciable) must be added as a correction to $K \rho^5$. The condition that this correction does not exceed 10 per cent. is $T < (\mu K / 10R) \rho^5$, or

$$T < [4.08] \mu \rho^5 \rho^5.$$
This is just about satisfied in the companion of Sirius if the temperature does not exceed that of a main series star. The formulæ of § 2 will then give sufficient accuracy.

If the current assumption of low opacity is wrong, so that the internal temperature is higher than we have supposed, \( R \) will be thereby increased; and the observed value should be correspondingly reduced before applying the black dwarf formulæ of § 2. By (2.6) \( \mu_1 \) is increased and the hydrogen content reduced. But this should apply, not only to the companion of Sirius, but also to \( \theta^2 \) Eridani and Van Maanen No. 2, in which \( \mu_1 \) is so near the limit that no important increase is possible. The latter stars therefore give some support to the view that the opacity in degenerate matter is low enough to avoid temperatures much above those of the main series.

In smaller masses it is necessary to take account of incomplete ionisation of the material, as Kothari has shown. But in white dwarfs the pressure is sufficient to make the ionisation practically complete.

7. The Transition between a Main Series Star and a White Dwarf.—It is generally taken for granted that white dwarfs are formed by the contraction of ordinary stars; but serious difficulties confront us when we examine the circumstances of the transition.

For definiteness consider the contraction of the Sun into a white dwarf. The gravitational energy liberated is sufficient to maintain the present rate of radiation for more than \( 10^8 \) years, and the time is greatly extended if we allow for the reduced radiation in the later stages. Thus we are threatened with grave difficulty in regard to the time-scale. The amount of energy is incomparably greater than that radiated in a Nova outbreak; hence it seems clear that the passage to white dwarf conditions is not catastrophic.

During the slow contraction the internal temperature rises and then falls. A lower limit to the maximum temperature can be calculated in the following way. During the contraction the model changes from \( n > 1.5 \) to \( n = 1.5 \); but, since the central temperature decreases with \( n \), we shall not exaggerate it if we adopt \( n = 1.5 \) throughout. We have at the centre

\[
P = B \rho^\mu, \tag{7.1}
\]

where, for \( n = 1.5 \) and a mass equal to the Sun, \( B = [14.702] \). Also (radiation pressure being negligible)

\[
P = \frac{\mathcal{R}}{\mu} \rho T + K \rho^\mu. \tag{7.2}
\]

From (7.1) and (7.2)

\[
\mathcal{R} T/\mu = B \rho^\mu - K \rho^\mu, \tag{7.3}
\]

which has a maximum value when \( \rho^\mu = B/2K \). We then have

\[
T_{\text{max}} = \frac{\mu}{\mathcal{R}} \frac{B^2}{4K} = [7.891] \mu \mu_1^\mu. \tag{7.4}
\]

According to the formula, \( T_{\text{max}} \) occurs when the ordinary gas pressure and degeneracy pressure are equal and the radius is twice the ultimate radius.
Actually, if we take account of the changing model, it occurs rather earlier. Thus more than half the gravitational energy is liberated after the maximum temperature is passed. It is unfortunately impossible to estimate how much of the contraction occurs during the final stage when the luminosity is decreasing from that of the Sun to that of the observed white dwarfs—a stage greatly prolonged by the slowness of the radiation.

The time occupied by the transition is reduced by the retention of part of the energy in the star, at first as heat energy and later as exclusion energy. In the gaseous stage approximately half the energy is retained; and, according to the usual derivation of the degeneracy formulae, this applies also to the degenerate state. But the latter result involves assumptions which may not be warranted; and I do not think we can exclude the possibility that as zero temperature is approached the whole gravitational energy is being converted into exclusion energy. This would considerably alleviate the time-scale difficulty. The position seems to be that if we strain every doubtful point in favour of rapid transit we can just about fit the evolution of the companion of Sirius into the accepted time-scale. This refers to gravitational energy only, and makes no provision for the disposal of the subatomic energy that may be liberated.

We turn to a still more striking difficulty. The temperature given by (7.4) is some four or five times that of the main series. It is generally agreed that this would enormously increase the rate of liberation of subatomic energy.* The radiation on the other hand increases only moderately. The stars would thus be gaining energy thousands of times faster than it radiates. But that means that it cannot be contracting. It appears that there is no way for an ordinary star to reach the white dwarf state, because the intermediate state is incompatible with contraction.

This difficulty is so fundamental that we have to inquire whether there is any conceivable way of meeting it. At present the most plausible theory of liberation of subatomic energy (due to Bethe †) is that carbon and nitrogen act as catalysts for the conversion of hydrogen into helium. It is claimed that most of the steps of the cycle have been observed in the laboratory, and that the calculation that the process occurs in the Sun at a rate sufficient to maintain the radiation is no more than a mild extrapolation of observational data. In solar conditions the rate of production of heat by this process varies as $T^{17}$. If this is right, the possible conclusions are narrowed down to two alternatives which will be discussed in §§ 8, 9. Serious objections can be found against both alternatives, and it may well be that the ideas now favoured by nuclear physicists require further revision.

8. First Alternative.—We have seen that between the ordinary star and the white dwarf there is an intermediate stage of high temperature, and the star will not contract through this stage unless the expected liberation of subatomic energy is in some way inhibited. This can only mean

* A rough calculation with Bethe’s formula gives an increase in the ratio 1 to $10^{16}$. This is probably an unjustified extrapolation, but it gives an idea of the increase to be expected.

that, when the contraction occurs, either there is no hydrogen to be transmuted or the necessary catalysts are absent.

The calculation of hydrogen content of the companion of Sirius shows that the hydrogen has not been eliminated before the contraction. Moreover, the elimination of hydrogen by transmutation is strictly controlled by the fact that the resulting energy can be got rid of by radiation and cannot have proceeded faster in the small star than in Sirius itself. Since the hydrogen is present, it follows that the catalysts must be absent during the contraction.

We conclude that there is a process (either building up or breaking down) which transmutes carbon and nitrogen into other elements. This must begin at about the temperature of the main series and, like all such processes, increase very rapidly with any further rise of temperature. When the catalysts have been burnt out, the contraction can proceed without opposition.

An objection appears when we try to apply this theory to stars of different masses. The main series temperature has to satisfy two conditions: (1) hydrogen transformation must proceed at the rate required to maintain the radiation; (2) carbon-nitrogen destruction must proceed at a slow but not negligible rate. (If the latter rate were zero the star would remain on the main series until its hydrogen was exhausted.) It is easily seen that the more massive the star the shorter will be the time it remains on the main series. Accordingly, white dwarfs would mostly be massive stars.

A more fundamental objection is that the theory is essentially a "special creation" theory, which supposes the elements (apart from the hydrogen-helium transformation and the breakdown of radio-active elements) to have existed in their present proportions from the beginning; for the conditions at present insisted on by nuclear physicists for transmutation of the elements (with the exceptions above specified) are obviously such as cannot have existed in the universe at any time.

9. Second Alternative. — Instead of supposing the catalysts to be destroyed before the contraction occurs, we may suppose that they are not evolved until after it has occurred. If carbon and nitrogen have not then been evolved, it is unlikely that higher elements have been evolved. Pursuing this idea, we arrive at the following theory.

The first stage is a globe of hydrogen (and very little else) contracting without transmutation of energy owing to the absence of catalysts. When the density approaches that of a white dwarf, evolution of the complex elements begins, and continues in spite of the falling temperature, the key process being presumably more dependent on density than on temperature. Carbon and nitrogen are formed along with the other elements; and, as the amount increases, the consequent catalytic transformation of hydrogen gives an increasing liberation of energy which presently exceeds the radiation. The star then expands. With decreasing density, evolution of the complex elements stops, and the carbon-nitrogen content thereafter remains fixed. (There is no burning out of the carbon-nitrogen on this theory.) The star expands very rapidly through the maximum temperature stage; indeed the enormous rate of transmutation of the hydrogen would
probably make the expansion catastrophic. The star then reaches a state of balance on the main series. A white dwarf stage thus precedes the main series stage; but, if the time-scale permits, the star may return a second time to the white dwarf stage after it has exhausted its hydrogen. The two white dwarf stages are distinguished by the large hydrogen content in the first stage and the vanishingly small content in the second stage.

A number of astronomical considerations favour this view. The formation of a first-stage white dwarf does not take unduly long; because the contracting star, being practically pure hydrogen, will be highly transparent and have great luminosity. We see why in the Sirius system the small star is the white dwarf, whilst the massive, and therefore more rapidly evolving, component is a main series star—a paradox very difficult to explain in any other way. Equally the reverse condition in ε² Eridani is explained, if the larger component is a second-stage white dwarf as its low hydrogen content indicates; but unfortunately the time required seems to be prohibitive. One of the most puzzling facts has been the comparative constancy of hydrogen content of stars of the same mass, which is shown (independently of the rather unreliable absolute determinations of hydrogen content) by their remarkably small scatter from a mean mass-luminosity curve. This is explained if the main transmutation occurs in a white dwarf state, which is a constant feature of the history of every star which has reached the main series.

Of special interest is the high temperature intermediate stage, which is passed through three times before the second white dwarf stage is reached. At the first passage we have a star, highly luminous for its mass, with small radius and therefore high effective temperature. It suggests itself that the stars which are the centres of planetary nebulae may belong to this period. The second passage is probably catastrophic. It is difficult to imagine any other conditions in which development is catastrophic, and we may therefore attempt to associate Novae with this period. The third passage is presumably made only by very massive stars which have had time to exhaust their hydrogen; it is somewhat similar to the first passage but slower, and the luminosity is not so great. Owing to the rapidity of the second passage and the time-lag of the transmutation processes, the star may at the end of it overshoot the main series and become a giant for a time.

This account does not profess to go further than to show that the theory has a superficial attractiveness, which may well become less favourable on closer examination. It is presented, not on its merits as a speculative explanation, but because the claims at present made as to what has been demonstrated by nuclear experiment and theory seem to leave us no alternative—except the theory of §7, which has not even a superficial attractiveness. Of the numerous objections which may be urged, we consider two which are of a very general character.

If the main series stars have all passed through a white dwarf stage, their angular momentum is limited by the condition that the star must have been stable with a very small radius. (It is safest to exclude binary stars in which
a subsequent transfer of angular momentum may have occurred.) I think that the observational evidence of rotation of single stars is difficult to reconcile with this limit. But an angular momentum difficulty has appeared in every evolutionary astronomical problem yet investigated, so that we have come to look on it almost as a pro forma objection. Presumably the explanation, when it is found, will be a general one covering all the problems in which the difficulty has appeared.

The second objection refers to a general unsuitability of the white dwarfs as a place of evolution of the complex elements.

10. Where are the Complex Elements Evolved?—By complex elements I mean elements other than hydrogen and helium. The considerations which have led nuclear physicists to reject the evolution of complex elements in ordinary stars obviously apply to conditions which are both cooler and more rarefied. We are therefore left with white dwarf stars, or conditions approaching thereto, as the only alternative to assuming that uranium atoms, for example, first come into being fully formed.

Unless we make the very artificial assumption that the complex elements are formed principally from helium in conditions in which there is relatively little transmutation of hydrogen, their evolution must be accompanied by liberation of energy. The most important consideration then is that on the white dwarf side of the stage of maximum temperature the conditions are unstable as regards energy liberation, since expansion gives an increase of temperature; whereas on the main series side of the maximum temperature stage the conditions are stable (or at the worst have only pulsatory instability), since expansion gives a decrease of temperature. Obviously we shall encounter great difficulties if we place the most important part of the evolution of the elements in the unstable period. On the other hand, if we place it in the stable period it would be (according to the Bethe theory) impossible for the white dwarf stage to be reached—as explained in § 7.

In some respects a white dwarf seems the very worst place to choose for evolution of the elements. Its low luminosity indicates that transmutation of hydrogen is proceeding a thousand times slower than in the Sun. The difficulty of time-scale, always serious, becomes so exaggerated as to seem prohibitive. The maximum temperature phase seems much more favourable, since high temperature and high density (§ of the white dwarf density) are associated together, and the radiation is great enough to allow a reasonable speed of transmutation. Let us suppose then that evolution of the complex elements first begins when the pure hydrogen star is approaching maximum temperature. It then becomes a critical question whether the evolution proceeds fast enough for the consequent liberation of energy to overtake the rate of radiation before the maximum temperature is reached. If not, the star will pass through the maximum temperature and become a white dwarf. Unfortunately we cannot say that the stars which fail to reach the maximum temperature stage will return to the main series; the conditions are stable, and the star should remain almost stationary in the state at which the balance is first reached.

11. Conclusion.—One thing emerges plainly from this survey of evolu-
tionary possibilities, namely, the decisive importance of an entirely trustworthy determination of the hydrogen content of the companion of Sirius. For theories of transmutation and stellar evolution no other present-day problem is so crucial. Everything turns on whether it is necessary to provide for white dwarfs with considerable hydrogen content or whether the white dwarf state is compatible only with nearly complete elimination of hydrogen. Realising now that a great deal depends on the security of the determination in § 4, we return to consider it. In so doing, we put out of mind the extreme difficulty of conceiving any way in which the companion could have lost its hydrogen whilst the principal star retained it; this should be balanced against the difficulties which seem to beset every attempt to reach a consistent view of evolution.

The observational side of the question is, Can the Einstein shift really be 50 km. per sec. instead of the value 20 found by two observers? To conform to this we should have to attribute to the star an effective temperature of about 13,000°. Allowing for possible uncertainty of the mass we might perhaps reduce the demand to 44 km. per sec., but not less. We need consider only the values for zero hydrogen content, since 5 per cent. hydrogen would be as decisive as 60 per cent. on the evolutionary questions raised.

If there is no observational error, it remains only to consider whether the radius may have been increased, not by hydrogen content, but by temperature as considered in § 6. To reduce α from 50 (or 44) to 20 the star must have more than double the “cold” radius. By § 7 this places it nearly at the maximum temperature stage. Here the degeneracy is only half complete, the gas-pressure being equal to the degeneracy pressure. Every known consideration indicates that the star will be highly super-luminous at this stage, and we are totally at a loss to account for its having the under-luminosity of a white dwarf.

It may seem suspicious that the companion of Sirius is the only one of three white dwarfs to show considerable hydrogen content. But really the evidence is considerably strengthened by this fact. Any kind of correction to the theory which would reduce its hydrogen content would almost inevitably give the other two white dwarfs (one more massive and the other less massive) a negative hydrogen content.

Though the large hydrogen content may not be established with the ideal security which we should desire in a fact on which far-reaching conclusions are to be based, it must, I think, be admitted that the evidence for it is difficult to shake.

Summary.—Most discussions of our theoretical and observational knowledge of white dwarfs have been based on the Stoner-Anderson degeneracy formula which the author considers to have been disproved in his earlier papers. There is therefore need for a study of these stars based on the earlier formula which appears to be correct. In § 2 the elementary formulæ are put into suitable form for practical use. The chief application is to determine the hydrogen content of three white dwarfs for which data are available.

Special attention is directed to the large hydrogen content found for the companion of Sirius. This is crucial in theories of transmutation and
stellar evolution, since it is often assumed in such theories that stars only pass to the white dwarf state after having exhausted their hydrogen content. The difficulty of reconciling Bethe's transmutation theory with the existence of hydrogen-containing white dwarfs is considered, and alternative ways of meeting the difficulty are discussed; one alternative in particular (§ 9) has interesting astronomical consequences. But serious difficulties remain outstanding, and in §11 we re-examine the trustworthiness of the evidence for large hydrogen content. The evidence, however, proves to be very difficult to upset.