Disc stability and neutral hydrogen as a tracer of dark matter

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ABSTRACT
We derive the projected surface mass distribution \( \Sigma_M \) for spherically symmetric mass distributions having an arbitrary rotation curve. For a galaxy with a flat rotation curve and an interstellar medium (ISM) disc having a constant Toomre stability parameter, \( Q \), the ISM surface mass density \( \Sigma_g \) and \( \Sigma_M \) both fall off as \( R^{-1} \). We use published data on a sample of 20 well-studied galaxies to show that ISM discs do maintain a constant \( Q \) over radii usually encompassing more than 50 per cent of the \( \text{H}_I \) mass. The power-law slope in \( \Sigma_g \) covers a range of exponents and is well correlated with the slope in the epicyclic frequency. This implies that the ISM disc is responding to the potential, and hence that secular evolution is important for setting the structure of ISM discs. We show that the gas-to-total mass ratio should be anticorrelated with the maximum rotational velocity, and that the sample falls on the expected relationship. A very steep fall-off in \( \Sigma_g \) is required at the outermost radii to keep the mass and angular momentum content finite for typical rotation curve shapes, and is observed. The observation that \( \text{H}_I \) traces dark matter over a significant range of radii in galaxies is thus due to the discs stabilizing themselves in a normal dark matter dominated potential. This explanation is consistent with the cold dark matter paradigm.

Key words: galaxies: evolution – galaxies: irregular – galaxies: spiral – galaxies: structure – dark matter.

1 INTRODUCTION
\( \text{H}_I \) has long been the best tracer of dark matter (DM) in galaxies (e.g. Bosma 1981; van der Hulst et al. 1993). This is because it typically extends much farther than the optically bright portion of a galaxy, in a fairly regular disc. But this ability to trace DM seems uncanny: typically the projected DM surface density scales very well with the measured \( \text{H}_I \) surface density (Bosma 1981; Sancisi 1983; Carignan & Beaulieu 1989; Carignan et al. 1990; Carignan & Puche 1990a,b; Jobin & Carignan 1990; Broeils 1992; Meurer et al. 1996; Hoekstra, van Albada & Sancisi 2001). Mestel (1963) had already shown that the flat rotation curve (RC) typically seen in the outer parts of spiral galaxies requires a surface mass density \( \Sigma(R) \propto R^{-1} \) fall-off in the disc, if that is where the dominant mass is located. This is not what is observed in the optical but close to the behaviour of \( \text{H}_I \) discs. \( \text{H}_I \) not only is a good dynamical tracer of DM but its distribution scales linearly with the DM in the outskirts of galaxies. This has prompted some researchers to posit that DM may be gaseous, perhaps in a disc configuration (Penninger, Combes & Martinet 1994; Penninger & Combes 1994; Gerhard & Silk 1996; Pfenniger & Revaz 2005; Bournaud et al. 2007; Hensman & Ziebart 2011). The scaling between DM and \( \text{H}_I \) is also well explained by the Modified Newtonian Dynamics (MOND) hypothesis in which the gravitational force law is modified; in a MOND analysis, \( \text{H}_I \) is the only significant mass in the outskirts of galaxies (e.g. Sanders & McGaugh 2002). Either explanation poses a problem for the standard cold dark matter (CDM) scenario. A gaseous form of baryonic DM would be dissipative, whereas in the CDM scenario DM only interacts via gravity and so is non-dissipative. The MOND scenario requires no DM.

We propose an explanation for the linear scaling between \( \text{H}_I \) and DM that is consistent with the CDM scenario. The interstellar medium (ISM) distribution in discs is configured to maintain a uniform minimum stability over as much of the disc as possible. For a flat RC, this will result in a surface density profile having the same form as the dominant mass. In Section 2, we give the basis of our model. In Section 3, we gather recent \( \text{H}_I \) RC data to test this hypothesis. Section 4 discusses our results and presents our conclusions.

2 THE STRUCTURE OF DYNAMICALLY STABLE GAS DOMINATED DISCS
The stability of a purely stellar or purely gaseous disc is a well-studied problem starting with the work of Safronov (1960) and

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Toomre (1964). For a disc to be stable against axisymmetric perturbations, the support from a combination of random motions and centrifugal forces must be larger than the gravitational attraction. This is expressed as a ratio, the ‘Toomre $Q$’ parameter:

$$Q = \frac{\sigma \kappa}{\pi G \Sigma},$$

(1)

where $\Sigma$ is the mass density in the disc, $\sigma$ is the velocity dispersion and $\kappa$ is the epicyclic frequency given by

$$\kappa = \frac{V}{R} \sqrt{2 \left(1 + \frac{R dV}{dR}\right)},$$

(2)

with $V$ being the rotational velocity at radius $R$. For a purely gaseous disc to be stable, $Q > 1$, while a purely stellar disc requires $Q > 1.07 = 3.36/\pi$ for stability. Unstable discs result in the formation of bars and spiral arms which gather the ISM, enhancing star formation efficiency and thus reheating the disc through feedback (Hohl 1971; Sellwood & Carlberg 1984; Debbattista et al. 2006). The stability of a multicomponent disc, i.e. gas and stars, is more complex (Jog & Solomon 1984; Rafikov 2001; Romeo & Wiegert 2011). Here we are concerned primarily with the outer disc, where the ISM mass dominates. In those cases, the single-fluid $Q$ given by equation (1) is sufficient for our purposes. In some of the galaxies, we analyse the stellar disc is important, and we show that the results are usually not significantly different over the radii we are concerned with when a multicomponent disc analysis is employed.

As done commonly, we consider a galaxy where the dominant mass has a spherical distribution of mass with density $\rho_M(R)$, which also contains an embedded ‘light’ (low-mass) gas disc having surface mass density $\Sigma_g$. We take the projected total surface mass density $\Sigma_M(R)$ to be

$$\Sigma_M(R) = 2R \rho_M(R),$$

(3)

This is the projection of a spherical shell on to a ring of the same radius at the equator. This is the correct definition when we are concerned with the RC (or $V$) of the dominant mass. Under our assumptions, disc particles in circular orbits only feel the potential of the mass interior to their orbit. We assume this geometry precisely because it allows some simple derivations. Olling & Merrifield (2000) note that theory indicates that DM haloes should be somewhat flattened spheroids, but that the limited observations do not clearly state what the flattening typically is. If the DM is in a disc, as might be expected for the gaseous form of DM, then the spherical approximation does not hold. As can be discerned from fig. 1 of Hessman & Ziebart (2011), assuming a spherical geometry will cause us to underestimate $\Sigma_M(R)$ at large $R$ in this case. However, for typically assumed RC shapes and beyond a few core radii, the ratio of true to inferred $\Sigma_M(R)$ changes slowly and only by $\sim 10$ per cent. Hence, the simplifying assumption of a spherical halo is not critical to our results.

Assuming standard Newtonian gravity and pure circular orbits, it is straightforward to derive $\Sigma_M(R)$ as a function of the RC:

$$\Sigma_M = \frac{1}{2\pi G R} \frac{d(RV^2)}{dR},$$

(4)

which becomes

$$\Sigma_M = \frac{1}{2\pi G R} \left(2R \frac{dV}{dR} + V\right).$$

(5)

We define $\zeta$ to be the local gas-to-total mass ratio

$$\zeta = \frac{\Sigma_g}{\Sigma_M},$$

(6)

The case of $\zeta$ being constant corresponds to the gas disc tracing the total mass. We will also have occasion to consider the integrated quantity

$$Z = \frac{M_\odot}{M(R_{\text{max}})},$$

(7)

where $M(R_{\text{max}})$ is the mass within the maximum radius, $R_{\text{max}}$, of the HI distribution.

We emphasize that the mass densities $\Sigma_M$ and $\rho_M$ in equation (3) and throughout are the projected and spherically averaged mass densities of all mass, respectively, i.e. disc (stars and gas), bulge, and halo (luminous and dark). Likewise $\zeta$ and $Z$ are the local and integrated gas-to-total mass ratios, respectively. We adopt these definitions to simplify the analysis, avoiding the need to fit the RC into contributions from the different components. The quantity $\zeta$ is thus the reciprocal of the ratio plotted in fig. 7 of Bosma (1981) where it first became apparent that HI traces whatever dominates the mass in the outskirts of galaxies. The implications of our results stem from the well-established result that at large radius the dominant form of mass in galaxies is DM (e.g. Freeman 1970; Rubin, Thomard & Ford 1978; Faber & Gallagher 1979; Kent 1987; Carignan & Beaulieu 1989; Carignan et al. 1990; de Blok et al. 2008).

Solving equation (1) for $\Sigma_g$ and employing equations (2) and (5) yields

$$\zeta = \frac{2\sigma}{Q} \sqrt{\frac{2(1 + (R/V) dV/dR)}{V + 2R (dV/dR)}}.$$  

(8)

It is instructive to adopt a power-law form to the RC:

$$V = kR^\gamma,$$

(9)

where $k$ is a constant. This approximation works fairly well over a limited range of radii in galaxies, which is sufficient for our purposes. This results in

$$\zeta = \sigma \frac{\sqrt{8(1 + \gamma)}}{Q \ kR^{\gamma/2} (2\gamma + 1)}. $$

(10)

The power-law index $\gamma$ has a narrow range of allowed values:

$$-1/2 \leq \gamma \leq 1.$$  

(11)

A $\gamma$ below this range means the RC drops faster than Keplerian, while a $\gamma$ above this range requires a ‘hollow’ mass distribution. $\gamma = -1/2$ corresponds to $\zeta = \infty$ since this requires $\Sigma_M = 0$. In the outer discs of many disc galaxies, the RC is flat at its maximum value; hence, $\gamma = 0$, $V = V_{\text{max}}$, and thus

$$\zeta = \frac{\sqrt{8}}{V_{\text{max}}/Q}. $$

(12)

If $\sigma$ and $Q$ are also constant, or their ratio is, then $\zeta$ is also constant and the gas disc will track the total mass distribution. From equation (5) we then have

$$\Sigma_M(R) = \frac{1}{2\pi G R} \frac{V_{\text{max}}^2}{R}, \quad \Sigma_g(R) = \frac{\sqrt{2} \sigma}{\pi Q G} \frac{V_{\text{max}}}{R}. $$

(13)

This is the well-known relationship of the total surface mass density falling off as $R^{-1}$ where the RC is flat, which is a fair approximation of what is observed in most spiral galaxies.

We posit that discs evolve towards maintaining a constant $Q$. Simple feedback should encourage such a condition. Over the optical face of a galaxy, star formation is likely be the regulating agent. Regions of the disc where $Q$ is higher than average have a disc that is a combination of hot or underdense compared to their
surroundings. In these regions, any star formation activity would decrease (relative to their surroundings), lowering σ, thus allowing more ISM to accumulate or cool and decreasing Q. In regions where Q is low, the disc is a combination of cold or overdense. Star formation will be enhanced in these areas, increasing σ as feedback from the newly born stars kicks in. While the outer disc is usually considered to be devoid of star formation, the discovery of outlying HII regions (Ferguson et al. 1998) and extended ultraviolet discs (Thilker et al. 2005, 2007) indicates that in many cases there are sources of new stars that can help regulate discs. Even in the case of pure gaseous discs, the simulations by Wada, Meurer & Norman (2002) show that a turbulent clumpy disc develops with a large range of Q over short scales, but with a quasi-steady equilibrium maintained with little variation in average Q with time or radius.

3 OUTER DISC Q MEASUREMENTS

We gathered data on 20 galaxies to test our hypothesis that outer discs maintain a nearly constant Q. The majority of the data are from The H1 Nearby Galaxy Survey (THINGS) for which detailed published RCs can be found in Oh et al. (2008) and de Blok et al. (2008). Data from individual studies of six additional galaxies with extended discs are included to test the robustness of the results (Warren, Jerjen & Koribalski 2004; Gentile et al. 2007; Cannon, Salzer & Rosenberg 2009; Elson, de Blok & Kraan-Korteweg 2010; Struve et al. 2010; Westmeier, Braun & Koribalski 2011). The main criteria for selection are that the galaxy has tabulated data available from recent studies (within ~5 yr) and H1 profiles extending beyond the optical radius R25 (where the B-band surface brightness is 25 mag arcsec−2). Table 1 lists the basic quantities of the sample and the data sources, arranged by the maximum rotational velocity Vmax. This sample covers 50 < Vmax < 375 km s−1 and the full range of late-type galaxy morphologies including spirals from types S0 (NGC 1167) to Sd (e.g. NGC 300), irregulars of types Sm (IC 2574) and Im (e.g. NGC 3741) as well as blue compact dwarf (BCD) galaxies (e.g. NGC 2915). The majority of the sample is nearby, with only two having distance D > 15 Mpc. Therefore, we adopt D values that are not based on the Hubble flow, where possible. The sources of D are given in Table 1. For the two cases where we use D based on redshift, we adopt the model given by NED1 for the Hubble flow corrected for inflow to the Virgo cluster, Great Attractor and Shapley supercluster (Mould et al. 2000), and standard cosmological parameters H0 = 73 km s−1 Mpc−1, Ωm = 0.27 and Ωvacuum = 0.73.

The main observational quantities of importance are the RC and the Σ profile. The RCs are shown in Fig. 1. We fit a cubic spline to the RCs using knots set by eye. In performing the fit, we keep the number of spline knots to a minimum and try to follow the data to within the errors. However, we smooth over small-scale fluctuations in the RCs, presumably due to spiral arms or non-circular motions. The fitted splines are shown as the continuous lines in Fig. 1. We chose this functional form for its flexibility and because it allows for easy evaluation of the derivative dV/dR needed for the calculation of κ and thus Q. For comparison, we plot a model RC using the functional form adopted by Leroy et al. (2008), with parameters Vlittle = 150 km s−1, Rlittle = 0.5R25 and R25 = 8 kpc. The sample includes many galaxies with flat RCs at large radii, like this model (DDO 154, ESO 215,2 NGC 2915, 2403, 3198, 6946 and 2841), or are still rising (IC 2574, NGC 3741 and 3621). However, about half the sample have RCs that have a substantial range of radii where they are declining (NGC 2366 and 300, ADBS J1138,3 NGC 7793, 4736, 5055, 2903, 3521, 3031 and 1167). To illustrate the range of shapes, we fit a power-law RC to the data between a limited range of shapes, we fit a power-law RC to the data between a limited

Table 1. Sample properties.

<table>
<thead>
<tr>
<th>Galaxy</th>
<th>Morphology</th>
<th>D (Mpc)</th>
<th>R25 (kpc)</th>
<th>Rmax (kpc)</th>
<th>M_HI (×10^9 M☉)</th>
<th>Vmax (km s⁻¹)</th>
<th>Data; distance source</th>
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</thead>
<tbody>
<tr>
<td>DDO 154</td>
<td>IB(s)m</td>
<td>4.3</td>
<td>1.24</td>
<td>8.28</td>
<td>0.45</td>
<td>50</td>
<td>de Blok et al. (2008); Makarova et al. (1998)</td>
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<tr>
<td>NGC 374</td>
<td>Im</td>
<td>3.2</td>
<td>0.97</td>
<td>6.97</td>
<td>0.22</td>
<td>52</td>
<td>Gentile et al. (2007); Dalcanton et al. (2009)</td>
</tr>
<tr>
<td>ESO 215</td>
<td>Im</td>
<td>5.2</td>
<td>0.80</td>
<td>10.82</td>
<td>1.46</td>
<td>52</td>
<td>Warren et al. (2004); Karachentsev et al. (2007)</td>
</tr>
<tr>
<td>NGC 2366</td>
<td>IB(s)m</td>
<td>3.4</td>
<td>2.20</td>
<td>8.20</td>
<td>0.81</td>
<td>58</td>
<td>Oh et al. (2008); Dalcanton et al. (2009)</td>
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<tr>
<td>IC 2574</td>
<td>SAB(s)</td>
<td>3.8</td>
<td>7.26</td>
<td>11.07</td>
<td>1.67</td>
<td>78</td>
<td>Oh et al. (2008); Dalcanton et al. (2009)</td>
</tr>
<tr>
<td>NGC 2915</td>
<td>BCD</td>
<td>4.1</td>
<td>1.23</td>
<td>10.12</td>
<td>0.49</td>
<td>86</td>
<td>Elson et al. (2010); Meurer et al. (2003)</td>
</tr>
<tr>
<td>NGC 300</td>
<td>S(A)</td>
<td>2.0</td>
<td>5.81</td>
<td>19.36</td>
<td>2.55</td>
<td>99</td>
<td>Westmeier et al. (2011); Freedman et al. (2001)</td>
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<tr>
<td>ADBS J1138</td>
<td>BCD</td>
<td>50.0</td>
<td>2.27</td>
<td>24.20</td>
<td>1.92</td>
<td>106</td>
<td>Cannon et al. (2009); NED</td>
</tr>
<tr>
<td>NGC 7793</td>
<td>S(A)</td>
<td>3.9</td>
<td>5.78</td>
<td>7.74</td>
<td>1.11</td>
<td>118</td>
<td>de Blok et al. (2008); Karachentsev et al. (2003b)</td>
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<tr>
<td>NGC 2403</td>
<td>SAB(s)cd</td>
<td>3.2</td>
<td>7.52</td>
<td>18.01</td>
<td>3.59</td>
<td>144</td>
<td>de Blok et al. (2008); Freedman et al. (2001)</td>
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<tr>
<td>NGC 3198</td>
<td>SAB(s)</td>
<td>13.8</td>
<td>13.22</td>
<td>37.51</td>
<td>14.50</td>
<td>159</td>
<td>de Blok et al. (2008); Freedman et al. (2001)</td>
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<tr>
<td>NGC 3621</td>
<td>S(A)</td>
<td>6.6</td>
<td>9.57</td>
<td>25.77</td>
<td>9.43</td>
<td>159</td>
<td>de Blok et al. (2008); Freedman et al. (2001)</td>
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<tr>
<td>NGC 4736</td>
<td>(R)SA(r)ab</td>
<td>4.7</td>
<td>5.41</td>
<td>9.61</td>
<td>0.52</td>
<td>198</td>
<td>de Blok et al. (2008); Karachentsev et al. (2003a)</td>
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<tr>
<td>NGC 6946</td>
<td>SAB(rs)cd</td>
<td>6.8</td>
<td>10.79</td>
<td>22.08</td>
<td>7.00</td>
<td>224</td>
<td>de Blok et al. (2008); Karachentsev, Sharina &amp; Huchtmeier (2000)</td>
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<tr>
<td>NGC 5055</td>
<td>SA(rs)bc</td>
<td>7.9</td>
<td>14.30</td>
<td>38.15</td>
<td>8.29</td>
<td>212</td>
<td>de Blok et al. (2008); Tully et al. (2009)</td>
</tr>
<tr>
<td>NGC 2903</td>
<td>SA(rs)bc</td>
<td>8.9</td>
<td>15.50</td>
<td>29.34</td>
<td>6.45</td>
<td>215</td>
<td>de Blok et al. (2008); Drozdovsky &amp; Karachentsev (2000)</td>
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<tr>
<td>NGC 3203</td>
<td>SA(rs)bc</td>
<td>10.7</td>
<td>11.49</td>
<td>31.17</td>
<td>12.70</td>
<td>233</td>
<td>de Blok et al. (2008)</td>
</tr>
<tr>
<td>NGC 3031</td>
<td>S(A)ab</td>
<td>3.6</td>
<td>11.42</td>
<td>14.80</td>
<td>4.05</td>
<td>260</td>
<td>de Blok et al. (2008); Freedman et al. (2001)</td>
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<tr>
<td>NGC 2841</td>
<td>SA(rs)b</td>
<td>14.1</td>
<td>28.90</td>
<td>51.68</td>
<td>13.90</td>
<td>324</td>
<td>de Blok et al. (2008); Saha et al. (2006)</td>
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<tr>
<td>NGC 1167</td>
<td>SA0</td>
<td>66.0</td>
<td>23.49</td>
<td>63.88</td>
<td>12.20</td>
<td>377</td>
<td>Struve et al. (2010); NED</td>
</tr>
</tbody>
</table>

1 The NASA/IPAC Extragalactic Database (NED) is operated by the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration.
3 Full name: ADBS J113845+0808 (Cannon et al. 2009).
Figure 1. RC, ordered and scaled by maximum rotational velocity, $V_{\text{max}}$, plotted against radius $R$ scaled by the maximum radius of the data $R_{\text{max}}$. We show the measured points and their errors (blue in the online version), the adopted spline RC fits as the connecting curve (magenta online), with the middle 50 per cent of the H I mass highlighted (green online), and a power-law fit to the data in this region as a dashed line (green online). Each panel lists the galaxy name at upper left-hand corner, the optical radius $R_{25}$ in kpc at the lower left-hand corner, $V_{\text{max}}$ in km s$^{-1}$ at the upper right-hand corner and the power-law slope $\gamma$ at the lower right-hand corner. The vertical (grey) lines mark intervals of $R_{25}$, while the horizontal lines mark intervals of 50 km s$^{-1}$. The toy model plotted in the lower right-hand panel is an RC of the form used by Leroy et al. (2008) with $R_{\text{flat}} = 4$ kpc.
range of radii, defined below. These and the other power-law fits in this paper were performed as linear fits in log–log space using the IDL procedure MPFIT (Markwardt 2009). The fits are shown as the dashed lines in Fig. 1, with the fit parameters, including zero-point, slope $\gamma$, and dispersion of the residuals $\epsilon$, listed in Table 2.

The $\Sigma_\epsilon$ profiles are shown in Fig. 2. They were derived from the inclination-corrected $H_\alpha$ profiles assuming $\Sigma_\epsilon = 1.3 \Sigma_{H_\alpha}$ to account for elements heavier than hydrogen. Since we are primarily interested in the outer disc, we assume that the molecular content is negligible. The profiles are drawn in log–log space, to highlight any power-law behaviour. In general, the profiles are nearly flat or decreasing towards small radii, and steeply dropping at the largest radii, leaving a ‘knee’ where the profile has an approximately power-law form. We examined each profile and determined the inner and outer radii at which the power-law portion begins and ends, respectively, and the fraction of the total $H_\alpha$ mass $M_{H_\alpha}$ within those radii. On average 21(\pm 10) per cent of $M_{H_\alpha}$ is interior to the inner radius and 75(\pm 15) per cent is interior to the outer; hence, the power-law knee contains a bit over 50 per cent of $M_{H_\alpha}$ on average. In order to treat the galaxies consistently, we determine the radii enclosing 25 and 75 per cent of $M_{H_\alpha}$, $R_1$ and $R_2$, respectively, and fit a power-law

$$\Sigma_\epsilon \propto R^\gamma$$  \hspace{1cm} (14)

to the points between these radii. The fits are shown in Fig. 2 with the parameters listed in the corners of the panels and in Table 2. The range $R_1$ to $R_2$ is highlighted in our figures by using thicker lines to plot the profiles. $N$ spans the range $-2.01$ (NGC 300) to $-0.32$ (NGC 3621). The power-law fit is reasonable. The average $\epsilon_{\Sigma_\epsilon} = 0.042$ dex (10 per cent), and in only two cases is $\epsilon_{\Sigma_\epsilon} > 0.1$ dex (in NGC 4736 and 5055).

While we use a power-law approximation of $\Sigma_\epsilon$ for convenience, $H_\alpha$ profiles are not always characterized as such in the literature. Ferguson & Clarke (2001), for example, use an exponential profile $\Sigma_\epsilon = \Sigma_{\epsilon,0} e^{-\alpha R}$,

$$\Sigma_{\epsilon,0}$$ is the central ISM surface mass density and $\alpha$ is the disc scalelength. They show that the ISM in their viscous disc models is well characterized by such a profile overall a wide range in radii, with the ISM scalelength greater than that of the stellar disc (which is well known to follow an exponential profile; Freeman 1970). The bottom right-hand panel of Fig. 2 shows an exponential profile having $\Sigma_{\epsilon,0} = 10 M_{\odot}$ pc$^{-2}$ and $\alpha = R_{\odot}$. This profile is similar in shape to that of our sample. We fit a power law to this profile in the same manner as the sample galaxies, and report the results in Table 2 and Fig. 2. Over the relevant $R_1$ to $R_2$ a power-law fit to an exponential profile gives an $\epsilon_{\Sigma_\epsilon}$ of 0.03 dex (7 per cent). In comparison, 55 per cent of our sample have $\epsilon_{\Sigma_\epsilon}$ less than or equal to this. Thus, a power law is at least as good a functional form as an exponential for about half the sample.

Fig. 3 shows the $\zeta$ profiles in order to test the premise that $H_\alpha$ traces the total mass. To form the ratio, we take $\Sigma_\epsilon$ from the observed profile and $\Sigma_{M_\odot}$ from equation (5). In general the profiles are highly structured. This is due to irregularities in both the $\Sigma_\epsilon$ and the $\Sigma_{M_\odot}$ profiles. The $\zeta$ profiles are particularly sensitive to the latter since it depends on the derivative of the RC (equation 5). The toy model, which combines the smooth RC shown in Fig. 1 and the smooth $\Sigma_\epsilon$ profile shown in Fig. 2, has $\epsilon_{\zeta} = 0.07$ dex. Six of the galaxies in the sample have $\zeta$ profiles that have $\epsilon_{\zeta}$ at this level or smaller.
Figure 2. Radial profiles of ISM surface mass density $\Sigma_g$ (blue online) plotted on a log–log scale. The line colour (in the online version) and highlighting are the same as in Fig. 1. Power-law fits to the highlighted portion of the profile are shown with the dashed lines (dark green online). Each panel lists the dispersion of the residuals of the fits at the upper right-hand corner, the power-law index $N$ at the lower left-hand corner and $\log (\Sigma_g) at R_{25}$ at the lower right-hand corner. The toy model shown in the lower right-hand panel is an exponential profile (equation 15) with scalelength $\alpha^{-1} = R_{25}$ and central density $\Sigma_{g,0} = 10 M_\odot pc^{-2}$.
Figure 3. Profiles of the ratio $\zeta$ of gas surface mass density $\Sigma_g$ to projected total surface mass density $\Sigma_M$, plotted on a log scale, against linear scaled radius. The line colour (in the online version) and highlighting are the same as in Fig. 1. The average log ($\zeta$) and the dispersion about the average in the highlighted region are marked with the solid and dashed horizontal (grey) lines and listed in the lower left-hand and lower right-hand corners, respectively. The vertical lines show intervals of $R_{25}$.

Fig. 4 shows the $Q$ profiles, for an assumed constant velocity dispersion $\sigma = 8$ km s$^{-1}$. The average log ($Q$) between $R_1$ and $R_2$ and the $\epsilon$ in this value are listed in Table 2 and the panels of Fig. 4. For the majority of the sample, $Q$ is quite flat between $R_1$ and $R_2$, and somewhat beyond for many cases. Over this range, the average $\epsilon_Q$ is 0.06 dex (15 per cent), while in two cases it is more than twice that: NGC 4736 and 5055, for which the variation in $Q$ corresponds largely to irregular structure in the $\Sigma_g$ profile (Fig. 2).
Figure 4. Toomre stability parameter $Q$ profiles on a log scale, plotted against linear scaled radius. The line colour and highlighting are the same as in Fig. 1. The average log($Q$) in the highlighted region is marked at the lower left-hand corner and the rms about this average is given at the lower right-hand corner, and these are shown with the horizontal (grey) lines. The vertical lines show intervals of $R_{25}$. In the six cases where the fit range is largely within $R_{25}$, we show profiles of the two-fluid stability parameter $Q_{2f}$ (Romeo & Wiegert 2011) as the dotted lines (thicker over the fit range, shown in red online); the average log($Q_{2f}$) over the fit range and the rms about this average are indicated with a thick line and two dashed lines (shown in pink online).
For NGC 2366, \( Q \) becomes undefined for much of the disc beyond \( R_2 \) because in this case its RC is declining more steeply than a Keplerian decline. Since this is unphysical, it is likely that a warp or non-circular motions cause the RC of Oh et al. (2008) to be underestimated. In the bottom right-hand panel, we combine the toy models shown in Figs 1 and 2. This model has \( \epsilon_0 = 0.09 \) dex, while 70 per cent of our sample have values less than or equal to this. As expected, \( Q \) is rising beyond \( R_2 \) for the majority of the sample.

The \( \sigma = 8 \text{ km s}^{-1} \) we adopt is in the middle of the pack compared to what has been adopted in other studies; e.g. Kennicutt (1989) adopts \( \sigma = 6 \text{ km s}^{-1} \), while Leroy et al. (2008) use \( \sigma = 11 \text{ km s}^{-1} \). Changing to a different constant \( \sigma \) will only change \( Q \) by a constant multiplicative value. Following Tamburo et al. (2009) we also performed calculations where \( \sigma \) declined linearly with radius, using their profiles for the few overlapping cases between our study and theirs, and otherwise setting \( \sigma = 10 \text{ km s}^{-1} \) at \( R_2 \) and falling linearly with radius to \( \sigma = 5 \text{ km s}^{-1} \) at the last measured point of the radial profiles. The resultant \( Q \) profile does not look very different from Fig. 4; in particular, the decline in \( \sigma \) does not remove the rise in \( Q \) often found at large \( R \).

While Fig. 4 shows that \( Q \) is fairly constant over radii incorporating about half of the \( H_\text{i} \), usually it increases outside this range. The explanation of what is happening at large radii is fairly simple. There, RCs typically are flat or become flatter as can be seen in Fig. 1. As shown by equation (13), for \( \epsilon \) to be constant in this limit requires \( \Sigma_g \propto R^{-1} \). Such a profile cannot be maintained to an arbitrary radius because it would require infinite \( M_{\text{H}_\text{i}} \). To have a disc with finite mass and angular momentum requires a more rapid drop-off in \( \Sigma_g \), as is observed beyond \( R_2 \) (Fig. 2); hence, \( Q \) typically rises at the largest radii. Hoekstra et al. (2001) point out that the sharp drop-off in \( \Sigma_g \) at large \( R \) does not correspond to an expected decline in RCs, thus casting doubt on the ability of \( H_\text{i} \) to trace DM. Hessman & Ziebart (2011) counter that the fitting technique of Hoekstra et al. (2001) overemphasizes the RC fits at large \( R \) where much of the hydrogen is ionized. Neutral or ionized, there is a finite ISM mass, and this will limit the ability of the ISM to trace DM.

There are multiple causes for the rising \( Q \) profiles towards small radii. The central parts of galaxies typically have a significant molecular content, which we have not included here; hence, \( Q \) is overestimated. In addition, the stellar disc usually dominates the mass distribution, so that a single-component \( Q \) is inadequate for determining the true disc stability (Leroy et al. 2008; Romeo & Wiegert 2011). Our assumption has been that the \( H_\text{i} \) largely resides in the outer disc. However, in six of the sample galaxies (IC 2574, NGC 7793, 4736, 2903, 3031 and 2841) the fitting range is largely interior to \( R_{25} \); hence, the stellar disc is likely to play an important role in determining the stability of the disc in these cases.

In order to determine the true disc stability for these cases, we calculated the two-fluid (stars and gas) stability parameter \( Q_{2f} \). There are various formulations of the two-fluid stability parameter (e.g. Jog & Solomon 1984; Wang & Silk 1994; Rafikov 2001; Romeo & Wiegert 2011). Here, we calculate \( Q_{2f} \) from equation (9) of Romeo & Wiegert (2011) which accounts for the thickness of the stellar and gaseous discs. It can be rewritten in a simplified form as

\[
\frac{1}{Q_{2f}} = \frac{p_e}{Q_e} + \frac{p_g}{Q_g},
\]

where \( Q_e \) and \( Q_g \) are the stability parameters for the stellar and gaseous components of the disc calculated separately using equation (1), and \( p_e \) and \( p_g \) are weight factors which depend on the velocity dispersions of the stars and gas and the value of \( Q_e \) compared to \( Q_g \), respectively. Calculation of \( Q_{2f} \) requires the stellar mass density \( \Sigma_\star \) in the disc which we derive from the stellar mass radial profiles of de Blok et al. (2008), and the radial component of the stellar velocity dispersion ellipsoid \( \sigma_\star \). We take \( \sigma_\star \approx \sigma_\star, R_0/0.6 \) following Shapiro, Gerssen & van der Marel (2003) where \( \sigma_\star, R_0 \) is the vertical component of the stellar velocity dispersion. This is given by

\[
\sigma_\star = \sqrt{2\pi G \Sigma_{\text{disc}} h_\star},
\]

where \( \Sigma_{\text{disc}} = \Sigma_\star + \Sigma_g \) is the total disc surface mass density and \( h_\star \) is the stellar disc scaleheight. Following Leroy et al. (2008) we estimate the scaleheight from the disc scalelength using \( h_\star \approx l/\sqrt{3} \) and fit the exponential scalelength \( l \) over the same fit region as highlighted in our figures. While we are primarily concerned with \( Q_{2f} \) over the fitted region, it is instructive to also see its behaviour beyond this range. In the central regions, in cases where the measured \( \Sigma_\star \) is greater than the extrapolated exponential fit, we replace \( \Sigma_\star \) with this fit, under the assumption that the excess light represents a bulge or thick disc with a scaleheight larger than the disc and thus has a lesser contribution to the disc potential than is expected for the surface brightness. Similarly, at large \( R \), beyond the last measured \( \Sigma_\star \), we also adopt the extrapolated fit when calculating \( Q_{2f} \). The results of the \( Q_{2f} \) calculations are shown in Fig. 4 as the dotted lines (shown in red online). The average \( \log(Q_{2f}) \) over the fitted range is shown as the thick lines (pink online) while the dashed lines (also pink online) are offset from this line by its rms.

Fig. 4 demonstrates that the \( Q_{2f} \) profiles are flatter than the \( Q \) profiles when the entire radial range is considered. The biggest difference is for NGC 4736 where the average \( \pm \) dispersion is \( \log(Q_{2f}) = 0.53 \pm 0.06 \) over the fitted region; the dispersion is a factor of 3 lower than that for the gas \( Q \). For the remaining cases, the differences between \( Q \) and \( Q_{2f} \) are more subtle, over the fitted range where we find \( \log(Q_{2f}) = 0.17 \pm 0.05, 0.46 \pm 0.02, 0.052 \pm 0.04, 0.30 \pm 0.04 \) and \( 0.59 \pm 0.12 \) for IC 2574, NGC 7793, 2903, 3031 and 2841, respectively. The dispersion in \( \log(Q_{2f}) \) is smaller than that of \( \log(Q) \) for half the cases: NGC 4736, NGC 2903 and NGC 3031. In most cases, \( Q \) is an adequate proxy for the total disc stability parameter \( Q_{2f} \) at the radii we are interested in here, even when much of the \( H_\text{i} \) is within the optical radius. However, a more sophisticated analysis, such as using \( Q_{2f} \), is required to extend the analysis even closer to the centre, or in cases like NGC 4736 where the stellar disc strongly dominates the gaseous disc at all radii. In Zheng et al. (2012) we perform such an analysis over the optically bright portion of galaxies, using various prescriptions for \( Q_{2f} \). There we show that the assumption of a constant stability disc can explain the relative distributions of gas, stars and star formation over much of the optically bright portion of galaxies.

Close examination of Figs 1 and 2 shows that another of our premises is not exactly correct: the RCs are not always flat. Instead there is a rather wide distribution of RC power-law indices \( r \). What then causes the nearly constant \( Q \) seen in Fig. 4? This requires the numerator and denominator in the defining equation (equation 1) to have the same shape, i.e. the same slope. For a constant \( \sigma \), this requires the \( \kappa \) and \( \Sigma_g \) profiles to have the same shape. To test this, we fit a power law \( \kappa \propto R^M \) to the epicyclic frequency profile. The \( \kappa \) profiles and the fits to them are shown in Fig. 5, with the relevant fit parameters listed in Table 2 and in the panels of Fig. 5. A power-law form is a reasonable approximation to the \( \kappa \) profiles between \( R_i \) and \( R_2 \).

Fig. 6 plots the power-law index in \( \Sigma_g, N \), against the power-law index in \( \kappa, M \). There is a crude correlation between the indices;
Pearson’s correlation coefficient $r_{xy} = 0.45$, with a 2 per cent chance that the correlation is random. Examination of the figure shows that there are two obvious outliers, and we can see plausible reasons for their discrepant behaviour in each case. NGC 3031 (M81) is in a nearby highly interactive group with three close companions (M82, NGC 3077 and HolX). These may affect the outer ISM distribution of NGC 3031 through either stripping material or having material stripped from them. NGC 300 has a very extended and lopsided H\textsc{i} distribution and the steepest $N = −2.01$ in our sample. Westmeier et al. (2011) note that there are morphological signs of ram-pressure
stripping of the outer disc, which could steepen the $\Sigma_g$ profile. They also note that their ATCA data may not capture the total H I flux due to missing short spacings. The $\Sigma_g$ profile we use was derived from a data cube that combined the ATCA data of Westmeier et al. (2011) and single-dish H I data from the Parkes 64-m telescope obtained for the GASS project (Kalberla et al. 2010), thus recovering the H I flux missing from the ATCA observations.\(^4\) Excluding these two galaxies, $r_{20} = 0.63$, with a 0.3 per cent chance of being random. We conclude that there is a modest correlation between the indices which scatter about the line $N = M$, that is, the $\kappa$ and $\Sigma_g$ profiles have the same shape. This is exactly what is needed for a constant $Q$ disc.

A further test of the hypothesis that galaxy discs evolve to constant $Q$ is given by equation (12) which predicts that $\kappa$ and $V_{\max}$ should be anticorrelated. These quantities are plotted (in the log) in Fig. 7. The filled circles show the average log ($\kappa$) (i.e. $\langle \log (\kappa) \rangle$), between $R_1$ and $R_2$, as marked in Fig. 3. The expected anticorrelation is clearly present, with $r_{xy} = -0.60$; the probability that this is due to random sampling of uncorrelated data is less than 0.3 per cent.

The solid line shows the expected anticorrelation for galaxies with a flat RC, equation (12), and for the assumed $\sigma = 8$ km s\(^{-1}\) and $Q = 3$ (this corresponds to the average log ($Q$) for our sample). The dashed line is a $\chi^2$ fit to the data:

$$\langle \log (\kappa) \rangle = (0.83 \pm 0.13) - (0.93 \pm 0.06) \log (V_{\max}).$$

(18)

The dispersion about this fit $\epsilon_{\log (\kappa)} = 0.28$ dex, while the average offset between the expected relation and this fit is 0.08 dex, very close to what is expected from the standard error on the mean, 0.06 dex. This demonstrates that equation (12) provides a reasonable model for the average gas-to-total mass ratio between $R_1$ and $R_2$.

The hollow circles in Fig. 7 show the ratio $Z$ plotted against $V_{\max}$. These quantities are even more strongly anticorrelated with $r_{xy} = -0.80$ and a less than 0.002 per cent chance that the correlation is due to random sampling of uncorrelated data. The dotted line shows an equally weighted least-squares fit to the data

$$\log (Z) = (1.17 \pm 0.19) - (1.12 \pm 0.09) \log (V_{\max});$$

(19)

the dispersion about this fit $\epsilon_{\log (Z)} = 0.24$ dex. This is tighter than equation (18), probably because $\sigma$ is better defined than $\langle \log (\kappa) \rangle$ which is more susceptible to noise in the $\Sigma_g$ and $V$ profiles.

### 4 DISCUSSION AND CONCLUSIONS

The correlation between the shape of the $\kappa$ and $\Sigma_g$ profiles shown in Fig. 6 is profound. Since $M$ is determined purely by the potential, this implies that the ISM disc is responding to the potential, and hence that secular evolution is driving the correlation. An alternate hypothesis is that the structure of galactic discs is set by the mass and angular momentum accretion history (Barnes 2002; Sancisi et al. 2008). However, it is not clear why there should be any such correlation under this scenario. CDM simulations indicate that Milky Way mass galaxies have had typically only two major merger events since $z \sim 2$ (Cole et al. 2007; D’Onghia, Mapelli & Moore 2008), 10 Gyr ago, which agrees with observations of mergers (Conselice, Rajgor & Myers 2008). So, to the extent the accretion is from major mergers, the outer discs should be very lumpy. Our sample does include some lumpy discs (e.g. NGC 4736 and 5055) and also some galaxies that may currently be interacting (NGC 300 and 3031) perhaps contributing to the scatter and outliers in Fig. 6. The fact that the majority of the sample falls on the $N = M$ line suggests that most are not strongly interacting and that prior interactions happened far enough in the past that the disc has re-stabilized and smoothed out to trace $\kappa$. The time-scale for doing this is the orbital time $t_{\text{orb}}$ ($\sim 3t_{\text{dyn}}$, where $t_{\text{dyn}}$ is the dynamical...
time-scale). For our sample, the average $t_{\text{orb}}(R_2) = 0.6$ Gyr. Taking the separation between major merger events to be 3 Gyr, there should be approximately five orbits between mergers, sufficient time for structure in the disc to smooth out. Alternatively, if accretion is slow ‘cold accretion’ (Kereš et al. 2005; Sancisi et al. 2008), the disc would not be expected to be lumpy. However, that scenario does not provide an obvious explanation for the $\Sigma_r$ and $\kappa$ profiles following each other. Of course, cold accretion combined with feedback to equalize $Q$, as advocated here, is consistent with our results.

Previous studies have noted the large scatter in the H$_I$-to-total mass or DM ratio (e.g. Bosma 1981; Hoekstra et al. 2001). This is hard to explain in the context of H$_I$ being a linear tracer of baryonic DM (Hoekstra et al. 2001). We show that an inverse correlation between $\zeta$ and $V_{\text{max}}$ is expected from constant $Q$ discs (equation 12), and indeed is observed (Fig. 7). This avoids the need to place implausibly large quantities of molecular gas in discs. By linking a luminous component of the disc to the rotational velocity, the $Z$-$V_{\text{max}}$ relation is reminiscent of the Tully–Fisher relation (Tully & Fisher 1977) and the baryonic Tully–Fisher relation (Freeman 1999; McGaugh et al. 2000). However, since $Z$ is a ratio, it does not provide a means to estimate ISM mass from $V_{\text{max}}$. To do so, one needs to know the extent of the H$_I$ mass distribution, since in the case of $\gamma = 0$ the local $\zeta$ and the integrated gas-to-total mass ratio within $R$ remain constant and equal to each other. Alternatively, if one has $M_{\text{H}_I}$ and $V_{\text{max}}$, then the extent of the ISM distribution can be estimated. This is typically the case in blind H$_I$ surveys, such as HIPASS (Barnes et al. 2001) and ALFALFA (Giovanelli et al. 2005), where the H$_I$ flux is known but the source is unresolved. Then one can use equation (19) to estimate $Z$ and from that the maximum H$_I$ extent:

$$R_{\text{max}} = \frac{GM_{\text{H}_I}}{2V_{\text{max}}^2}. \quad (20)$$

This may be useful for determining whether follow-up observations of a particular galaxy are likely to be fruitful, or estimating the covering factor of H$_I$ absorbers (e.g. Zwaan et al. 2005).

Our results imply that secular processes are important for setting disc structure. Lynden-Bell & Fringle (1974) noted that viscous discs should evolve so that the mass is concentrated in the centre and the angular momentum goes to infinity. As dissipative encounters cause ISM mass to be funnelled towards the galaxy centre, the disc must also spread to conserve angular momentum, and the ISM disc size should grow with time (e.g. Ferguson & Clarke 2001). High-resolution simulations of galactic discs show that transient density waves can increase the size of discs and alter their metallicity distributions (Roškar et al. 2008a,b). The flattening of metallicity gradients (Werk et al. 2010, 2011) in outer discs may be evidence of ISM circulation in spreading discs. If discs are spreading now, they should have been denser and more compact in the past. This would imply a higher molecular fraction from the increased hydrostatic pressure, as well as increased total gas content (to account for the stars that have since formed). Braun et al. (2011) noted an order of magnitude increase in the molecular mass for the most luminous star-forming galaxies from now to $z \sim 0.4$, mirroring the increase in the cosmic star formation rate density $\rho_{\text{SFR}}(z)$ (e.g. Hopkins & Beacom 2006). They argue that this implies that recent star formation evolution is largely due to the run-down in the available ISM supply. Hanish et al. (2006) made a similar argument based on the slope in $\rho_{\text{SFR}}(z)$ being similar to that expected from the star formation law. As outer discs spread and evolve, $Q$ should remain constant with radius. The exact values of $Q$ and $\sigma$ will be set by feedback (with less efficient star formation in the thinning disc) and the available angular momentum.

Finally, our results do not disprove MOND, nor do they rule out the possibility that some or most of the DM is in a gaseous form. However, some of the appeal of these theories is that it was not clear under the prevailing CDM paradigm why dissipative gaseous discs should trace the non-dissipative CDM halo which resides in a spheroid. Our work provides this linkage by showing that ISM discs will trace DM as a natural consequence of disc stabilization, and our tests of the H$_I$ dominated outer discs are consistent with that interpretation.

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NOTE ADDED IN PRESS

We note a previous paper that reached similar conclusions that had escaped our attention. Struck-Marcell (1991, ApJ, 368, 348) used a related approach, the assumption that gas disks maintain a balance of hydrostatic forces, to derive a disk structure of $\Sigma_\zeta \propto R^{-1}$ for disks having a flat RC. That paper briefly alludes to the relationship with DM, but does not generalize the problem to arbitrary RC shape as we have done.

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