Scaling relations of the slightly self-interacting cold dark matter in galaxies and clusters

M. H. Chan

Department of Physics, The Chinese University of Hong Kong, Shatin, New Territories, Hong Kong, China

ABSTRACT

Recent observations in galaxies and clusters indicate that dark matter density profiles exhibit core-like structures which contradict the numerical simulation results of collisionless cold dark matter. The idea of self-interacting cold dark matter (SICDM) has been invoked to solve the discrepancies between the observations and numerical simulations. In this paper, I derive some important scaling relations in galaxies and clusters by using the long-range SICDM model. These scaling relations give good agreement with the empirical fittings from observational data in galaxies and clusters if the dark matter particles are only slightly self-interacting. Also, there may exist a universal critical optical depth $\tau_c$ that characterizes the core-like structures. These results generally support the idea of the SICDM to tackle the long-lasting dark matter problem.

Key words: dark matter.

1 INTRODUCTION

The nature of dark matter remains a fundamental problem in astrophysics and cosmology. The rotation curves of galaxies and the mass profile probed by the hot gas in clusters indicate the existence of dark matter. It is commonly believed that dark matter is collisionless and becomes non-relativistic after decoupling. Therefore, they are regarded as cold dark matter (CDM). The CDM model can provide excellent fits on large-scale structure observations such as Ly$\alpha$ spectrum (Croft et al. 1999; Spergel & Steinhardt 2000), 2dF Galaxy Redshift Survey (Peacock et al. 2001) and cosmic microwave background (Spergel et al. 2007).

However, on the cluster and galactic scales, the CDM model shows discrepancies from observations. N-body simulations based on the CDM theory predict that the density profile of the collisionless dark matter halo should be singular at the centre ($\rho \sim r^0$). Navarro, Frenk & White (1997) first obtained $\alpha = -1$ (the NFW profile). Later, different values of $\alpha$ ranging from $-0.75$ to $-1.5$ were obtained (Moore et al. 1999; Klypin et al. 2001; Taylor & Navarro 2001; Colín et al. 2004; Diemand et al. 2005). Recently, high-resolution numerical simulation indicates $\alpha = -0.8$ for $r \approx 120$ pc and $\alpha = -1.4$ for $r \approx 2$ kpc (Stadel et al. 2009). Nevertheless, observations show us core-like structures instead of singular density profile in many clusters and galaxies. For example, H$\alpha$ observations indicate cores present in over a hundred of disc galaxies and dark-matter-dominated galaxies (Borriello & Salucci 2001; Salucci 2001). de Blok, Bosma & McLaugh (2003) obtained a mildly cuspy slope $\alpha = -0.2 \pm 0.2$ based on modelling the presence of realistic observational effects. In cluster scale, observational data from gravitational lensing also show that cores exist in some clusters (Tyson, Kochanski & Dell’Antonio 1998; Newman et al. 2011). In particular, Sand (2008) obtain $\alpha = -0.45 \pm 0.2$ by the combination of gravitational lensing and dynamical data of clusters MS2137-23 and Abell 383. Clearly, observations do not support the numerical small-scale predictions by the CDM model. This discrepancy is known as the core–cusp problem (de Blok 2010).

In addition, computer simulations predict that there should exist thousands of small dark haloes or dwarf galaxies in the Local Group if the dark matter particles are collisionless (Cho 2012). However, observations of the Local Group only reveal less than one hundred galaxies (Spergel & Steinhardt 2000). Such a discrepancy is known as the missing satellite problem (Cho 2012).

Many theories have been invoked to solve the core–cusp problem and the missing satellite problem. One of the most spectacular ideas is that the dark matter is not cold. The existence of keV sterile neutrinos, as a candidate of warm dark matter (WDM), has been proposed to solve the discrepancies (Xue & Wu 2001). However, recent observations tend to reject the keV sterile neutrinos to be the major component of dark matter since the observational bound of sterile neutrino mass in Lyman $\alpha$ forest contradicts that in X-ray background (Abazajian & Kouzhiappas 2006; Seljak et al. 2006; Viel et al. 2006). Also, the WDM model alone cannot get good agreement on the large-scale power spectrum (Spergel & Steinhardt 2000; Boyarsky et al. 2009). The WDM model is likely to be ruled out in standard cosmology. Therefore, the success of the CDM model on large scales suggests that a modification of the dark matter properties may be the only approach to solve the discrepancies (Spergel & Steinhardt 2000). Spergel & Steinhardt (2000) proposed that the conflict of observations and simulations can be
reconciled if the CDM particles are self-interacting. Later, Burkert (2000) performed the numerical simulation of the self-interacting cold dark matter (SICDM) and showed that core-like structures can be produced. On the other hand, the analysis of the metallicity distributions of globular clusters indicates that the existence of the SICDM is able to solve the missing satellite problem (Côté, West & Marzke 2002). The earliest estimated range of the cross-section per unit mass of the self-interacting dark matter particle is \( \sigma/m = (0.45 - 4.50) \, \text{cm}^2 \, \text{g}^{-1} \) (Spengler & Steinhardt 2000). This ratio has been estimated several times by some model-dependent observations of clusters and galaxies and numerical simulations. For example, Randall et al. (2008) and Bradac et al. (2008) obtained \( \sigma/m \approx 0.7 \) and \(<4 \, \text{cm}^2 \, \text{g}^{-1} \), respectively, by using the observational data from the clusters 1E 0657–56 and MACS J0025.4–1222. On the galactic scales, Ahn & Shapiro (2005) and Koda & Shapiro (2011) show that \( \sigma/m \approx 100 \, \text{cm}^2 \, \text{g}^{-1} \) can explain the core-like structures.

However, gravitational lensing and X-ray data indicate that the cores of clusters are dense and ellipsoidal where the SICDM model predicts them to be shallow and spherical (Loeb & Weiner 2011). Therefore, the dark matter cross-section may either be smaller than expected or depend on velocity. Nevertheless, Peter et al. (2012) show that the discrepancies can still be solved even if the cross-section is velocity independent. The latest numerical simulations with SICDM indicate that the cross-section per unit mass should be \( \sigma/m \approx 0.01-0.1 \, \text{cm}^2 \, \text{g}^{-1} \) in order to produce the reported core sizes and central densities of galaxies and clusters (Buckley & Fox 2010; Peter et al. 2012; Rocha et al. 2012; Zavala, Vogelsberger & Walker 2013).

In this paper, I will show in another way that the slightly long-range interaction of dark matter can naturally generate some model-independent scaling relations in galaxies and clusters which agree with the observations. Lastly, I will comment on this small interaction of dark matter.

2 OPTICAL DEPTH OF THE DARK MATTER PARTICLES

In the SICDM model, the size of a core in a structure depends on the self-interacting rate of the dark matter particles. This rate is closely related to a physical quantity ‘optical depth of the dark matter particles’ \( \tau \). The optical depth for dark matter is defined as \( \tau \equiv \sigma \, d \), where \( d \) is the distance travelled by a dark matter particle and \( \sigma \) is the mean number density of the dark matter particles. Therefore, the optical depth within the core radius \( r_c \) is given by \( \tau = \sigma \, r_c \). The dark matter particles can be considered as collisionless if \( \tau \approx 0 \). Spengler & Steinhardt (2000) propose that \( \tau \approx 1 \) within the core, which corresponds to the ‘photosphere’ of the dark matter. However, this optical depth is too large to match the observational data. Here, we assume that the size of the core is characterized by a critical optical depth \( \tau_c \) such that \( n \sigma \, r_c = \tau_c \), where \( 0 \leq \tau \leq 1 \). Since the core mass is given by \( M_c = 4 \pi m n \sigma r_c^3 / 3 \), we have

\[
\tau_c = \frac{3 M_c}{4 \pi m r_c^2} = \frac{\sigma}{\Omega_c}.
\]

This equation indicates a rough scaling relation \( M_c \propto r_c^2 \) if \( \tau_c \) is a constant. This relation is generally consistent with the recent result in galaxies obtained by Gentile et al. (2009): \( M_c = 72^{+32}_{-20} \, \sigma/r_c^2 \, M_\odot \, \text{pc}^{-2} \).

3 THE SCALING RELATIONS IN CLUSTERS AND GALAXIES

3.1 Baryonic Tully–Fisher relation

The orbital speed in a galaxy is given by

\[
V = \sqrt{\frac{GM}{R}},
\]

where \( M \) and \( R \) are the total enclosed mass and radius of luminous matter, respectively. From equation (2), the observed flat rotation curves in most galaxies give \( M/R \approx M_c/r_c \). By combining equations (1) and (2), we get

\[
M_c = \left( \frac{3}{4 \pi \tau_c} \right) \left( \frac{\sigma}{m} \right) G^{-2} V^4.
\]

The density profile of the SICDM can be approximately given by the Burkert profile (Burkert 1995; Rocha et al. 2012)

\[
\rho(r) = \frac{\rho_0 r_c^3}{(r + r_c)(r^2 + r_c^2)},
\]

where \( \rho_0 \) is the central density of dark matter. Therefore, the integrated mass profile is given by

\[
M(r) = \int_0^r 4 \pi \sigma^2 \rho(r) dr = \pi \rho_0 r_c^3 f(r),
\]

where \( f(r) = \ln[(r^2 + r_c^2)/(r_c^2)] + 2 \ln(r + r_c)/r_c - 2 \tan^{-1}(r/r_c) \). The size of luminous matter \( R \) can be regarded as the radius \( r_{max} \) where the rotation curve peaks in the simulations, i.e. \( R \approx r_{max} \). Since the numerical simulations indicate that \( r_{max} \approx 3 r_c \) (Rocha et al. 2012), by equation (5), the integrated total mass to core mass ratio is about \( M/M_c \approx 5 \). Assuming that the ratio of total baryonic mass to total mass is nearly a constant for all galaxies (\( M_b/M \approx M_{\odot}/M_\odot \approx 0.17 \), where \( M_\odot \) and \( M_{\odot} \) are the cosmological density parameters of baryonic matter and total matter, respectively), the total baryonic mass of a galaxy is given by

\[
M_b = \left( \frac{15}{4 \pi \tau_c} \right) \left( \frac{\sigma}{m} \right) \left( \frac{\Omega_b}{\Omega_m} \right) G^{-2} V^4.
\]

If \( \tau_c \) and \( \sigma/m \) are constant for all galaxies, we have \( M_b \propto V^4 \). This scaling relation is indeed the baryonic Tully–Fisher relation (Tully & Fisher 1977; McGaugh 2005, 2012). Latest observations indicate \( M_b = (47 \, M_\odot \, \text{km}^2 \, \text{s}^{-1} \, V^4 \) (McGaugh 2012). If \( \sigma/m = 0.1 \, \text{cm}^2 \, \text{g}^{-1} \), we get \( \tau_c = 0.005 \). In fact, Mo & Mao (2000) have already shown that the Tully–Fisher relation can be obtained by assuming a particular form of core density profile. Here, I use another independent and simpler way to show that the Tully–Fisher relation is consistent with the SICDM scenario.

Furthermore, from equation (1), we have \( \rho_0 r_c = \tau_c (\sigma/m)^{-1} \), which would be a constant if \( \tau \) and \( \sigma/m \) are constants. Surprisingly, a recent analysis indicates that \( \rho_0 r_c = 141^{+32}_{-53} \, M_\odot \, \text{pc}^{-2} \), which is a constant for a large sample of dwarf and late-type galaxies (Gentile et al. 2009). If \( \tau = 0.005 \) and \( \sigma/m = 0.1 \, \text{cm}^2 \, \text{g}^{-1} \), we get \( \rho_0 r_c \approx 240 \, M_\odot \, \text{pc}^{-2} \), which is generally close to the empirical fits from observations.

3.2 Size–temperature relation in clusters

Reiprich & Böhringer (2001) studied more than 100 clusters’ hot gas profiles and probed the total mass of each cluster. The mass
profile of a cluster can be approximately given by (Reiprich & B"ohringer 2001)
\[ M(r) \approx \frac{3 \beta k T r^3}{G m_c (r^2 + r_c^2)^{\beta}}. \]  
(7)

where \( \beta \) is the parameter ranging from 0.4 to 1.1 in the King's \( \beta \)-model (King 1972), \( T \) is the hot gas temperature and \( m_c \) is the mean mass of a hot gas particle. Here, we have used the fact that the hot gas profiles are nearly isothermal in most clusters (Reiprich & B"ohringer 2001). From equation (7), the central density of the dark matter is given by
\[ \rho_0 = \frac{9 \beta k T}{4 \pi G m_c r_c^2}. \]  
(8)

Since the central density of hot gas is just \( 10^{-20} \text{ g cm}^{-3} \) (Mohr, Mathisens & Evrard 1999), which is much less than the total central density \( 10^{-23} \text{ g cm}^{-3} \), the effect of the baryons at the centre is ignored. By combining equations (1) and (8), we get
\[ r_c \approx \left( \frac{9}{4 \pi \rho_0} \right) \left( \frac{\sigma}{m} \right) \left( \frac{\beta k}{G m_c} \right) T. \]  
(9)

In fact, \( r_c \) represents the core sizes of both total matter (dominated by dark matter) and baryonic matter (Reiprich & B"ohringer 2001). Therefore, the size of the hot gas in cluster can be characterized by \( r_c \). If \( \tau_c \) and \( \sigma/m \) are constant for all clusters, we have a scaling relation \( r_c \propto T \), which agrees with the empirical fits \( R' = 0.9(T/6\text{ keV})^{1/3} \text{ Mpc} \) from observational data of some nearby clusters (Mohr et al. 2000; Sanders 2007), where \( R' \) is the isophotal size of a cluster. For a \( 10^{15} \text{ M}_\odot \) cluster, \( r_c \approx 300 \text{ kpc} \) (Rocha et al. 2012), which is \( \approx 0.7 R' \). If \( \sigma/m = 0.1 \text{ cm}^2 \text{ g}^{-1} \), by using equation (9) and the mean \( \beta \) for all clusters, we have \( \tau_c = 0.006 \).

### 3.3 Mass–temperature relation in clusters

Besides the size–temperature relation, we can also obtain a scaling relation of the total cluster mass and hot gas temperature. At large radii, the hot gas in clusters may not be isothermal. The total cluster mass will be close to the Burkert mass profile in equation (5):
\[ M \approx 7.9 \pi \rho_0 r_c^3, \]  
(10)

where we have assumed that \( R_{200} \approx 15r_c \) and \( R_{200} \) is the radius when the mean total mass density is 200 times the cosmological critical density. By putting equations (8) and (9) into the above equation and assuming \( M_c/M \approx \Omega_h/\Omega_m \), we have
\[ M_h \approx \left( \frac{40}{\pi c^2} \right) \left( \frac{\sigma}{m} \right) \left( \frac{\Omega_h}{\Omega_m} \right) \left( \frac{\beta k}{G m_c} \right)^2 T^2. \]  
(11)

Since the hot gas dominates the baryonic mass in most clusters, the total hot gas mass \( M_h \) in a cluster is close to the total baryonic mass \( M_b \). This scaling relation \( M_h \approx M_b \propto T^2 \), again, agrees with the empirical fitting from clusters \( M_h/10^{14} \text{M}_\odot = 0.017(T/1 \text{ keV})^2 \) (Mohr et al. 1999; Sanders 2007). By putting all the known numerical values and \( \sigma/m = 0.1 \text{ cm}^2 \text{ g}^{-1} \) into equation (11), we get \( \tau_c = 0.002 \).

### 4 DISCUSSION

In this paper, I show that the long-range interaction of CDM can naturally obtain some important scaling relations, including the baryonic Tully–Fisher relation for galaxies (\( M_b \propto V^4 \)), the size–temperature relation (\( r_c \propto T \)) and mass–temperature relation (\( M \propto T^2 \)) in clusters. These scaling relations get remarkably good agreement with the empirical fits from observations. If the cross-section per unit mass is \( \approx 0.1 \text{ cm}^2 \text{ g}^{-1} \), the characteristic critical optical depth \( \tau_c \approx 0.002–0.006 \), which is the nearly same value as found in different scaling relations for galaxies and clusters. Moreover, we can get \( \text{pc} \approx 240 \text{M}_\odot \text{pc}^{-2} \), which is a constant for all galaxies. This result is generally consistent with the recent analysis from the observations of dwarf and late-type galaxies (Gentile et al. 2009). It means only a slight dark matter interaction is enough for producing core-like structures. Therefore, when the central density is high enough such that \( \tau = \tau_c \), a core would be produced. It may explain why some clusters do not exhibit core-like structures as their central densities are too low such that \( \tau \leq \tau_c \) within a resolvable radius.

In the past decade, it is believed that the dark matter cross-section is velocity dependent (Koda & Shapiro 2011; Loeb & Weiner 2011). Nevertheless, recent results in simulations show that it is possible to have a velocity-independent cross-section \( \sigma/m \approx 0.1 \text{ cm}^2 \text{ g}^{-1} \) (Peter et al. 2012; Rocha et al. 2012). In this model, the scaling relations derived may support this idea and enable us to measure this small cross-section by using observational data. Although only a small window of constant cross-section is remained (Zavala et al. 2013), our results provide more pieces of evidence to support the SICDM scenario, which can successfully address the dark matter problem, core–cusp problem and the missing satellite problem.

To conclude, the derived scaling relations by using the SICDM scenario can obtain good agreement with observations. It generally supports the idea of the SICDM and the velocity-independent dark matter cross-section. More simulations and observations will be needed to confirm the existence of the universal critical optical depth \( \tau_c \), which characterizes the core-like structures in galaxies and clusters.

### ACKNOWLEDGEMENTS

I am grateful to the referee for helpful comments on the manuscript.

### REFERENCES

Scaling relations of the SICDM model

Peacock J. A. et al., 2001, Nat, 410, 169

This paper has been typeset from a \TeX\LaTeX file prepared by the author.