A Seismic Solar Model Deduced from the Sound-Speed Distribution
and an Estimate of the Neutrino Fluxes

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Abstract

We have deduced the density, pressure, temperature, and hydrogen profiles in the solar interior by solving the basic equations governing the stellar structure with the imposition that the sound-speed profile is that determined by the helioseismic data of Libbrecht et al. (1990; AAA 52.080.103) and Jiménez et al. (1988; AAA 45.080.041). This approach is completely different from that of the standard solar model, and is based on more experimentally well-determined data. We solved the equations by requiring that the mass and mean molecular weight at the surface match the solar mass and a certain fixed value, respectively, as the outer boundary conditions. Together with these conditions and the appropriate inner boundary conditions, these equations were reduced to a boundary-value problem. We examined whether the luminosity at the surface matches the observed value. The error levels were estimated by a Monte-Carlo simulation with Gaussian noise on the sound-speed profile. The thus-constructed seismic model marginally satisfies the luminosity condition, $L(\frac{R_s}{\alpha}) = L_\odot$, at the 3σ level. Using this seismic model, we estimated the neutrino fluxes, and found that the $^8$B neutrino flux is about 60% of that of the standard solar model. This model does not seem to contradict the Kamiokande neutrino detection experiment. The $^7$Be neutrino flux of the model is about 20% smaller than the standard solar model, and the pp-neutrino flux of the model is almost the same as that of the standard solar model. We estimated the total neutrino capture rate of the chlorine experiment (Homestake) and that of the gallium experiments (GALLEX and SAGE), except for a contribution from the CNO cycle, by scaling the capture rates based on the standard solar model. The thus-estimated capture rates are 5.62 SNU and 117 SNU, respectively, and are higher than those observed at the 3σ error level.

Key words: Helioseismology — Neutrinos — Sun: interior — Sun: oscillations

1. Introduction

The sun is oscillating by itself with a typical period of five minutes, although the amplitude is so tiny that we cannot recognize with our naked eyes that it is a pulsating star. The oscillation was discovered in 1960; we now know that it is a manifestation of a superposition of many eigenmodes of the sun. Helioseismology is a sort of remote sensing, by which we can probe from oscillation data the invisible interior of the sun, ranging from the core up to the subphotospheric region.

One of the scientific goals of helioseismology is to clarify whether or not modelling of the sun is responsible for the solar-neutrino problem, rather than neutrino physics. Experiments for detecting the solar neutrinos were begun by Davis and his colleagues in the 1960s; the detected neutrino flux, which is mainly due to the $^8$B neutrino, in this case, has been deficient compared with the theoretical expectation based on the standard solar models (Davis 1993). This is the solar-neutrino problem, which has remained unsolved for a long time (see, e.g., Bahcall 1989, Haxton 1995). In the late 1980s the Kamiokande experiment group independently succeeded in detecting neutrinos from the sun; however, the flux is again about 1/2 of the theoretical expectation (Hirata et al. 1991; Suzuki 1993). Since the two independent groups using different detection techniques reached the same result, in the sense that the detected $^8$B neutrino flux is deficient compared with theoretical estimates based on the solar models constructed by the standard evolution theory, experiments do not seem to be responsible for the solar-neutrino problem. Furthermore, experiments to detect lower energy neutrinos generated by the pp-reaction succeeded in 1990, and, according to the latest results, the detected flux is also deficient compared with the theoretical estimates (Abrashitov et al. 1994; Anselmann et al. 1994). These facts mean that either the neutrino physics is incorrect, or that the modelling of the solar
interior is incorrect. Although the elementary assumptions and physics have been carefully examined, we have not yet found a definite solution to the problem (e.g., Bahcall, Pinsonneault 1992, 1995; Turck-Chièze et al. 1988; Turck-Chièze, Lopes 1993; Dzitko et al. 1995). In this situation, it is desirable that helioseismology should provide us with an independent probe of the solar interior.

Although the eigenfrequencies of the standard solar models are in fairly good agreement with the observed frequencies, the agreement is not complete. There remain some discrepancies larger than the observational errors. We therefore definitely need some improvements in modelling the interior of the sun. Since the eigenmodes of the sun so far detected are the acoustic modes, the sound-speed distribution inside the sun has been successfully well determined. The discrepancies between the eigenfrequencies lead to a discrepancy between the sound-speed profile obtained by performing an inversion of the observed frequencies and that of the models. The sound speed of the sun seems to be slightly faster near the center than that given by the models. However, whether removing such a discrepancy might resolve the solar-neutrino problem has not been clarified, since the physical quantities, such as the temperature and density, have not been determined based on the sound-speed profile. The physical quantities usually used in stellar-evolution theory are the density \( \rho(r) \), pressure \( P(r) \), temperature \( T(r) \), luminosity \( L_r \), and mass \( M_r \) as functions of the distance from the stellar center \( r \), and not the sound-speed distribution \( c(r) \). Obviously, we need \( T(r) \), \( \rho(r) \), and the chemical-composition profile to estimate the neutrino fluxes; also, the sound-speed profile \( c(r) \) is insufficient to do so. It is thus desired to determine the solar-interior structure from the helioseismic data, and to compare it with the evolutionary models and with the detected neutrino fluxes (cf. Gough, Kosovichev 1990; Shibahashi 1993, 1995; Dziembowski et al. 1994; Antia, Chitre 1995; Goode 1995; Kosovichev 1995).

In this paper we report on our attempt to produce a solar model based on the helioseismic data (Shibahashi 1993, 1995). The preliminary result was presented at the 4th SOHO workshop (Shibahashi et al. 1995). We describe here our method in more detail, and present the latest results, including an estimate of the neutrino fluxes.

2. The Procedure to Make a Standard Solar Model

It is instructive to be reminded how to make a standard solar model. The assumptions adopted in constructing the standard solar model are:

- the model is in hydrostatic equilibrium,
- the model is in thermal balance,
- the model is chemically homogeneous at zero age,
- the initial abundance ratio \( Z/X \) is the same as the present abundance observed at the photosphere,
- abundance changes are caused only by nuclear reactions,
- the standard theory of stellar evolution is followed,
- the age of the sun is \( 4.5 \times 10^9 \) yr,
- \( M = 1M_\odot \), \( R = 1R_\odot \), and \( L = 1L_\odot \) at present,
- no mass is lost during evolution,
- the model is spherically symmetric and we ignore the effects of rotation and the magnetic field, and
- the updated micro-physics is used in calculations of the nuclear reactions, opacity, and convection.

Among them, the first assumption and the values of the present solar mass, radius, and luminosity should be definitely accepted. The real sun must be hydrostatic; otherwise, the sun would shrink or expand within its dynamical timescale, which is on the order of one hour. As for the mass, the radius, and the luminosity, they have been very precisely measured as compared with the other astrophysical quantities. In comparison with these, the other assumptions have less experimental support. Although the standard theory of stellar evolution has succeeded in explaining many observational properties of stars, its success has been in treating stars as a statistical group. There is no guarantee that a specific star, the sun in this case, follows this theory precisely. There are uncertainties in the solar helium abundance \( Y \) and the efficiency of the convective energy transport. In making standard solar models, these uncertainties are treated as free parameters; they are determined so that the luminosity and the radius of the evolutionary model match with \( L_\odot \) and \( R_\odot \) at \( t = 4.5 \times 10^9 \) yr. Therefore, it is fair to say that the standard solar model is not always complete, nor always experimentally well supported.

3. The Procedure to Make a Seismic Solar Model

In the following we depart from the standard construction of a solar model described in the previous section, and try to reconstruct a solar model by using only experimentally well measured quantities. These quantities are the mass \( M_\odot \), radius \( R_\odot \), photon luminosity \( L_\odot \), and the sound-speed distribution \( c(r) \) obtained from helioseismology. We also assume that the sun is in hydrostatic equilibrium. Whether the sun is in thermal balance is...
uncertain, since even if the real sun is not in thermal balance, it takes about $10^7$ yr for the sun to recover its equilibrium state. The justification of this assumption is made only by the solar neutrino flux measurement, since the detected neutrinos must have been generated near to the solar center only eight minutes ago. In this paper, however, we assume that the sun is in thermal balance, since our present purposes are to construct a solar model which is as consistent with various observational data as possible, and to see if the modelling of the sun is responsible for the solar-neutrino problem. In summary, our assumptions in reconstructing a solar model are as follows (Shibahashi 1993, 1995):

- the mass is $M_\odot$,
- the radius is $R_\odot$,
- the photon luminosity is $L_\odot$,
- the sound speed distribution $c(r)$ is that obtained from helioseismology,
- the model is in hydrostatic equilibrium,
- the model is in thermal balance, and
- the model is spherically symmetric, and we can ignore the effects of rotation and the magnetic field.

It should be emphasized that we do not need any assumptions concerning the history of the sun, and that we make use of the data for the present sun. The basic equations for constructing a model with the above assumptions are:

\[
\frac{dM_r}{dr} = 4\pi r^2 \rho,
\]

\[
\frac{dP}{dr} = -\frac{GM_r \rho}{r^2},
\]

\[
\frac{dL_r}{dr} = 4\pi r^2 \rho c,
\]

\[
\frac{dT}{dr} = \begin{cases} 
-\frac{3}{4\pi c^2 L_r} & \text{if radiative} \\
-\frac{\kappa_{\text{conv}}}{4\pi c^2 L_\odot} & \text{if convective},
\end{cases}
\]

and

\[
\frac{\Gamma_1 P}{\rho} = c^2_{\text{obs}}(r),
\]

with the following boundary conditions:

\[
M_r = \begin{cases} 
0 & \text{at } r = 0 \\
M_\odot & \text{at } r = R_\odot
\end{cases}
\]

and

\[
L_r = \begin{cases} 
0 & \text{at } r = 0 \\
L_\odot & \text{at } r = R_\odot
\end{cases}
\]

Here, $M_r$, $L_r$ are the mass and the luminosity at the radius $r$, $c^2_{\text{obs}}(r)$ denotes the sound-speed distribution inferred from helioseismology, and $\kappa(\rho, T, X_i)$ and $\frac{\partial \ln \kappa}{\partial \ln \rho}$ are the nuclear reaction rate and the opacity as functions of the density, temperature, and chemical composition. The other symbols have their usual meanings.

It should be noted that the difference between the present approach and the conventional programs for the stellar structure is only the subject condition; that is, equations (1)-(4) are solved using the given chemical-composition distribution $X_i(r)$ in the conventional programs for stellar evolution, while they are solved with the given sound-speed distribution $c_{\text{obs}}(r)$ in the present approach.

4. Determination of the Hydrostatic Structure

Equations (1)-(4) form a fourth-order differential equation with the auxiliary relations concerning the equation of state, the opacity, and the nuclear reaction rate. In the case of a fully ionized perfect gas for the equation of state, with the help of the constraint (5), the equations governing the hydrostatic balance [equations (1) and (2)] are decoupled from those governing the thermal structure [equations (3) and (4)]. To solve equations (1)-(4), we assumed that this is the case. This was only to simplify the problem; there is no difficulty in solving equations (1)-(4) with a more physical equation of state, such as that of Mihalas-Hummer-Däppen (Hummer, Mihalas 1988; Mihalas et al. 1988; Däppen et al. 1990). The departure from this simple law is expected to be small in the bulk of the sun, except for near to the photosphere; it may be smaller than the uncertainties of the other physical quantities, such as the opacity or the nuclear-reaction rates. We can then determine the pressure and density distribution solely from the helioseismic observational data by using equations (1), (2), and (5) with the boundary condition equation (6). It should be noted that, in doing this, we do not need information about the opacity nor the nuclear reaction rate, which are more uncertain than the equation of state. Our only assumption is the value of the adiabatic exponent $\Gamma_1$, which we set to be 5/3. In reality, $\Gamma_1$ must be very close to 5/3 in the bulk of the sun, except for near the surface. Starting from the center and parameterizing the value of the central density, $\rho_c$, we have only to integrate equations (1) and (2) subject to equation (5) and to search for the parameter value satisfying condition (6).

To verify that the numerical program works correctly, we constructed a stellar model, and then solved equations (1) and (2) using the sound speed of this model.
to see whether the model was reproduced. Since it was, we are convinced that the program is correct. We then solved equations (1) and (2) using the sound-speed distribution, which was inverted by Vorontsov and Shibahashi (1991) from the observational data of the solar p-mode frequencies obtained by Libbrecht et al. (1990) and Jiménez et al. (1988). It is important to estimate the error in the inverted results. To see the effect of any observational errors in the eigenfrequencies on the sound-speed inversion, Vorontsov and Shibahashi (1991) carried out a Monte-Carlo simulation. They constructed 100 sets of frequencies with white noise added with limiting amplitudes corresponding to the 1σ level error of the observed frequencies. They then inverted these data sets and estimated the statistical error as the envelope of this set of the sound-speed profile. We estimate here that the 1σ observational error when Gaussian noise is added is larger than the statistical error estimated by Vorontsov and Shibahashi (1991) for white noise by a factor of $\sqrt{3}$. Besides the observational error, there is also an intrinsic systematic error which depends on the inversion method, itself. To estimate such a systematic error, Vorontsov and Shibahashi inverted the frequencies of two different theoretical models to obtain the sound speeds, and compared the relative difference of these inverted sound speed profiles with that of the true profiles. Although the agreement was fairly good, there remained discrepancies, which were as large as a few percent in the deep interior, where $r/R_\odot \leq 0.2$. This systematic error is about three- to five-times larger than the statistical error in the deep interior. We estimated the 1σ level of the total error in the sound-speed profile inverted by Vorontsov and Shibahashi (1991) to be five-times larger than the statistical error estimated by them. If we accept this error level, the inverted sound-speed distribution is consistent with the results obtained by different inversion techniques (cf. Dziembowski et al. 1990). The sound-speed profile and the 3σ level error are shown in figure 1.

In the following we discuss our assumption that the error follows a Gaussian distribution, and a subsequently performed Monte-Carlo simulation. We constructed 100 sets of sound-speed profiles with Gaussian noise added with the limiting amplitudes corresponding to the thus-estimated error level; that is, to construct each sound speed profile, we added Gaussian noise to the most likely sound speed at every five meshes, corresponding to a step of $\Delta r/R = 0.1$, and then interpolated the sound speed at the other mesh points smoothly. We then solved equations (1) and (2) with each sound-speed profile, and obtained the most likely profiles of the density and pressure, as well as their error levels. The density profile and pressure profile obtained as solutions of equations (1) and (2) are shown in figures 2 and 3, respectively. The solid curves represent the most likely profiles, and the 3σ error levels are shown by the dashed curves. As the sound speed at the center decreases, the central density and central pressure of the solution tend to increase. Roughly speaking, the density profile and the pressure profile close to the upper (lower) 3σ error levels in figures 2 and 3, respectively, are close to a solution based
on the lower (upper) 3σ error level of the sound-speed profile in figure 1. Although the density and pressure obtained in this way are in fairly good agreement with the standard solar models for $r/R > 0.2$, they are substantially lower than those of the standard solar models for the deeper region. For example, the central density $\rho_c$ of the present seismic model is $112 - 149$ g cm$^{-3}$, while that of Bahcall and Pinsonneault's (1995) model is $156$ g cm$^{-3}$. The central pressure $P_c$ of the present model is $(1.78 - 2.30) \times 10^{17}$ dyn cm$^{-2}$, while Bahcall and Pinsonneault's (1995) model gives $2.40 \times 10^{17}$ dyn cm$^{-2}$.

Once we obtain the density profile, we can calculate the mass distribution $M_r$ and the gravity profile. By combining these and evaluating the squared Brunt-Väisälä frequency,

$$N^2 = -g \left( \frac{d \ln \rho}{dr} - \frac{1}{T_1} \frac{d \ln P}{dr} \right),$$

we can determine whether the solar core is convectively stable ($N^2 > 0$) or not ($N^2 < 0$). The thus-obtained squared Brunt-Väisälä frequency profile is shown in figure 4; we conclude that the solar core is convectively stable, as in the standard solar models. As can be clearly seen in figure 4, the bottom of the convective envelope is at $r/R \approx 0.71$; this is consistent with an earlier investigation (Christensen-Dalsgaard et al. 1991).

5. Determination of the Thermal Structure

We now consider the thermal equations (3) and (4) subject to the given sound-speed distribution $c_{\text{obs}}(r)$. Since the perfect-gas law has been assumed as the equation of state, the squared sound speed is expressed in terms of the temperature and chemical abundance as

$$c_{\text{obs}}^2 = \frac{\Gamma_1 \mathcal{R} T}{\mu},$$

where $\mathcal{R}$ is the gas constant, and $\mu$ denotes the mean molecular weight, which is given by

$$\mu = \frac{1}{2X + \frac{1}{2}Y + \frac{1}{2}Z}.$$ 

In this step, in equations (3) and (4), the pressure $P(r)$ and density $\rho(r)$ can be treated as known functions as a result of the previous section. Also, the opacity and nuclear-reaction rate are treated as known functions of the temperature, density, and chemical composition. Then, the independent quantities appearing in these equations are the temperature $T(r)$, luminosity $L_r$, and either hydrogen abundance $X(r)$, or the heavy element abundance $Z(r)$. Since the number of independent quantities is larger than the number of equations, we must fix a certain functional form of one of these quantities. We assume that $Z$ is constant over the entire region of the solar interior; that is, we assume that the abundance ratios of the various heavy elements in the solar interior are the same as those observed spectroscopically near to the solar surface; that is, the mass ratio of the heavy elements to hydrogen $Z/X$ is on the range of $0.0245 \leq Z/X \leq 0.0277$ (Grevesse 1984; Aller 1986; Grevesse, Noels 1994). Then, the number of unknowns coincides with the number of equations, and we can now
solve these equations for $T(r)$ and $L_r$ and determine the hydrogen abundance profile $X(r)$. Starting from the center and parameterizing the value of the central temperature, $T_c$, we can, in principle, integrate equations (3) and (4) subject to equation (9) and search for the parameter value which satisfies condition (7). It should be remembered that the luminosity and radius should be determined as eigenvalues in solving the equations for the stellar structure. However, in our case, both of them are fixed. Hence, there is not always a solution which satisfies condition (7). If there is no solution satisfying condition (7), this means, as far as the sound-speed profile is supposed to be correct, that either the present sun is in thermal imbalance, or our knowledge about the nuclear-reaction rates is still poor.

To verify that our numerical program to determine the thermal structure works correctly, we solved equations (3) and (4) with the profiles of the sound speed and density of the same stellar model as that used to check our numerical program in order to determine the density and pressure profiles in the previous section. The model was reproduced quite well, and we are confident that the program is correct.

We then solved equations (3) and (4) with the sound-speed distribution shown in figure 1 and the density profile shown in figure 2. Starting from the center and parameterizing the value of the central temperature, $T_c$, we can integrate equations (3) and (4) subject to equation (9) and search for a parameter value which satisfies condition (7). As noted previously, there is not always a solution which satisfies condition (7). Therefore, for practical purposes, we adopted the condition concerning the mean molecular weight $\mu$ as the outer boundary condition. As for $Z/X$, we also tried two cases: $Z/X = 0.0245$ and $0.0277$. To fix $X$ and $Z$, we fixed the helium abundance $Y$ and examined two cases: $Y = 0.23$ and 0.25. The helium abundance $Y$ has been estimated from helioseismology (e.g., Antia, Basu 1994); the values adopted here are consistent with the helioseismologically determined value of $Y$. Furthermore, since we assume that the convection zone is chemically homogeneous, the boundary condition can be set at the base of the convection zone rather than at $r = R_\odot$. The base of the convection zone was deduced from the kink of the sound-speed distribution, and is known to be at $r/R_\odot \approx 0.712$ (Christensen-Dalsgaard et al. 1991, see also section 4 of this paper). Therefore, instead of equation (7), we adopted the value of $\mu$ at $r/R_\odot = 0.712$ as the outer boundary condition:

$$\mu^{-1} = \left[ 2 + \frac{1}{2} (Z/X) \right] \frac{1 - Y}{1 + (Z/X)} + \frac{3}{4} Y$$

at $r/R_\odot = 0.712$. \hspace{1cm} (11)

An advantage of using this boundary condition is that we do not need to treat the convective energy transport, which still involves some theoretical uncertainties. By imposing this condition at $r/R_\odot = 0.712$ and $L_r = 0$ at the center, we solved equations (3) and (4) by the Heneyey method.

As for the opacity, we adopted the OPAL opacity library (Rogers, Iglesias 1992). Since the luminosity is sensitive to the nuclear-reaction rates, we need to carefully check the reaction rates. We adopted here the reaction rates adopted by Bahcall and Pinsonneault (1996), but ignored the CNO reaction. As for the $^3$He distribution, we treated it as the equilibrium distribution in the deep interior, and assumed that the distribution in the outside follows the accumulation of $^3$He due to the $D(p,\gamma)^3$He reaction without destruction (cf. Unno 1975).

Figures 5 and 6 show the temperature distribution and the thus-determined hydrogen-abundance distribution, respectively, corresponding to the sound-speed distribution shown in figure 1 and the density profile shown in figure 2, and $Z/X = 0.0277$ and $Y = 0.23$. As the sound speed at the center decreases, the central temperature of the solution tends to increase. Roughly speaking, the temperature profile close to the upper (lower) $3\sigma$ error levels in figure 5 is close to the solution based on the lower (upper) $3\sigma$ error level of the sound-speed profile in figure 1. On the other hand, as the sound speed at the center decreases, the central hydrogen abundance of the solution tends to decrease.

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is \((1.39-1.56) \times 10^7\) K, while that of Bahcall and Pinsonneault's (1995) model is \(1.58 \times 10^7\) K. The lower temperature implies a lower mean molecular weight, and, in turn, more abundant hydrogen near to the center. Since the hydrogen-abundance distribution is not assumed to be constant outside the core, as in standard evolution theory, but is obtained as a solution of the set of the differential equations, there is no guarantee that \(X\) is constant, even outside of the core. As can be seen in figure 6, even in the seismic model, hydrogen happens to be uniform outside the nuclear-reacting core, except near to the outer boundary. This implies that there seems to be no substantial error concerning opacity. The slight decrease in \(X\) with the depth near from the outer boundary may be interpreted as being a result of chemical diffusion. As shown later, the lower dashed curve indicating the 3σ error level corresponds to a model satisfying equation (7). In this model, hydrogen is slightly more abundant in the center \((X_c = 0.34)\) than in the standard solar model of Bahcall and Pinsonneault (1995) \((X_c = 0.33)\).

The corresponding luminosity profile is shown in figure 7. Equations (3) and (4) satisfy the boundary condition (7) for only some appropriate forms of the opacity \(\kappa\) and the nuclear reaction rate \(\varepsilon\). There is no guarantee that the opacity and nuclear-reaction rate adopted here can satisfy condition (7). Indeed, as can be seen in figure 7, the present combinations of \(Z/X, Y, \kappa, \) and \(\varepsilon\) do not reproduce the present solar luminosity with the density and pressure profiles shown in figures 2 and 3, respectively, at the 1σ level, and the seismic model marginally satisfies the luminosity condition, \(L(R_\odot) = L_\odot\), at the 3σ level.

6. Estimate of the Neutrino Fluxes

The neutrino fluxes at one astronomical unit can be estimated along with a calculation of the nuclear-reaction rates from the thus-determined temperature and chemical-composition distribution. Comparing the thus-estimated neutrino fluxes with the observations, we can judge whether the solar-neutrino problem arises from a deficiency in the solar models. Figure 11 shows the calculated neutrino fluxes for the seismic models for \(Z/X = 0.0277\) and \(Y = 0.23\). In this figure, the neutrino...
fluxes are separately drawn for individual processes generating neutrinos. The solid lines show the most likely fluxes corresponding to the thick curves of figures 1-7. The dashed lines indicate the 3σ error levels. Roughly speaking, the upper and the lower 3σ error levels correspond to the upper and lower 3σ error levels of the luminosity profile shown in figure 7, respectively. Note that the lower 3σ error level of 8B neutrino flux is too low to be distinguished from the zero flux in this diagram. The dotted lines show the fluxes expected from Bahcall and Pinsonneault’s (1995) standard solar model.

It should be noted that the most likely fluxes correspond to a model with \( L(R_\odot) > L_\odot \). The models with \( L(R_\odot) < L_\odot \) naturally generate low neutrino fluxes.

Since our purposes are, however, to construct a solar model which is consistent with various observational data as much as possible and to see if such a model is consistent with the observed neutrino fluxes, these models are less important in our treatment. We should pay attention to a model satisfying equation (7), which corresponds to
Table 1. Capture rates predicted by the seismic solar model for a chlorine detector and for the gallium detectors.

<table>
<thead>
<tr>
<th>Neutrino Source</th>
<th>Cl (SNU)</th>
<th>Ga (SNU)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{7}\text{Be}$</td>
<td>0.23</td>
<td>3.1</td>
</tr>
<tr>
<td>$^{8}\text{Be}$</td>
<td>1.01</td>
<td>30.8</td>
</tr>
<tr>
<td>$^{8}\text{B}$</td>
<td>4.38</td>
<td>9.6</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>5.62</strong></td>
<td><strong>116.5</strong></td>
</tr>
</tbody>
</table>

* We do not take account of contributions from $^{13}\text{N}$ and $^{15}\text{O}$ neutrinos.

The upper 3σ lines.

We now compare first the $^{8}\text{B}$ neutrino flux of this seismic solar model with that of the standard solar model. The $^{8}\text{B}$ neutrino flux is highly sensitive to the temperature near to the solar center. Since the present seismic solar model has a lower temperature there than does the standard solar model, the $^{8}\text{B}$ neutrino flux of the present model is lower than the standard solar model; it is $3.95 \times 10^6 \text{ cm}^{-2}\text{s}^{-1}$, which is about 60% of that of Bahcall and Pinsonneault (1995). Hence, the present seismic solar model does not seem to contradict the Kamiokande experiment concerning the detection of $^{8}\text{B}$ neutrino, which yields the observed $^{8}\text{B}$ neutrino flux as $[3.0 \pm 0.41(\text{stat}) \pm 0.35(\text{syst})] \times 10^6 \text{ cm}^{-2}\text{s}^{-1}$ (Hirata et al. 1991; Suzuki 1993).

Since the energy threshold of the chlorine experiment in the Homestake gold mine is lower than that of the Kamiokande experiment, the chlorine experiment can capture not only $^{8}\text{B}$ neutrinos, but also a part of the $^{7}\text{Be}$ neutrinos. The observations yield $2.28 \pm 0.23$ SNU, about one-third of the prediction of the standard solar models (Davis 1993). The $^{7}\text{Be}$ neutrino flux of the seismic solar model is about 20% less than that of the standard solar models. If we assume that the capture rates of the chlorine experiment are simply proportional to the neutrino fluxes, we can estimate the total neutrino capture rate of the chlorine experiment by scaling the capture rates based on the standard solar model (see table 1). The thus-estimated total capture rate is $5.62$ SNU plus a contribution from the CNO cycle, which is ignored in this paper, but is estimated to be about $0.5$ SNU; there remains a substantial discrepancy between the observations.

The two gallium experiments, GALLEX and SAGE, are expected to capture pp-neutrinos as well as $^{7}\text{Be}$ neutrinos and $^{8}\text{B}$ neutrinos. The observed capture rates, $79 \pm 10(\text{stat}) \pm 6(\text{syst})$ SNU for the GALLEX experiment and $69 \pm 10(\text{stat}) \pm 6(\text{syst})$ SNU for the SAGE experiment, are deficient compared with the theoretical expectation, $137 \pm 6$ SNU (Bahcall, Pinsonneault 1995), and are as little as the expectation value for pp-neutrinos alone (Abdurashitov et al. 1994; Anselmann et al. 1994). The pp-neutrino flux of the seismic solar model is almost the same as that of the standard solar model of Bahcall and Pinsonneault (1995). Assuming that the capture rates of the gallium experiments are also simply proportional to the neutrino fluxes, we estimated the total neutrino capture rate of the gallium experiments expected from the seismic solar model by scaling the capture rates based on the standard solar model (table 1). The thus-estimated total capture rate is about $117$ SNU plus a contribution from the CNO cycle, which is ignored in this paper, but is estimated to be about $10$ SNU. This value is outside the 3σ level of the observations.

Since the seismic solar model was constructed only with experimentally well-determined data for the present sun, we should recognize that the inconsistency between the neutrino fluxes of the seismic solar model and the observations is serious. The expected neutrino fluxes do not contradict the Kamiokande experiment, but are discrepant with the other experiments concerning the solar neutrino detection.

7. Discussion

We have demonstrated the possibility of constructing a solar model based on the sound-speed distribution deduced from helioseismic data. It should be emphasized that the present approach uses only the sound-speed distribution as a direct helioseismic output. Various inversion techniques to obtain the sound-speed distribution from the solar p-mode frequencies have so far been proposed. The inversion method, based on the variational principle, provides not only the sound speed, but also some additional information, such as the density profile (e.g., Gough, Thompson 1990; Gough, Kosovichev 1993; Antia, Basu 1994; Basu, Thompson 1996; Christensen-Dalsgaard et al. 1991; Dziembowski et al. 1990, 1994, 1995; Kosovichev 1995; Thompson 1991). An advantage of the present method for obtaining the density profile is that a reference model is not needed, and anyone can construct seismic solar models once the inverted sound-speed distribution is obtained (a file containing the inverted sound speed profile shown in figure 1 is available on request through e-mail to the address in the title page), while method based on the variational principle requires a good reference model and can be performed only by experts in inversion.

It should be noted that the present attempt is the first step to a full inversion. We assumed that the equation of state is that of an ideal gas, and treated $\Gamma_1 = 5/3$. Once we obtain the thermal structure, we can check the accuracy of this approximation. We can then repeat the procedure to obtain more consistent solutions. By iter-
The present combinations of $Z/X$, $Y$, $\kappa$, $\varepsilon$, and the sound-speed profile did not reproduce the present solar luminosity at the 1σ level. As described in section 5, this implies that: (i) the adopted nuclear reaction rate is inappropriate and/or (ii) the sun is substantially far from the thermal equilibrium and/or (iii) the sound-speed profile that we adopted in the present paper involves larger errors than we have estimated. We now discuss possibility (iii) among them. We used the sound-speed profile inverted by Vorontsov and Shibahashi (1991) from the frequency data compiled by Libbrecht et al. (1990) as a given set. However, we should perform an inversion to obtain the sound-speed profile, itself, as a part of the work to reconstruct a seismic solar model. By doing so, we will be able to estimate the error level more precisely. There are also excellent data sets other than those of Libbrecht et al.'s (1990); some of them have been obtained from ground-based networks or from satellites or by continuous observations from the terrestrial south pole. As can be seen in figure 1, the inverted sound speed in the deep interior ($r/R > 0.2$) is less accurate than in the outer region ($r/R > 0.2$). This is because the number of p-modes penetrating into the deep interior is small. The information concerning the deep interior is extracted mainly from the low-degree modes. A precise measurement of these modes and the use of good inversion techniques are important to seismologically determine the solar internal structure. It has been pointed out that different data sets and different treatments would give conflicting results about the core (Gough et al. 1995). Hence, we should examine various excellent data and various techniques to reach a more definite conclusion. The same frequency data as that which Vorontsov and Shibahashi (1991) used have been inverted by Dziembowski et al. (1994) and Antia and Basu (1994) to determine the hydrostatic structure of the sun. Although the density and pressure profiles determined by Dziembowski et al. (1994) and Antia and Basu (1994) are in good agreement with the present results for $r/R \geq 0.2$, the agreement is not complete for the central region. The central density $\rho_c$ and the central pressure $P_c$ of Dziembowski et al. (1994) are $141-147$ g cm$^{-3}$ and $(2.26-2.28) \times 10^{17}$ dyn cm$^{-2}$, respectively. Although these values are within the 3σ levels of our results, they are substantially different from the most likely values. This difference must arise due to the difference in the inversion techniques; we think that we should even further improve the inversion techniques.

It should also be remembered that the most effective information for a seismic investigation of the solar core is the eigenfrequencies of the g-modes, rather than the p-modes. Since the g-modes oscillate even near to the solar center, their frequencies reflect the structure of the solar core. If the detection of the solar g-modes becomes well established, the Brunt-Väisälä frequency profile can be obtained from an inversion of their frequencies. Then, with the constraint of the thus-determined Brunt-Väisälä frequency profile, we will be able to more reliably determine the solar internal structure and to determine whether our current knowledge about the nuclear-reaction rates is sufficiently good or if the sun is in thermal equilibrium.

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Fig. 1. Photograph of the GIS-S2 detector. The length is 58 cm, and the diameter of the main cylindrical section is 14 cm.

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Fig. 1. (a) X-ray image of the observed field in the energy range 2–10 keV (a mosaic of X-ray images of eight adjacent fields).  
(b) X-ray image of the central ~ 20' x 20' field of the galactic center in the energy range 2–10 keV, overlaid with VLA radio contours at 20 cm (Yusef-Zadeh et al. 1984). A striking correlation between the structures in X-rays and in the radio continuum is noted.

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Fig. 3. (b) The 6.4 keV line image near to the Sgr B2 cloud, obtained by a follow-up long-exposure observation, is overlaid with the CH$_3$CN line contours (Bally et al. 1988). The 6.4-keV brightness distribution is systematically shifted from the radio distribution by $\sim 2'$ to the galactic-center side (to the right in the figure).

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Fig. 3b. Simulation results of the typical model (model 1) for the horizontal-coronal-field case. In all of the panels, the solid lines are the magnetic field lines, and the arrows are flows, with the arrow length indicating the velocity at the foot of each arrow. The scale length of the velocity vectors is given by the arrow at the top-right corner of each panel, whose size is $V = 5.0$ in units of the sound speed in the cool layer $C_{s,pho}$. The spacing between the field line is 0.25 of the magnetic flux. The intensity levels are shown by the color bars. Panel (a) shows the initial evolution of the magnetic field lines. Panels (b) and (c) show the time evolution of (b) the temperature $T$, and (c) the density $\log_{10} \rho$. Panel (d) shows the current-density $J_y$ distribution at $t = 97.6$ near to the central loops ($35 \leq x \leq 115$, $-5 \leq z \leq 50$). The thick dashed curves show the slow-mode MHD shock fronts, and the thick solid curves denote the fast-mode MHD shock fronts. (See the main text for figures 3a, c, and d.)

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Fig. 9a. Simulation results of the typical model (model 7) for the oblique-coronal-field case, $\theta_{\text{cor}} = 3\pi/4$. The figure shows the region near to the central loops (lower half of the computation box), whose range is $0 \leq x \leq 80$ and $-5 \leq z \leq 50$. Panels (a) and (b) show the time evolution of the temperature $T$, and the density $\log_{10} \rho$, respectively. Panel (c) shows the current density $J_y$ distribution at $t = 105.0$. The remaining notation is the same as in figure 3. (See the main text for figures 9b and c.)