Radiative Collimation of Electron–Positron Jets II

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Abstract

We consider a collimation and acceleration mechanism for relativistic jets emanating from an inner region of a relativistic accretion disk surrounding a Kerr black hole, with Keplerian angular momentum under disk radiation fields, taking into account the effect of radiation drag. When the jets consist of electron–positron pair plasmas and the disk luminosity is of the order of the Eddington luminosity, we found that the terminal speed is about 0.9c for a Schwarzschild case, while it becomes \( \sim 0.96c \) for an extreme Kerr case. In such a case, the opening angle of jets is of the order of 5°. We also examined the opening angle and the listing degree when the disk radiation is not axisymmetric, but has a one-armed pattern. For the case of 0.3 amplitude, the opening angle is of the order of a few degrees, and the jet direction is bent by several degrees to 10°.

Key words: accretion, accretion disks — accretion-disk winds — astrophysical jets — galaxies: active — X-ray: stars

1. Introduction

Radiative acceleration of astrophysical jets from luminous disks has been extensively investigated by many researchers: as disk winds (Bisnovatyi-Kogan, Blinnikov 1977; Katz 1980; Icke 1980; Melia, Königl 1989; Misra, Melia 1993; Tajima, Fukue 1996, 1998; Watarai, Fukue 1999), as on-axis jets (Icke 1989; Sikora et al. 1996), as outflows confined by a gaseous torus (Lynden-Bell 1978; Davidson, McCray 1980; Sikora, Wilson 1981; Fukue 1982), or as jets confined by the outer flow or corona (Sol et al. 1989; Marcowith et al. 1995; Fukue 1999), and as numerical simulations (Eggum et al. 1985, 1988).

In these mechanisms, no one can succeed to produce ultra-relativistic jets with \( \sim 0.9c \), or well-collimated jets of the order of a few degrees for realistic jet flows, which emanate from the disk “surface” with “angular momentum”, and are accelerated under radiation fields produced by the standard accretion disk model.

For example, Melia and Königl (1989) performed detailed calculations of the spectra from ultrarelativistic jets, considering the Compton-drag deceleration of initially hyper-relativistic jets. They found that terminal Lorentz factors of about 10 are readily produced in their model for plausible parameters. In their model, however, the jet flow, itself, was one-dimensional along the rotational axis, and the angular momentum was not included. In addition, they ignored the gravitational effects of both relativistic and Newtonian gravities. Finally, they assumed that the jet is launched with very high initial Lorentz factor of about 1000, and did not consider the “acceleration”, but “deceleration”. In Misra and Melia (1993), they applied their earlier model of jets to the galactic center object 1E 1740.7–2942. They examined electron–positron pair flows under radiation fields of the disk, and showed several trajectories as well as velocity fields along the flow. However, they failed to obtain ultrarelativistic jets of \( \sim 0.9c \), but the terminal velocity of their jets is \( \sim 0.7c \). Furthermore, collimation of their flow is about 10°. Further, Donea and Biermann (1996) considered disk and jets surrounding a Kerr black hole. They assumed that the jet starts between a marginally stable orbit and some larger radius, on a relativistic standard accretion disk. They examined the disk structure, modified by the existence of jets, which extract mass, energy, and angular momentum from the disk. The dynamics of jet flows, however, was beyond the scope of their paper.

Thus, as far as we know, no one has shown such a result that collimation occurs to a few degrees and terminal velocities of \( \sim 0.9c \) exist.

In a previous paper (Fukue et al. 2001), hence, we proposed a mechanism for the collimation of radiatively driven jets, which is effectively applicable to electron–positron jets ejected from an inner hot region surrounded by an outer luminous disk.

The aim of this paper is to extend the previous study, where a Schwarzschild black hole is supposed and axisymmetry is assumed, taking the Kerr hole into considerations and examining the non-axisymmetry.

In the next section we describe the situations and the gravitational and radiation fields. In section 3 we present basic equations and the flow properties; in particular, our attention is focused on the terminal speed. The final section is devoted to concluding remarks. The results for non-axisymmetric properties are given in the appendix.

2. Situations and Model

We consider a black-hole accretion-disk system (figure 1). The black hole is assumed to be a Kerr black hole with mass \( M \), surrounded by a geometrically thin, optically thick accretion disk. In addition, it is supposed that electron–positron pair plasmas are produced in the inner hot region of the disk (cf. Kusunose 1991; Yamasaki et al. 1999). If so, the electron–positron gas is initially pushed outward by the thermal pressure, and is then greatly influenced by the
radiation fields of the luminous disk. Moreover, realistic jet flows have angular momentum, which is reasonably equal to the Keplerian values at the footpoint of jets. In other words, we do not consider on-axis jets, but jets emanating from the disk “surface” with “angular momentum”, and accelerated under radiation fields produced by a relativistic disk.

2.1. Gravitational Field

In order to mimic the general relativistic effect of the central Kerr black hole, we adopt the pseudo-Newtonian force (Artemova et al. 1996; cf. Paczyński, Wiita 1980), which can well describe the dynamical properties of general-relativistic effects around a rotating black hole. The force for free-fall acceleration is given by

$$F = \frac{GM}{R^{2-\beta}(R-r_H)^{\beta}},$$

where the parameter $\beta$ is

$$\beta = \frac{r_{in}}{r_H} - 1.$$  

(1)

Here and hereafter, $r_g$ is the Schwarzschild radius of the central object, $r_H$ the radius of the event horizon, and $r_{in}$ the inner radius of the disk, which is equal to the radius of a marginally stable orbit. These are all determined by exact expressions for general relativity, as follows:

$$r_g = \frac{2GM}{c^2},$$

$$r_H = \frac{r_g}{2}(1 + \sqrt{1-a^2}),$$

$$r_{in} = \frac{r_g}{2}\left[3 + Z_2 - \sqrt{(3-Z_1)(3+Z_1+2Z_2)}\right],$$

$$Z_1 = 1 + (1-a^2)^{\frac{3}{2}}[1+(1+a)^{\frac{3}{2}} + (1-a)^{\frac{3}{2}}],$$

where $Z_2 = \sqrt{3a^2 + Z_1^2}$.  

(2)

(3)

(4)

(5)

(6)

2.2. Disk Model

Before calculating the trajectories of jets, we should calculate the surface temperature $T_d$ of the disk for the present case (Hirai, Fukue 2001). As a luminous disk, we adopt the standard picture, where viscous heating is balanced by radiative cooling (Shakura, Sunyaev 1973; Kato et al. 1998 for a review). Using the pseudo-Newtonian force mentioned above, the surface temperature of the accretion disks is expressed as a function of $r_d$ on the disk plane by

$$2\sigma T_d^4 = \frac{3M}{4\pi} \Omega_K^2 hf,$$

where $\sigma$ is the Stephan–Boltzmann constant and $M$ the constant accretion rate in the disk. The relativistic Keplerian angular speed $\Omega_K$, the relativistic correction factor $h$, and the correction factor due to the boundary condition $f$ are given on the disk plane, respectively, as

$$\Omega_K^2 = \frac{GM}{r^{1-\beta}(r-r_H)^{\beta}},$$

$$h = \frac{r - (3-\beta)r_H/3}{r-r_H},$$

$$f = 1 - \frac{\ell_{in}}{\ell_K} = 1 - \sqrt{\left(\frac{r_{in}}{r}\right)^{1+\beta}\left(\frac{r-r_H}{r_{in}-r_H}\right)^{\beta}},$$

(9)

(10)

(11)

where $\ell_K = r_d^2\Omega_K$ and $\ell_{in} = \ell_K(r_{in})$. 

Fig. 1. Schematic view of a disk–jet system. The jet gas emanates from an inner hot region and is accelerated by disk radiation fields. Various forces work on jet particles: gravity of the central object, the centrifugal force, the radiation pressure force, and the radiation drag force. All components are incorporated under the pseudo-Newtonian treatment and within the special relativity.

Fig. 2. Several quantities in the Kerr black hole case as a function of a spin parameter $a$. The inner radius $r_{in}$, the radius of the marginally stable orbit, is shown by a dashed curve. The radius $r_0$ at the maximum temperature of the disk is shown by a solid curve. The Keplerian rotation speed $v_0$ at $r_0$, which is assumed to be the initial ejection velocity, is shown by a thin solid curve. The radius and velocity are measured in units of $r_g$ and $c$, respectively.

$$Z_2 = \sqrt{3a^2 + Z_1^2}.$$

Figure 2 shows the inner radius $r_{in}$ of the disk as a function of the spin parameter $a$. The inner radius $r_{in}$ varies from $3r_g$ to $0.5r_g$ with $a$. 

![Diagram of disk–jet system](https://example.com/diagram.png)
Assuming that the disk surface is a blackbody radiator, the specific intensity \( I_0 \) of the disk radiation in the comoving frame with the disk is given by

\[
I_0 = \frac{1}{\pi} \sigma T_d^4 = \frac{1}{\pi} \frac{3GMM}{8\pi r_d^3} g f,
\]

where

\[
g = \frac{r^\beta}{(r - r_H)^\beta} \left( \frac{r - (3 - \beta)r_H/3}{r - r_H} \right).
\]

Because of the boundary condition, the surface temperature \( T_d(r_0) \) has a maximum at some radius \( r_0 \). This radius of the maximum temperature (and intensity) shifts inward with a spin parameter \( a \). In figure 2 the radius \( r_0 \) of the maximum temperature as well as the Keplerian rotation velocity \( v_0 \) there are shown as a function of \( a \).

2.3. Disk Radiation Fields

Using the surface intensity of the disk (12), we can calculate the components of radiation fields at point \( P \) over the disk (cf. Tajima, Fukue 1998; Watarai, Fukue 1999; Hirai, Fukue 2001; Fukue et al. 2001). According to the definition (e.g., Kato et al. 1998, appendix), the radiation energy density \( E \), the radiative flux \( F^\alpha \), and the pressure stress tensor \( P^{\alpha \beta} \) are calculated, respectively, as

\[
E = \frac{1}{c} \int \frac{I_0}{(1 + z_{\text{red}})^2} d\Omega
\]

\[
= \frac{1}{c} \int \frac{3GMM}{8\pi r_d^3} g f \frac{1}{(1 + z_{\text{red}})^2} d\Omega,
\]

\[
F^\alpha = \frac{1}{c} \int \frac{I_0}{(1 + z_{\text{red}})^2} l^\alpha d\Omega
\]

\[
= \frac{1}{c} \int \frac{3GMM}{8\pi r_d^3} g f \frac{1}{(1 + z_{\text{red}})^2} l^\alpha d\Omega,
\]

\[
P^{\alpha \beta} = \frac{1}{c} \int \frac{I_0}{(1 + z_{\text{red}})^2} l^\alpha l^\beta d\Omega
\]

\[
= \frac{1}{c} \int \frac{3GMM}{8\pi r_d^3} g f \frac{1}{(1 + z_{\text{red}})^2} l^\alpha l^\beta d\Omega.
\]

Here, \( l^\alpha \) is the direction cosine between point \( P \) over the disk and a small surface element \( Q \) on the disk, \( z_{\text{red}} \) the redshift factor associated with the motion of the disk element \( Q \) relative to point \( P \), and \( d\Omega \) the solid angle subtended by element \( Q \) \((d\Omega = z_{\text{red}} d\Omega_{d\varphi} d\varphi_d)/D^3 \), \( D \) being the distance between element \( Q \) and point \( P \). In these expressions the relativistic correction factors \( f \) and \( g \), and the redshift factor \( z_{\text{red}} \) is fully evaluated up to the order of \((\nu/c)^2\).

In order to normalize the expressions, in several works (Tajima, Fukue 1998; Fukue et al. 2001) we use the disk luminosity \( L_d \) normalized by the Eddington luminosity \( L_E \). Since the disk luminosity depends on the inner radius and changes as a function of the spin parameter \( a \), we use the accretion rate \( M \) as a parameter in the text.

Using the critical accretion rate (cf. Kato et al. 1998), the “normalized” accretion rate \( \dot{m} \) is defined by

\[
\dot{m} = \frac{M c^2}{L_E} = \frac{L_d}{\eta L_E},
\]

where \( \eta \) \((= 1/12)\) is the efficiency. In the Schwarzschild case, for normal plasmas, this normalized accretion rate \( \dot{m} \) becomes 12 when the disk luminosity is equal to the Eddington one, or the mass accretion rate \( M \) is the critical one. Then, equations (14)–(16) are rearranged as

\[
E = \frac{m_p}{\sigma_T} \frac{3GMM}{4r_g^2} \dot{m} \varepsilon,
\]

\[
F^\alpha = \frac{m_p c}{\sigma_T} \frac{3GMM}{4r_g^2} \dot{m} f^\alpha,
\]

\[
p^{\alpha \beta} = \frac{m_p}{\sigma_T} \frac{3GMM}{4r_g^2} \dot{m} p^{\alpha \beta},
\]

where \( m_p \) is the proton mass, and \( \varepsilon \), \( f^\alpha \), and \( p^{\alpha \beta} \) are the components of radiation fields in the following nondimensional form:

\[
E = \frac{1}{\pi} \int \frac{g f}{r_d^3} (1 + z_{\text{red}})^4 \frac{z_{\text{red}} d\Omega_{d\varphi} d\varphi_d}{D^3},
\]

\[
f^\alpha = \frac{1}{\pi} \int \frac{g f}{r_d^3} (1 + z_{\text{red}})^4 z_{\text{red}} d\Omega_{d\varphi} d\varphi_d,
\]

\[
p^{\alpha \beta} = \frac{1}{\pi} \int \frac{g f}{r_d^3} (1 + z_{\text{red}})^4 z_{\text{red}} d\Omega_{d\varphi} d\varphi_d.
\]

where the radii are measured in units of \( r_g \).

Using equations (21)–(23), we numerically calculate the disk radiation fields spreading above and below the disk. At point \( P \) the value of any component of the disk radiation fields are obtained by simply summing up the contribution from all small elements on the disk surface over the entire disk (see also Hirai, Fukue 2001; Fukue et al. 2001).

3. Radiatively Driven Jets

Let us now examine jet flows ejected from the inner hot disk under radiation fields produced by the luminous disk. After describing the basic equations for jet flows, we first show typical velocity fields, and then summarize the terminal speed of jets.

The equations of motion for jet particles under the influence of radiation fields are generally written as (Kato et al. 1998 and references cited therein)

\[
\frac{d u^\alpha}{d \tau} = -\frac{\partial \Phi}{\partial x^\alpha} + \frac{\sigma_T}{mc} \left[ \gamma F^\alpha - c \gamma^2 E u^\alpha - c P^{\alpha \beta} u_\beta \right]
\]

\[
+ cu^\alpha \left( 2 \gamma F^\beta u^\beta / c - p^{\alpha \beta} u_\beta u_\gamma \right),
\]

where \( u^\alpha \) \((= \gamma u^\alpha)\) is the four velocity, \( \tau \) the proper time, \( \gamma \) the Lorentz factor, \( \Phi \) the gravitational potential, \( \sigma_T \) the Thomson-scattering cross section, and \( m \) the particle mass; \( E \), \( F^\alpha \), and \( P^{\alpha \beta} \) are the energy density, flux, and pressure stress tensor of the disk radiation fields, respectively.

Recalling the dimensionless quantities introduced above, and measuring the radius and velocity in units of \( r_g \) and \( c \), respectively, equation (24) is rewritten explicitly as follows:

\[
\frac{d u^\alpha}{d \tau} = -\frac{\partial \Phi}{\partial x^\alpha} + \frac{\ell^2}{r^3} + \frac{3 m_p}{8 m} \left[ \gamma f^\alpha - \gamma^2 E u^\alpha - p^{\alpha \beta} u_\beta \right]
\]

\[
+ u^\alpha \left( 2 \gamma f^\beta u^\beta - p^{\alpha \beta} u_\beta u_\gamma \right). \tag{25}
\]
In this paper we have examined astrophysical jets emanating from an inner hot region of a relativistic accretion disk around a Kerr black hole. The jets are collimated and accelerated by the radiation fields of the accretion disk.

In the Kerr black hole case, as the spin parameter $a$ increases, the inner radius decreases and the initial velocity becomes large. In addition, the disk temperature increases with $a$. Consequently, the gas particle is further accelerated and the terminal velocity also becomes high. Moreover, as the accretion rate (and luminosity) increases, the terminal velocity
Fig. 4. Terminal velocity \( v_\infty \) as a function of a spin parameter \( a \) for several values of \((m_p/m_e)\dot{m}\).

Fig. 5. Schematic view of a disk–jet system under a non-axisymmetric disk-radiation pattern. The listing degree \( \phi \) of jets is measured from the rotation axis of the disk to the central axis of jets, while the opening angle \( \theta \) from the central axis to the outside trajectory of jets.

Fig. 6. Listing degree \( \phi \) as a function of the normalized luminosity \((m_p/m_e)\Gamma_d\) for several amplitude \( A \).

becomes high. When the jets consist of electron–positron pair plasmas and the disk luminosity is of the order of the Eddington one, the terminal velocity is about \( 0.9c \) for a Schwarzschild case, while it becomes \( \sim 0.96c \) for an extreme Kerr case.

We have been investigating electron–positron pair jets, relating to accretion disk winds, for several years (e.g., Tajima, Fukue 1998; Fukue et al. 2001). It has been found that well collimated (opening angle is a few degrees) and subrelativistic \((0.9c-0.96c)\) jets can form. However, under the assumptions of uniform, continuous, and steady flows, it is very difficult to produce ultrarelativistic jets of Lorentz factor 10, because of the radiation drag and angular momentum. In order to break this radiation-drag limit, it should be necessary to introduce alternative approaches, including unsteadiness and inhomogeneity (Fukue 2000, 2003).

We also examine the effect of the non-axisymmetric disk luminosity on the jets. For the case of 0.3 amplitude, the opening angle is of the order of a few degrees, and the jet direction is bent by several degrees to \( 10^\circ \).

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Appendix. Non-Axisymmetric Case

In this appendix we briefly discuss the case where disk radiation fields are not axisymmetric but have a non-axisymmetric pattern. In general the luminosity distribution on the disk could be inhomogeneous, e.g., patchy. As a result, the axisymmetry of jet flows is lost.

In the present analysis, for simplicity, a central black hole is assumed to be a Schwarzschild one. As a non-axisymmetric disk radiation field, we assume a one-armed pattern: \( I_0 \propto (1 + A\cos \phi_d) \), where \( A \) is the amplitude of a non-axisymmetric pattern. The jet flow affected by such a non-axisymmetric field leans by some listing angle \( \phi \) with an opening angle \( \theta \) (figure 5). We calculated the trajectories of particles from an inner disk for several values of the normalized disk luminosity \( \Gamma_d (= \dot{m}/12) \) and the amplitude \( A \), and estimated the opening angle \( \theta \) and the listing degree \( \phi \) of jets.

Figure 6 shows the listing degree \( \phi \) as a function of the normalized luminosity \( \Gamma_d \) for several amplitudes \( A \). The listing degree remarkably depends on the amplitude, and increases with the amplitude. It also increases with the luminosity. When the amplitude is about 0.3 and the jet gas consists of electron–positron pair plasmas \([ (m_p/m_e)\Gamma_d \sim 1000] \), the listing degree...
is of the order of $8^\circ$.

Figure 7 shows the opening angles $\theta$ as a function of the normalized luminosity $\Gamma_d$ for several amplitudes $A$. As can be seen in figure 7, the opening angle depends mainly on the luminosity and is almost independent of the amplitude. The jets are collimated as the luminosity increases. When the jet gas consists of electron–positron pair plasmas \((m_p/m_e)\Gamma_d \sim 1000\), the opening angle is of the order of $2^\circ$ for a sufficiently larger luminosity.

In this appendix we have proposed a mechanism for the precession of astrophysical jets due to the nonaxisymmetry of the radiation field of the accretion disk. The magnitude of collimation and the opening angle strongly depend on the luminosity, and the jets emanated from the inner disk are well collimated when \((m_p/m_e)\Gamma_d \sim 100$–$1000$. The listing degree depends on the luminosity and amplitude. Astrophysical jets list due to an inhomogeneity of the disk radiation field as the luminosity increases. On the other hand, the terminal speed of jets does not depend on the amplitude, and is mainly determined by the luminosity. When the jets consist of electron–positron pair plasmas and the disk luminosity is of the order of the Eddington one \[i.e., (m_p/m_e)\Gamma_d \sim 1000\], the terminal speed of the present well-collimated jets becomes \(\sim 0.9c\), as shown in the text and in Fukue et al. (2001).

Fig. 7. Opening angle $\theta$ as a function of the normalized luminosity \((m_p/m_e)\Gamma_d\) for several amplitude $A$.

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