
The star known as 61 Geminorum was entered in the Bedford Catalogue as double, 7\text{\textfrac{1}{2}} and 9 magnitudes, with a position-angle of 110°, and a distance of 60", the colour being recorded as deep yellow, and yellowish.

I examined it with a 3\text{\textfrac{7}{8}}-inch achromatic, March 23, 1852, and entered it "white, single." There could be little doubt as to its identification, from the neighbourhood of the double star \text{H} III. 48. February 14, 1855, having taken pains as to its identification, I again found it, with the same instrument, single and white. I noticed, however, on this latter occasion an exceedingly minute star, not above 11 mag., which might agree as to distance, but with a very roughly estimated angle of 185° or 190°.

In 1861 and 1871 the comet was invisible to Mr. Knott, with all the advantage of a 7\text{\textfrac{1}{2}}-inch Alvan Clark object-glass. I believe I have never looked for it since, but have been lately interested by finding that it has been recovered this spring by Herbert Sadler, Esq., of Honiton Rectory, the extreme acuteness of whose vision, in the use of a 6\text{\textfrac{1}{2}}-inch silvered mirror by Calver, is attested by his recognition of several most delicate and difficult objects. He gives it only 12.5 mag. at about the right distance, but with an estimated angle of 160° to 165°.

It seems, therefore, probable that, unless we can suppose an error in the figure expressing Smyth's magnitude, we have here a variable star, which it would be desirable to examine closely, and with instruments capable of giving a definite value to the angle of position. The possible change of colour also in the principal star merits attention. My own 9\text{\textfrac{1}{8}}-inch speculum being at the present time dismounted, with a view to the ultimate perfection of its figure, though previously very good, I am unable to contribute any information on the subject.

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Note on the Theory of Precession and Nutation.

By Prof. Cayley.

We have in the dynamical theory of Precession and Nutation (see Bessel's Fundamenta (1818), p. 126),

\[ C \frac{dp}{dt} + (B-A) qr = L.S \left( \frac{x' y' - x y'}{\Delta s} \right) dm' \left( \frac{1}{\Delta^2} - \frac{1}{r^2} \right), \]

\[ A \frac{dq}{dt} + (C-B) rp = L.S \left( \frac{y' z' - y z'}{\Delta s} \right) dm' \left( \frac{1}{\Delta^2} - \frac{1}{r^2} \right), \]

\[ B \frac{dr}{dt} + (A-C) pq = L.S \left( \frac{x' z' - x z'}{\Delta s} \right) dm' \left( \frac{1}{\Delta^2} - \frac{1}{r^2} \right). \]
where $L$ is the mass of the Sun or Moon, $x, y, z$ the coordinates of its centre referred to the centre of the Earth as origin,

$$r = \sqrt{x^2 + y^2 + z^2},$$

the distance of its centre, and

$$\Delta = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2},$$

the distance of its centre from an element $dm'$, coordinates $(x', y', z')$ of the Earth's mass, the sum or integral $S$ being extended to the whole mass of the Earth [I have written $dm'$, $r$ for Bessel's $d\varpi, r_1$], we have

$$\Delta^2 = r^2 - 2 (xx' + yy' + zz') + x^2 + y^2 + z^2;$$

and thence

$$\frac{1}{\Delta^3} - \frac{1}{r^3} = \frac{3}{r^3} (xx' + yy' + zz')$$

$$- \frac{3}{2} \frac{1}{r^3} \left\{(xx^2 + yy^2 + zz^2)(xx'^2 + yy'^2 + zz'^2) - 5(xx' + yy' + zz')^2\right\}$$

$$+ \text{etc.}$$

The principal term is the first one,

$$\frac{3}{r^3} (xx' + yy' + zz');$$

but Bessel takes account also of the second term,

$$- \frac{3}{2} \frac{1}{r^3} \left\{(xx^2 + yy^2 + zz^2)(xx'^2 + yy'^2 + zz'^2) - 5(xx' + yy' + zz')^2\right\},$$

viz. considering the Earth as a solid of revolution (as to density as well as exterior form), he obtains in regard to it the following terms of $\sin \omega \frac{d\omega}{dt}$ and $\frac{d\omega}{dt}$ respectively;

$$\frac{3L}{4r^4} \frac{1}{C_n} \cdot 2 (C - \Lambda) K (5 \sin^2 \delta - 1) \cos \delta \sin a,$$

$$- \frac{3L}{4r^4} \frac{1}{C_n} \cdot 2 (C - \Lambda) K (5 \sin^2 \delta - 1) \cos \delta \cos a.$$
where

$$2 (C - A) K = S (3\mu - 5\mu^3) 2\pi q R^3 d R d\mu,$$

K being in fact a numerical quantity, relating to the Earth only, and the value of which is by pendulum observations ultimately found to be = 0.13603.

Writing, for shortness,

$$(x^2 + y^2 + z^2) (x'^2 + y'^2 + z'^2) - 5 (x x' + y y' + z z')^2 = \Omega,$$

then the foregoing terms of \( \omega \frac{d\psi}{dt} \) and \( \frac{d\omega}{dt} \) depend as regards
their form on the theorem, that for any solid of revolution (about the axis of z) we have

$$S (x'y - xy') \Omega d m', \quad S (y'z - yz') \Omega d m', \quad S (z'x - xz') \Omega d m'$$

$$= \alpha,$$

$$- \frac{1}{2} \int \frac{y (x^2 + y^2 + z^2 - 5 x')}{S [3 (x'^2 + y'^2 + z'^2) - 5 z''] x' d m'},$$

$$- \frac{1}{2} \int \frac{x (x^2 + y^2 + z^2 - 5 y')}{S [3 (x'^2 + y'^2 + z'^2) - 5 z''] x' d m'},$$

respectively: viz. writing \( x'^2 + y'^2 + z'^2 = R^2 \), and \( z' = R \mu \), also \( x^2 + y^2 + z^2 = r^2 \) and \( a = r \cos \delta \cos \alpha, \quad y = r \cos \delta \sin \alpha, \quad z = r \sin \delta \), the values would be

$$\alpha,$$

$$+ \frac{1}{2} \int \frac{x^2 \cos \delta \sin \alpha (1 - 5 \sin^2 \delta)}{S (3 - 5 \mu^2) \mu R^3 d m'},$$

$$- \frac{1}{2} \int \frac{y^2 \cos \delta \sin \alpha (1 - 5 \sin^2 \delta)}{S (3 - 5 \mu^2) \mu R^3 d m'},$$

which are of the form in question.

The verification is easy: the solid being one of revolution about the axis of z, any integral such as \( S x'z' d m' \) or \( S x'y' z' d m' \) which contains an odd power of \( x' \) or of \( y' \) is = \( \alpha \); while such integrals as \( S x'^2 z' d m' \), \( S y'^2 z' d m' \) are equal to each other, or, what is the same thing, each = \( \frac{1}{2} S (x'^2 + y'^2) z' d m' \). That we have \( S (x'y - xy') \Omega d m' = \alpha \) is at once seen to be true; considering the next integral \( S (y'z - yz') \Omega d m' \), the terms of \( (y'z - yz') \Omega \) which lead to non-evanescent integrals are .

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\[-y \delta' \left( x^2 + y^2 + z^2 \right) \left( x'^2 + y'^2 + z'^2 \right),\]
\[-5 y' \delta \cdot 2 y = y' \delta',\]
\[+5 y \delta' \left( x^2 x'^2 + y^2 y'^2 + z^2 z'^2 \right);\]

giving in the integral the several terms.

\[-y \left( x^2 + y^2 + z^2 \right) S \left( x'^2 + y'^2 + z'^2 \right) \delta' \, d m',\]
\[-10 y z^2 \cdot \frac{1}{2} S \left( x'^2 + y'^2 + z'^2 - z'^2 \right) \, d m',\]
\[+5 y \left( x^2 + y^2 + z^2 - z^2 \right) \cdot \frac{1}{2} S \left( x'^2 + y'^2 + z'^2 - z'^2 \right) \delta' \, d m'\]
\[+ y z^2 S z'^2 \, d m,\]

viz., collecting, the value is

\[\left( -1 + \frac{5}{2} = \frac{3}{2} \right) \left( x^2 + y^2 + z^2 \right) y S \left( x'^2 + y'^2 + z'^2 \right) \delta' \, d m',\]
\[\left( -\frac{5}{2} = -\frac{5}{2} \right) \left( x^2 + y^2 + z^2 \right) y \underline{z} z'^2 \, d m',\]
\[\left( -\frac{5}{2} - 5 = \frac{15}{2} \right) y z^2 S \left( x'^2 + y'^2 + z'^2 \right) \delta' \, d m',\]
\[\left( +\frac{5}{2} + 5 + 5 = \frac{25}{2} \right) y z^2 S z'^2 \, d m';\]

which is

\[= \frac{1}{2} y \left( x^2 + y^2 + z^2 - 5 z^2 \right) S \left[ 3 \left( x'^2 + y'^2 + z'^2 \right) - 5 z'^2 \right] \delta' \, d m';\]

and similarly the last term is

\[-\frac{1}{2} \left( x^2 + y^2 + z^2 - 5 z^2 \right) S \left[ 3 \left( x'^2 + y'^2 + z'^2 \right) - 5 z'^2 \right] \delta' \, d m',\]

which completes the proof.