Compressive Effects on Globular Clusters by Gravitational Disk-Shocking

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Projected images of the Galactic globular clusters are not necessarily circular but elliptical in many cases. To study the possibility that the gravitational disk-shocking is the principal cause of the observed figures, we carry out a series of numerical experiments of disk-shocking on model clusters. We measure the axial ratios of the projected density contours of the model clusters, and find that the disk-shocking does not produce the values of axial ratios obtained by observations in the literature.

§ 1. Introduction

Elliptic features of projected images of the Galactic globular clusters (GGC) were noted already in the time of Shapley, and discussion is given in his book. Since that time, observations of projected ellipticities were made by many authors, however, the sample clusters were not particularly abundant. Recently, Frenk and Fall measured the projected ellipticities of 93 GGC and 52 LMC clusters, and White and Shawl provided the newest data of 100 GGC's axial ratios of projected figures. These authors reported the values of the axial ratios from approximately 0.8 to 1.0, with a mean value ~0.9.

There are three possible mechanisms for the ellipticity: 1. anisotropic velocity dispersion in xyz-coordinates, 2. rotation and 3. tidal force from the Galaxy.

The first mechanism is thought to apply to giant elliptical galaxies. Two-body relaxation times of the elliptical galaxies are much longer than their ages, and the anisotropic velocity dispersion in the epoch of galaxy formation may remain. However, a typical value of two-body relaxation time of GGC (~10^9 yr) is shorter by nearly one order than their ages, and therefore GGC are thought to represent relaxed systems and the first mechanism does not seem to apply to GGC.

The second mechanism is considered to be probable in general. In particular for a rotating cluster ω Cent., Meylan and Mayor showed a correlation between the differential rotation velocities and the ellipticities at some radius from the rotation axis. However this kind of observations has been made for only a few clusters, and this mechanism has not been established as the cause of the ellipticity.

In Ref. 4), apparent correlations between the axial ratios and positions of GGC relative to the Galaxy are shown. In Fig. 1 we plot (both circles and triangles) the axial ratio vs distance Z from the Galactic plane of 72 GGC that are within 15 kpc from the Galactic center. The axial ratios are from Ref. 4) and Z are from Ref. 8). We can see that the clusters with lower axial ratio (more elliptic) are seen only near the Galactic plane (low Z). This suggests that the third mechanism may be the principal cause of the ellipticities.

Tidal forces from the Galaxy contain three elements: 3a. tidal force from the...
Fig. 1. Observed axial ratios $b/a$ vs heights $Z$ from the Galactic plane of 72 GGC. The axial ratios are from White and Shawl,$^6$ and the heights are from Djorgovski.$^8$ Circles are used for clusters with $\bar{r} \leq r_h$, and triangles are used for clusters with $r_h < \bar{r} \leq 2r_h$. Here, $\bar{r}$ is the mean radius in the observation and $r_h$ is the half-mass radius of each cluster.

galactic disk, 3b. from the bulge, and 3c. from giant molecular clouds. Historically speaking, the effects of 3a is called "disk-shocking" or "compressive shock", and the effect of 3b and 3c are called "tidal shock".$^5$

When globular clusters go through the Galactic disk, the tidal force from the disk acts on them to compress in the direction perpendicular to the disk, and ellipticities of globular clusters may be produced.$^9$

Considering Fig. 1, we are interested in this compressive disk-shocking due to the Galactic disk, and we devote ourselves to a numerical study of the effects in this paper.

Until now, however, the tidal effects were thought not to be the principal cause of the observed ellipticities. One reason for this is that the major axis orientations have correlation with the direction of neither the Galactic center nor the Galactic plane. Another reason is that the tidal forces were thought not to be so strong to distort the inner part of GGC.

To clarify whether the second point is correct or not, we plan a quantitative study on the disk-shocking. If the disk-shocking were to produce the observed ellipticities, and if we were to find a mechanism for removing the correlation mentioned at the first point above, the disk-shocking would become a candidate of the principal cause of the ellipticities.

Several authors carried out numerical experiments on the disk-shocking. Leon et al.$^{10}$ reported a result of their simulations. They solved the perturbed collisionless Boltzmann equation by the particle perturbation method. They showed an elliptic feature of a cluster with the axial ratio $\sim 0.9$, but unfortunately, we can see only one result in their brief paper. A detailed description of the method is seen in Ref. 11).

In IAU symposium 174, Ramamani et al.$^{12}$ reported the results of systematic $N$-body simulations of disk-shocking. The conspicuous figure of their compressed cluster
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seemed to suggest that the disk-shocking plays an important role in producing the ellipticities to the clusters. However there was no quantitative discussion on the axial ratio of projected density contours in their brief report.

In the present study we carry out numerical experiments with a new method suitable for our problem. We use a self-consistent field (SCF) method described briefly in § 5. One advantage of SCF is that a relatively large number of particles (stars) can be used. This is a great advantage to make projected density contours. To clarify the effects of compressive shock, we use somewhat simplified models as follows. The Galactic disk is modeled by a plane parallel potential with a scale height, and the cluster perpendicularly passes through the potential. At some times on single passage, the produced axial ratios are measured.

We define our problem in the following section. In §§ 3 and 4, the disk model and the cluster model are shown, respectively. The numerical integration method is described in § 5. Results of numerical experiments are shown in § 6 followed by concluding remarks in § 7.

§ 2. Statement of our problem

Tidal force from the disk varies linearly across a cluster, while the gravitational mean force of the cluster to a member star at a distance $r$ from the cluster center, varies as $\sim r^{-2}$. Therefore, the relative importance of the tidal force varies as $\sim r^{-3.14}$ Namely, the effect of the tidal force decreases inward to the cluster center. Concerning the compressive effect, it is apparent that the tidal force has less effect in the inner part of a cluster and has a relatively stronger effect in the outer part.

As for the observational results, each axial ratio in Fig. 1 is a mean value of the axial ratios measured at some distances from the cluster center. Circles represent the clusters with the mean distance $\bar{r}$ smaller than the half-mass radius $r_h$ of the cluster, and triangles represent the clusters with $r_h < \bar{r} < 2r_h$.

Here, we set up our problem. “Can the compressive disk-shocking produce the observed axial ratios 0.8~0.9 around the half-mass radius?” We study this problem by a series of numerical experiments.

§ 3. Simple disk models

The mass density distribution

$$\rho(R, Z) = \rho(R_0, 0) \exp(-Z/h) \exp[-(R-R_0)/h_h]$$

is used by many authors as the standard model of the Galactic disk. Here, $Z$ is the height from the Galactic plane, and $R$ is the galactocentric distance in the plane. $R_0$ is the $R$ value of the Sun.

In this paper, we neglect the $R$ dependence and use a simple, plane parallel disk model,

$$\rho(Z) = (\sigma_h/2h) \exp(-Z/h),$$
where $\sigma_R$ is the surface density of the disk. With this density distribution, the disk force $F_z$ is

$$F_z = 2\pi G \sigma_R [1 - \exp(-Z/h)],$$

where $G$ is the gravitational constant. The surface density $\sigma_R$ is obtained by

$$\sigma_R = \int_{-\infty}^{\infty} \rho(R, Z) dZ.$$  \hspace{1cm} (2)

As many authors, \textsuperscript{15} we set $h=325$ pc, $h_R=3.5$ kpc, $R_0=8$ kpc, and $\rho(R_0, 0) \sim 0.13 \text{M}_\odot/\text{pc}^2$ in (1). Then, from (2), we obtain $\sigma_R=265, 150, 84.5, \text{and } 47.7 \text{M}_\odot/\text{pc}^2$ at $R=4, 6, 8$ and 10 kpc, respectively. We use these four values for $\sigma_R$.  

§ 4. Initial cluster models

We use two types of cluster models, each of which consists of $10^5$ particles.

**Isotropic Plummer model**

Plummer\textsuperscript{17} showed that the density distribution

$$\rho(r) = \frac{3M}{4\pi b^3} \left(1 + \frac{r^2}{b^2}\right)^{-5/2},$$ \hspace{1cm} (3)

fits some globular clusters, where $r$ is the distance from the cluster center and $b$ is the scale length of the cluster.\textsuperscript{6} The total mass of the cluster is represented by $M$. We distribute $10^5$ equal-mass particles according to (3). In fact, (3) is the density profile of the polytrope distribution function with the index 5,\textsuperscript{6} and we set the velocities according to the distribution function. Here, we set the velocity dispersion to be isotropic at all radii. We use the values $r_h \sim 1.3b \sim 7.7$ pc and $M=10^5 \text{M}_\odot$. These values are typical values of the half-mass radius and the total mass, respectively. In this model, a typical orbital time of stars at $\sim r_h$ is $\sim 0.6 \times 10^7$ yr.

**Anisotropic model**

A typical value of 2-body relaxation time of GGC is $10^9$ yr, shorter than their age by nearly one order. Therefore GGC are considered to have experienced the 2-body relaxation. It is already known that the radial velocity dispersion must have grown up in outer regions of such clusters. In other words, radial orbits are dominant in the outer regions.\textsuperscript{5} This orbital character may have some effect on our results, so we set up an anisotropic model as the second type of the initial cluster model. This model is more realistic than the isotropic model.

To set up such a realistic model, we evolve the isotropic Plummer model using Spitzer’s Princeton Monte Carlo methods\textsuperscript{5,18} with the effects of 2-body encounters. To clarify the effect of the anisotropic orbital structure, we develop the radial velocity dispersion as far as we can. For this purpose, we evolve the model cluster for an extremely long time, from $T=0$ to $T \sim 14 \times t_{rh}$. Here, $t_{rh}$ is the half-mass relaxation time, and it is $\sim 3 \times 10^8$ yr in our model. The resultant cluster has the half-mass radius $r_h \sim 10$ pc, and a typical orbital time of stars at $\sim r_h$ is $\sim 0.9 \times 10^7$ yr.
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Fig. 2. Profile of $\left< v_r^2 \right>/\left< v_t^2 \right>$ of the anisotropic cluster model. $\log\left[ \left< v_t^2 \right>/\left< v_r^2 \right> \right] \approx \log 2 \approx 0.3$ denotes isotropy, and the radial velocity dispersion dominates as $\log\left[ \left< v_t^2 \right>/\left< v_r^2 \right> \right] \to -\infty$.

By the effect of 2-body encounters, the radial velocity dispersion grows up in the outer region of the cluster. In Fig. 2, we plot $\left< v_r^2 \right>/\left< v_t^2 \right>$ at some radii, where brackets represent the mean value on 4000 particles that lie around each radius. When the velocity dispersion is isotropic, the value of $\log\left[ \left< v_t^2 \right>/\left< v_r^2 \right> \right]$ is equal to $\log 2$. When the radial velocity dispersion is dominant, the value of $\log\left[ \left< v_t^2 \right>/\left< v_r^2 \right> \right]$ becomes small. From Fig. 2, we see that the radial orbits are dominant in the outer regions, and that the anisotropy begins to grow at $r \sim 10\,\text{pc} \sim r_h$.

§ 5. Numerical integration

We numerically simulate the motion of the cluster stars under the influence of the tidal force from the disk. Of course we take into account the cluster's accelerated motion in the disk potential. In all cases, the initial position of the cluster is 2 kpc below the disk plane, and the cluster begins to move toward the disk plane with some initial velocity. The simulation parameters are listed in Table I. We use non-inertia coordinates fixed to the cluster.

We simulate single passages of the clusters through the disk, and, as shown in Table I, the integration times are less than $10^8\,\text{yr}$. Therefore, we must treat the cluster as a collisionless system, and we must use such a numerical integration scheme. The SCF method provides one such scheme. Here, we briefly describe the basic idea of the SCF algorithm (for details, see Ref. 13).

Brief description of SCF algorithm

From the spatial distribution of the point-masses (stars), we obtain a smoothed density distribution $\rho(r)$ of the cluster in the form

$$\rho(r) = \sum_{n,l,m} A_{nlm} \rho_{nlm}(r).$$
Table I. Simulation parameters and resultant figure numbers.

<table>
<thead>
<tr>
<th>Cluster Model</th>
<th>R (kpc)</th>
<th>Initial Velocity (km/s)</th>
<th>Maximum Velocity (km/s)</th>
<th>Integration Time (10^7 yr)</th>
<th>Output Interval (10^7 yr)</th>
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<td>0.39</td>
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<td>200</td>
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<td>5(a)</td>
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<td></td>
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<td></td>
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Here, \( \rho_{nm}(r) \) are “density bases-functions” and \( A_{nm} \) are expansion coefficients labeled with non-negative integers \( n, l \) and \( m \). To describe the positions of cluster stars, we use spherical coordinates \((r, \theta, \phi)\) with the polar axis perpendicular to the disk plane. The integer \( n \) is the radial “quantum” number and represents the order of the radial expansion, and the integers \( l \) and \( m \) are the “quantum” numbers in \( \theta \)- and \( \phi \)-direction, respectively.

With the Poisson equation and \( \rho(r) \), we obtain a smoothed potential \( \Phi(r) \), and the force field \( F(r) \) is calculated as

\[
F(r) = -\nabla \Phi(r).
\]  

In simulations of isolated clusters, cluster stars are integrated by one step according to this force field and their velocities.

To take into account the tidal force, we add the tidal force term to (4) and we use

\[
F(r) = -\nabla \Phi(r) + [F_D(x+r) - F_D(x)]
\]

for numerical integration. Here, \( F_D(x) \) is the disk force at a position \( x \) relative to the disk, and \( c \) is the position of the cluster center. In the coordinate system fixed to the cluster, the added term represents the tidal force. The position vector \( c \) varies as the cluster moves in the disk potential. We numerically integrate the equations of motion of the cluster at the same time in our code.

Because of the axial symmetry around the polar-axis (z-axis), we may take only the 0th order term in the \( \phi \) direction, i.e., we can put \( m_{\text{max}} = 0 \). After some test experiments, we set \((n_{\text{max}}, l_{\text{max}}, m_{\text{max}}) = (8, 4, 0)\). Higher order expansions produce no difference in our results substantially.\(^{13,19}\) Since deviations from the spherical sym-
§ 6. Results

At first, we drew projected density contour maps of the clusters. Figure 3 shows an example of such contour maps. Here, the projection plane (zx-plane) is perpendicular to the disk plane, which is parallel to the xy-plane.

Next, we measured the axial ratios of the contour “ellipses”. As expected, the axial ratios vary with the distance from the cluster center. Then, we made two measurements for each contour map; one is the axial ratio of a contour with its semi-major axis \( \sim r_h \), and another is the axial ratio of a contour with its semi-major axis \( \sim 2r_h \). As before, \( r_h \) is the half-mass radius of the cluster at that time. We denote the former axial ratio as \((b/a)_1\) and the latter as \((b/a)_2\). Specifically, the axial ratio \( b/a \) is the ratio of the axis \( b \) in the direction \( z \) perpendicular to the disk plane to the axis \( a \) in the direction \( x \) parallel to the disk plane. Therefore, \( b/a < 1 \) represents the expected “compression” in the \( z \)-direction, and \( b/a > 1 \) represents the elongation in the \( z \)-direction.

Figures 4~7 show the variation of \((b/a)_1\) and \((b/a)_2\) along the journey of the cluster from left to right. When the height is equal to zero, the cluster is on the disk plane. In the figures, circles represent the \((b/a)_1\), and triangles represent \((b/a)_2\).

Comparison between the isotropic and anisotropic clusters

In Figs. 4 and 5 we set the initial velocities so that the maximum velocity \( V_{\text{max}} \) of the cluster to the disk becomes 200 km/s, which is a typical velocity of the cluster at disk-passing.5) The isotropic cluster model is used in Fig. 4 and the anisotropic model is used in Fig. 5.

It is evident that the anisotropic cluster has the tendency to suffer compression more severely than the isotropic cluster. However, the values of axial ratios at \( \sim r_h \), i.e., \((b/a)_1\) satisfy the inequalities,

\[ 0.94 < (b/a)_1 < 1.04 \]

and these values are substantially larger than those from observations (see Fig. 1). The values of \((b/a)_2\) are relatively small, but many of them are larger than 0.9. According to Figs. 4 and 5, the compressive effects by disk-shocking do not produce the small values of the axial ratios observed in the real GGC.

Experiments with other conditions

With the impulse approximation used in Ref. 9), the magnitude of the velocity impulse \( \Delta v_z \) due to the disk-shocking is inversely proportional to the
Fig. 4. Axial ratios vs clusters’ height from the Galactic plane. In each panel, a cluster moves from left to right with $V_{\text{max}}=200$ km/s. The cluster has isotropic velocity dispersion. (a) $R=4$ (kpc), (b) $R=6$, (c) $R=8$ and (d) $R=10$.

Fig. 5. Same as in Fig. 4 but the cluster has anisotropic velocity dispersion.

velocity $V_{\text{ec}}$ of the cluster through the disk. The resultant axial ratios may also depend on the value of $V_{\text{ec}}$, and in particular, smaller values of $V_{\text{ec}}$ may produce larger values of $\Delta v_z$, and consequently, smaller axial ratios. Therefore, we carried out other sets of experiments with other values of $V_{\text{max}}$. Hereafter, we use the more
realistic anisotropic cluster model.

In the first place, we set the conditions so that $V_{\text{max}} = 300$ km/s, faster than that in the cases of Fig. 5. We show the results in Fig. 6. As expected, the values of the produced axial ratios are closer to 1 than those in Fig. 5.

GGC are thought to have the transversal velocity $V_{\text{trans}}$ of at least $\sim 100$ km/s when they pass through the Galactic plane. As a typical value, Spitzer used $V_{\text{max}} = 160$ km/s in his analysis. Therefore, in the second place, we set the conditions so...
that $V_{\text{max}}$ becomes slower than 200 km/s (see Table 1). If the impulse approximation is valid, slower $V_{\text{max}}$ may produce smaller axial ratios (more elliptic contours).

Figure 7 shows the results. Also in those cases, the disk-shocking seems not to produce the values of the axial ratios at $\sim r_h$ obtained by observations in Fig. 1.

§ 7. Concluding remarks

We carried out a series of numerical experiments of the compressive disk-shocking on globular clusters. Around the half-mass radius, the compressive disk-shocking does not produce the axial ratios of the projected figures of GGC. Therefore, the disk-shocking seems not to be the principal cause of the observed axial ratio.

One reason for this is, of course, that the importance of the disk force relative to the cluster force is lesser for inner stars. We used the cluster models with $r_h = 7.7$ pc (isotropic model) and 9.9 pc (anisotropic model). Although these are slightly large values in GGC, the compressive effects at $r_h$ should be smaller if $r_h$ is smaller than the values we used. Therefore, cluster models with smaller $r_h$ yield the same conclusion as ours. Moreover, in the inner part of the clusters, the random orbital motions of the stars are faster and the collective deviation from the spherical symmetry may be difficult to form.

Although these reasons for the ineffectiveness of the disk-shocking may have been discussed elsewhere,\textsuperscript{14} we have not seen any quantitative comparisons between observational results and numerical results. This is why we carried out this short study. According to our results, the rotation or the velocity dispersion in $xyz$-coordinates seems to be the principal cause of the ellipticities.

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References

3) P. N. Kholopov, Astr. Zh. 29 (1952), 671.