Hadronic Couplings of Open $b$-Flavour States

S. N. RAM and C. P. SINGH

Department of Physics, Banaras Hindu University, Varanasi

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Strong interaction coupling parameters of particles with beauty quantum number are obtained using dispersion sum rules in various forms, e.g., current algebra sum rules, superconvergence sum rules and finite energy sum rules, etc. These sum rules lead to a set of algebraic relations among masses and coupling constants. We compare the hadronic couplings of beauty particles as obtained from various techniques and discuss their implications on the hadronic production of these states.

§ 1. Introduction

Recent years have witnessed a great deal of activities in the study of hadrons due to exciting discoveries of charm and beauty quantum numbers. Consequently, there has been an increase of the parameters required to describe the interaction properties of these states, e.g., masses, coupling constants, form factors, etc. The main line to attack the problem of finding a consistent scheme for understanding and predicting these quantities is based on quantum chromodynamics (QCD) which is the candidate theory for strong interactions. This analysis is based on the fact that hadrons are composite objects of constituent quarks. It assigns binding of constituents together in the long distance scale. However, these attempts suffer from the lack of proper understanding of the binding mechanism, i.e., confinement. An alternative approach, which has so far played a central role in the strong interaction physics, is based on the $S$ matrix phenomenology. This approach has given us many calculational tools in the form of current algebra sum rules, superconvergence relations, etc. Recently hadronic couplings of charmed particles have been calculated using these techniques by many authors. In this paper we want to use these techniques to calculate hadronic couplings of open beauty particles. This theoretical study of hadronic parameters of beauty particles will throw considerable light on the dynamics of flavour symmetry breaking as well as on the hadronic production of these particles.

Recently we have used PCAC Adler's consistency conditions, finite energy sum rules and superconvergence sum rules to determine charmed particles coupling constants. We have further exploited these hadronic parameters to determine the suppression factors for the charmed particle productions in exclusive $\pi p$ and $p p$ interactions. Chung et al. have recently obtained experimental limits on $D^*$ production in $\pi^- p$ interaction. We find that the experimental data support the suppression factor predicted by our theory. In this paper we shall determine hadronic couplings of beauty hadrons which have beauty ±1, charm and strangeness quantum numbers zero. We then make use of these couplings to determine the suppression of beauty production process in hadronic interactions.

The plan of the paper is as follows. In §2 we discuss the use of PCAC Adler’s constraints on the invariant amplitudes for various elastic and inelastic processes involving beauty baryons. Uses of superconvergence sum rules and finite energy sum rules involving various amplitudes as well as their combinations have been given in §3. In §4...
we summarise the implications of our results on the production of open beauty processes and list the conclusions.

\section{PCAC Adler conditions}

We consider the following elastic as well as inelastic processes involving pion and beauty baryons:

\begin{align*}
\pi + \Lambda_b &\rightarrow \pi + \Lambda_b , \\
\pi + \Sigma_b &\rightarrow \pi + \Sigma_b , \\
\pi + \Lambda_b &\rightarrow \pi + \Sigma_b , \\
\pi + \Lambda_b &\rightarrow \pi + \Sigma_b^* .
\end{align*}

The derivations of Adler's consistency conditions on the invariant amplitudes of these processes are straightforward and have been discussed elsewhere.\textsuperscript{4} We get the following conditions:

\begin{align*}
A^{\pi\Lambda_b}(\nu = 0, \nu_B = 0, K^2 = 0) &= 0 , \\
A^{\pi\Sigma_b}(\nu = 0, \nu_B = 0, K^2 = 0) &= \frac{4g^2_{\pi\Sigma_b\Sigma_b} K_{\pi\Sigma_b\Sigma_b}(0)}{m_{\Sigma_b}} , \\
A^{\pi\Lambda_b\pi\Sigma_b}(\nu = 0, \nu_B = 0, K^2 = 0) &= \frac{g_{\pi\Lambda_b\pi\Sigma_b} K_{\pi\Lambda_b\pi\Sigma_b}(0)}{m_{\Lambda_b} + m_{\Sigma_b}} , \\
a_2 A^{\pi\Lambda_b\pi\Sigma_b^*}(\nu = 0, \nu_B = 0, K^2 = 0) &= 0 ,
\end{align*}

where $g$ is the coupling constant, $K_{\pi\Lambda_b\pi\Sigma_b}(0)$ is the pionic form factor of the vertex evaluated at $K^2 = 0$ and

\begin{align*}
\nu &= - \frac{K \cdot (p_i + p_f)}{m_i + m_f} , \\
\nu_B &= \frac{K \cdot q}{m_i + m_f} .
\end{align*}

We have decomposed the matrix element for meson baryon scattering $M(K, m_{\pi}) + B(p_i, m_i)\rightarrow M'(q, m_{\pi}) + B'(p_f, m_f)$ into the invariant amplitudes as follows:

\begin{equation}
T = - A + i\gamma \cdot KB ,
\end{equation}

for processes (1)\textsuperscript{\textendash}(3) and

\begin{equation}
T_a = a_1 K_a + a_2 q_a + i\gamma \cdot K(b_1 K_a + b_2 q_a)
\end{equation}

for process (4).

In the numerical evaluation of the consistency conditions, we use unsubtracted dispersion relations for the invariant amplitude

\begin{equation}
A(\nu, t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \text{Im} \frac{A(\nu', t)}{\nu' - \nu} d\nu'
\end{equation}

and evaluate them by pole term contributions in $s$ as well as $u$ channels at fixed $t = m_{\pi}^2$. 

\[132\]
Hadronic Couplings of Open b-Flavour States

Table I.

<table>
<thead>
<tr>
<th>Particles</th>
<th>( J^P )</th>
<th>I</th>
<th>Charm</th>
<th>Strange-ness</th>
<th>Beauty</th>
<th>Mass (GeV)</th>
<th>Strong decay mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B )</td>
<td>0(^-)</td>
<td>( \frac{1}{2} )</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>5.28</td>
<td>-</td>
</tr>
<tr>
<td>( B^* )</td>
<td>1(^-)</td>
<td>( \frac{1}{2} )</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>5.36</td>
<td>-</td>
</tr>
<tr>
<td>( B^{**} )</td>
<td>2(+)</td>
<td>( \frac{1}{2} )</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>5.55</td>
<td>( B_s )</td>
</tr>
<tr>
<td>( \Lambda_b )</td>
<td>( \frac{1}{2} )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>5.625</td>
<td>-</td>
</tr>
<tr>
<td>( \Sigma_b )</td>
<td>( \frac{1}{2} )</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>5.846</td>
<td>( \pi \Lambda_b )</td>
</tr>
<tr>
<td>( \Sigma_b^* )</td>
<td>( \frac{3}{2} )</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>5.869</td>
<td>( \pi \Lambda_b )</td>
</tr>
</tbody>
</table>

(i.e., \( \nu_b = 0 \)). The usual procedure is to make a subtraction at threshold in writing the dispersion relation for the amplitude \( A \). However, in the absence of experimental information for the processes (1)\( \sim \) (4), we are unable to determine this subtraction constant. We have also shown that in the low-energy limits which we consider in evaluating the integral, our results do not change much even if we take the modified form for the dispersion relation based on the finite energy sum rule for the amplitude.\(^{10}\)

The masses and spin-parities used for open beauty particles have recently been given by many authors.\(^{11}\) These parameters are tabulated in Table I. Saturating condition (5) with the help of known \( I = 1 \) beauty baryon states \( \Sigma_b \) and \( \Sigma_b^* \) in the narrow width approximation we get

\[
1.4 \Gamma_{\Sigma_b} - \Gamma_{\Sigma_b^*} = 0, \tag{12}
\]

where \( \Gamma_{\Sigma_b} \) and \( \Gamma_{\Sigma_b^*} \) are the widths of \( \Sigma_b \) and \( \Sigma_b^* \) for the decay into \( \Lambda_b \pi \) channel, respectively. We can relate the widths of \( \Sigma_b^* \) and \( Y_{1*}(1385) \) on the basis of single quark transition scheme\(^{12}\) motivated by Melosh transformation and we get

\[
\Gamma_{\Sigma_b^* - \pi \Lambda_b} = 0.89 \Gamma_{Y_{1*} - \pi \Lambda}. \tag{13}
\]

Using the width for \( Y_{1*} - \Lambda \pi \) decay mode, we get the width for \( \Sigma_b^* \) as follows:

\[
\Gamma_{\Sigma_b^* - \Lambda_b \pi} = 27.6 \text{ MeV}. \tag{14}
\]

Now from (12) we can predict

\[
\Gamma_{\Sigma_b - \Lambda_b \pi} = 19.8 \text{ MeV}. \tag{15}
\]

The value can be compared with the value obtained from single quark transition scheme. We use the relation

\[
\frac{\Gamma_{\Sigma_b - \Lambda_b \pi}}{\Gamma_{\Sigma_b^* - \pi \Lambda_b}} = \left( \frac{p^*}{p^*} \right) \left( \frac{p_1}{p_1^*} \right)^2,
\]

where \( p \) and \( p^* \) are the c.m. momenta for pion in the decays \( \Sigma_b \rightarrow \Lambda_b \pi \) and \( \Sigma_b^* \rightarrow \pi \Lambda_b \), and \( p_1 \) and \( p_1^* \) are the corresponding quantities for massless pions. We finally get \( \Gamma_{\Sigma_b - \Lambda_b \pi} = 19.5 \text{ MeV} \) which agrees well with the value obtained from PCAC approach as given in (15).
The values of the corresponding coupling constants can be calculated and are given as

\[ \frac{g_{\pi \Lambda_b E_b}^2}{4\pi} = 275.7, \]  
\[ \frac{g_{\pi \Lambda_b E_b^*}^2}{4\pi m_{\pi}^2} = 5.8. \]

Similarly from the consistency conditions (6) ~ (8), after the saturation of dispersion integral with the known low-lying beauty baryon states, we get

\[ 750.4 g_{\pi \Xi_b \Sigma_b}^2 / m_{\pi}^2 + 0.17 g_{\Xi_b \Sigma_b}^2 = 0.68 g_{\Xi_b E_b}^2, \]  
\[ 754.05 g_{\pi \Xi_b \Sigma_b} g_{\Xi_b \Sigma_b} / m_{\pi}^4 - 0.087 g_{\Xi_b \Xi_b} g_{\Xi_b \Xi_b} = 0.087 g_{\Xi_b E_b}^2, \]  
\[ 0.623 g_{\Xi_b E_b} g_{\Xi_b E_b^*} - 1.99 g_{\Xi_b \Sigma_b} g_{\Xi_b \Sigma_b} = 0. \]

From these equations, we can find the values of remaining coupling constants

\[ \frac{g_{\Xi_b E_b}^2}{4\pi} = 96.4, \quad \frac{g_{\Xi_b E_b^*}^2}{4\pi m_{\pi}^2} = 1.15, \quad \frac{g_{\Xi_b E_b^*}^2}{4\pi} = 5.4. \]

Here this must be emphasized that we have ignored \( g_{\Xi_b \Sigma_b} \) coupling appearing in the Lagrangian because it is purely \( F \)-wave coupling and we are working in the low energy approximation. However, this appears essential here because we have four equations from which four unknowns could only be determined.

§ 3. Superconvergence sum rules

Let us consider the following processes involving pion and beauty baryons:

\[ \pi^+ + \Sigma_b^- \longrightarrow \pi^+ + \Sigma_b^+, \]  
\[ \pi^+ + \Sigma_b^- \longrightarrow \pi^+ + \Sigma_b^{*-}, \]  
\[ \pi + \Lambda_b \longrightarrow \pi + \Lambda_b, \]  
\[ \pi + \Lambda_b \longrightarrow \pi + \Sigma_b^*. \]

These processes are similar to the processes considered in the charmed baryon case. Therefore, corresponding superconvergence sum rules can be written as follows:

\[ \int_{-\infty}^{0} \text{Im} B^{\nu=2}(\nu, 0) d\nu = 0 \]  
\[ \int_{-\infty}^{0} \text{Im} a_2^{\nu=2}(\nu, 0) d\nu = 0, \]  
\[ \int_{-\infty}^{0} \text{Im} b_1^{\nu=2}(\nu, 0) d\nu = 0 \]

for process (23). Similarly we deduce
Hadronic Couplings of Open b-Flavour States

\[ \int_{-\infty}^{\infty} \text{Im} B^{i=0}(\nu, 0) d\nu = 0 \]  

(29)

for (24), and finally for the process (25)

\[ \int_{-\infty}^{\infty} \text{Im} b^{i=1}(\nu, 0) d\nu = 0. \]  

(30)

After saturating these sum rules with the beauty baryons listed in Table I and using the value of \( g^2_{\pi A_b E_1}/4\pi m^2 = 5.8 \) from (17), we get the following values of the coupling constants:

\[ g^2_{\pi A_b E_1} = 242.7, \quad g^2_{\pi E_1 E_2} = 124.5, \]
\[ g^2_{\pi E_1 E_2} = 0.82, \quad g^2_{\pi E_1 E_2} = 93.2, \]
\[ g^2_{\pi E_1 E_2} = 0.46. \]  

(31)

These values can be compared with the values given in Eq. (21) obtained from PCAC conditions. We find that all the values of the coupling constants agree within 20% except for the value of \( g^2_{\pi E_1 E_2} \). Similar difference we obtained in the charmed baryon case as well. This difference in \( g^2_{\pi E_1 E_2} \) arises because we have neglected \( g^2_{\pi E_1 E_2} \) coupling in our previous calculation. This signifies that simply in the name of low energy approximation we cannot neglect \( g^2_{\pi E_1 E_2} \).

Similarly, constructing the superconvergent combination from the invariant amplitudes for the processes \( \pi N \rightarrow \pi N, KN \rightarrow KN, BN \rightarrow BN, \pi \pi \rightarrow \pi \pi, \pi D \rightarrow \pi D \) and \( \pi B \rightarrow \pi B \), we find the following superconvergence sum rules:

\[ \int_{-\infty}^{\infty} \text{Im} (B^\pi N A^\pi N - A^\pi N B^\pi N) d\nu = 0, \]  

(32)

\[ \int_{-\infty}^{\infty} \text{Im} (B^\pi N A^\pi N - A^\pi N B^\pi N) d\nu = 0, \]  

(33)

\[ \int_{-\infty}^{\infty} \text{Im} (M^\pi N B^\pi N - M^\pi N B^\pi N) d\nu = 0, \]  

(34)

\[ \int_{-\infty}^{\infty} \text{Im} (M^\pi N B^\pi N - M^\pi N B^\pi N) d\nu = 0. \]  

(35)

In order to saturate sum rules obtained above, we retain the contributions from the low-lying states only in \( s \) as well as \( u \) channels. For example, we use \( N \) and \( \Lambda \) states for \( \pi N \rightarrow \pi N; \Lambda_b, \Sigma_b, \Sigma^*_b \) for \( BN \rightarrow BN; \Lambda, \Sigma, \Sigma^*_b \) for \( KN \rightarrow KN, \rho \) for \( \pi \pi \rightarrow \pi \pi, D^* \) for \( \pi D \rightarrow \pi D \) and \( B^*, B^{**} \) for \( \pi B \rightarrow \pi B \). We further use \( \Gamma^b_{8} \rightarrow \pi K = 93 \text{ KeV (Ref. 13))}, \ g^2_{\pi A_b} / 4\pi = 0.4(\text{Ref. 6}) \), \( g^2_{\pi A_b} / 4\pi = 2.9, \ g^2_{K N A_b} / 4\pi = 10, \ g^2_{K N A_b} / 4\pi = 3, \ g^2_{K N A_b} / 4\pi m^2 = 6.03, \ g^2_{K N A_b} / 4\pi = 14.6 \) and \( g^2_{K N A_b} / 4\pi m^2 = 18 \). We finally get the following values of the coupling constants:

\[ g^2_{\pi A_b} / 4\pi = 2.43, \quad g^2_{K N A_b} / 4\pi = 1317.8, \quad g^2_{K N A_b} / 4\pi = 1172.7, \]
\[ g^2_{K N A_b} / 4\pi m^2 = 32.4. \]  

(36)
§ 4. Conclusion

We find that the values of the coupling constants for the beauty particles are much larger than the corresponding values for the charmed and strange particles. It emphasizes the role of $SU(5)$ flavour symmetry breaking. As the masses of the particles become larger, their coupling strengths also increase. This is clearly shown in Fig. 1 where we have plotted the strength of the couplings like $g_{\Sigma \Xi \Xi}$, $g_{\Xi C C}$, and $g_{\Xi \Xi \Xi}$ against masses of the particles $\Sigma_b$, $C_1$ and $\Sigma$. We find that the points lie on a straight line. It should be mentioned that we have nowhere considered $SU(5)$ flavour symmetry in our calculations. But in our calculations, we are relating the symmetry breaking of particle couplings to the known symmetry breaking in particle masses because we are getting algebraic relations in masses and coupling constants after saturating the sum rules. It may be considered that this increased value of the coupling for the beauty baryons will play a dominant role in increasing the production cross-sections for these particles. However, a simple Regge minded calculation\textsuperscript{7} gives the suppression factor as $10^{-31}$ for the process $pp \rightarrow \Lambda_b \bar{A}_b$ in comparison to the process $p\bar{p} \rightarrow C_b \bar{C}_b$ at the total energy $\sqrt{s} = 62$ GeV. In estimating this value, we have used the trajectory $\alpha(t) = j + \alpha' (t - m^2)$. The slope for $B$ trajectory is taken as $\alpha'_B = 0.36$ [Ref.13] and that for $D$ trajectory is $\alpha'_D = 0.33$ [Ref. 7]. Assuming that Regge pole calculations do not pose any problem at this energy, we thus claim that open $b$-flavour production processes are difficult to be observed in the laboratory. Recently observation of beauty baryon $\Lambda_b$ in the reaction

\[
pp \rightarrow \bar{B} \xrightarrow{\rightarrow e^+ \cdots} \Lambda_b \xrightarrow{\rightarrow D^0 p \pi^-} \xrightarrow{\rightarrow K^- \pi^+} X
\]

at $\sqrt{s} = 62$ GeV has been claimed by CBF collaboration.\textsuperscript{14} The cross-section presented

![Fig. 1. The values of the couplings $g_{\Xi \Xi \Xi}$, $g_{\Xi C C}$, and $g_{\Xi \Xi \Xi}$ are plotted against the masses of $\Sigma$, $C_1$ and $\Sigma_b$ particles.](image-url)
for the signal is \((27\pm 1.7)\, \mu b\). However, the experiment performed with better sensitivity\(^{13}\) yields negative result for beauty baryon process. In addition, the cross-section corresponding to the \(\Lambda_b\) signal claimed by CBF collaboration is much larger (about 100 times) than an upper limit obtained from the \(e^+\pi\) ratio measured by the same collaboration. Thus, the experimental data are still incomplete and ambiguous. We hope that in the near future, we will have sufficient experimental information to test our theoretical results. However, we must admit that even if experiments demonstrate the suppression claimed here, it is mainly due to difference in intercepts of \(B\) and \(D\) trajectories. Therefore, it cannot be considered as any credit to our analysis. We propose that approximate equality of cross-sections for the processes \(p\bar{p} \rightarrow \Lambda_b\bar{\Lambda}_b\) and \(p\bar{p} \rightarrow \Sigma_b\bar{\Sigma}_b\) would lend support to our analysis because the couplings \(g_{B\Lambda\Lambda}\) and \(g_{B\Sigma\Sigma}\) are not much different. This is quite in contrast with the cross-sections for \(p\bar{p} \rightarrow \Lambda\bar{\Lambda}\) and \(p\bar{p} \rightarrow \Sigma\bar{\Sigma}\) which give\(^{16}\) the value for the coupling \(g_{K\Lambda\Lambda}\) much higher than \(g_{K\Sigma\Sigma}\).

In conclusion, superconvergence and current algebra sum rules give a significant tool to calculate hadronic coupling strengths involving particles and resonances. Although these sum rules are of great theoretical interest, it is difficult to give any convincing saturation scheme\(^{10}\) for them. In the processes involving beauty particles, this scheme involves a long extrapolation from the known region of approximate validity of these ideas which do not contain heavy quarks. However, the saturation scheme employed here, has been found to be successful in the past in predicting many coupling constants.\(^{2-8}\) Moreover, lack of experimental information on beauty baryon spectroscopy prompts us to stick to the saturation by low-lying states only. We hope that the values of these coupling constants obtained here will help us in drawing a dynamical scheme for the breaking mechanism of flavour symmetry.

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