Mass Degeneracy of Squarks and Sleptons in Supergravity

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Mass degeneracy of squarks and sleptons is discussed in the framework of Einstein supergravity. It is shown that under plausible assumptions, the mass degeneracy naturally occurs even in the supergravity with non-minimal couplings, which is phenomenologically required for suppression of flavor-changing neutral currents.

1. Supersymmetry (SUSY) provides an intriguing solution to the gauge hierarchy problem where the cancellation of all quadratic divergences takes place between boson and fermion loops. It is, however, well known that the presence of superpartners of quarks and leptons (squarks and sleptons) causes significant effects in the low-energy physics. Examples include the proton decay due to dimension five operators and the flavor-changing neutral currents (FCNCs) induced by one-loop diagrams including squarks and sleptons. Observed suppression of the $K^0\rightarrow\bar{K}^0$ mixing and $\mu\rightarrow e\gamma$ decay strongly implies that the masses of squarks and sleptons in different flavors (more precisely in the first and the second generations) with the same $SU(3)\times SU(2)\times U(1)$ quantum numbers should be highly degenerate. (For their masses $m \approx 1$ TeV, the mass differences $\delta m^2$ are constrained as $\delta m^2/m^2 \leq 10^{-2} - 10^{-3}$.)

In the minimal $N=1$ supergravity where a flat Kähler metric is postulated, all soft SUSY breaking masses of the scalar bosons are the same as a gravitino mass $m_{3/2}$ (for a review of supergravity, see Ref. 4). But there is no apparent reason why nature chooses the minimal coupling in the Kähler potential. Indeed, as we will see below, when fields which are singlet under $SU(3)\times SU(2)\times U(1)$ gauge symmetry non-minimally couple to squarks and sleptons, the soft SUSY breaking masses of the squarks and sleptons have additional terms and are not automatically degenerate.

In order to confirm that the non-minimal coupling is not a peculiar one, let us consider string theory. In a superstring theory (or more precisely a heterotic string theory) compactified into four dimensions, there exist moduli fields whose vacuum expectation values (VEVs) specify the shape (the complex structure) and the size (the Kähler class) of the six-dimensional compact space. They are gauge singlets and non-minimally couple to the non-singlet fields (squarks and sleptons) in the Kähler potential, as we will illustrate in an orbifold model. Thus the string theory appears problematic if the mass degeneracy is not realized.

The problem also exists in supersymmetric grand unified theories (GUTs). For example, in the standard $SU(5)$ SUSY GUT, some of the Higgs fields in adjoint
representation are singlet under the $SU(3) \times SU(2) \times U(1)$ and if they have non-minimal couplings to the non-singlet fields in the Kähler potential, the mass degeneracy does not in general occur.

In this paper, we examine under what circumstances the soft SUSY breaking mass terms for the sfermions are degenerate in the context of supergravity. As a working hypothesis, we assume that Einstein supergravity is valid, that is, (1) the gravity sector is dominated by the Einstein action and (2) higher derivative terms of matter fields are excluded. We also postulate that the supersymmetry is spontaneously broken in the hidden sector which is completely decoupled to the observable sector in the Kähler potential. Then it turns out that, with some plausible assumptions, the mass degeneracy of squarks and sleptons naturally occurs even in the non-minimal Kähler potential. The validity and limitation of our approach will be discussed at the end of this paper.

2. In the Einstein supergravity, a potential of scalar fields is written in terms of a total Kähler potential and its derivatives. In this paper, we consider the following form of the total Kähler potential,

$$G = H(z, z^*) + K(y, y^*) + \ln|\hat{h}(z) + g(y)|^2,$$

where we have set a gravitational constant $\sqrt{8 \pi / m_{\text{Planck}}} = 1$. $H(K)$ is a real function of $z(y)$ and a superpotential $h(g)$ is some analytic function of $z(y)$. $z$ denotes a hidden sector boson and $y = \{y^i, y^a\}$ include observable fields such as the squarks and sleptons. We will call this sector an observable sector. The Latin (Greek) indices represent $SU(3) \times SU(2) \times U(1)$ non-singlet (singlet) fields in this sector. When we consider the heterotic string theory compactified to four dimensions, the hidden sector includes a dilaton field and the moduli fields are singlet in the observable sector.

Using the total Kähler potential (1), the potential of scalar bosons is given by

$$V = \exp G\{(G^{-1})^a_B G^a_B - 3\} = \exp G\{G_i, i = 1, 2, 3\} = G_a(G^{-1})^a_i i^f G^i_G + G_a(G^{-1})^a_i G^i_G + G_a(G^{-1})^a_i G^a_i + G_a(G^{-1})^a_i G^i_G - 3\}.$$

Here $G_a = \partial G / \partial \varphi^A$, $G^A = \partial G / \partial \varphi_A$, and $(G^{-1})^a_B$ is the inverse matrix of $(G)_a^B = \partial G / \partial \varphi^B$, where we symbolically denote all kinds of fields $z$ and $y$ by $\varphi^A$.

Singlet fields may have their VEVs $\langle y^a \rangle = v^a$ keeping the $SU(3) \times SU(2) \times U(1)$ gauge symmetry. After shifting the fields so that $\langle y^a \rangle = 0$, the function $K(y, y^*)$ can be expanded as

$$K(y, y^*) = K^{(0)} + K^{(0)}_a y^a + K^{(0)}_{i\alpha} y_i^* y^\alpha + K^{(0)}_{i\beta} y_i^* y^\beta + K^{(0)}_{i\alpha} y_i^* y^\alpha + K^{(0)}_{i\beta} y_i^* y^\beta$$

where $\alpha, \beta, i = 1, 2, 3$ and $y_i^*$ denotes the complex conjugate of $y_i$.
Mass Degeneracy of Squarks and Sleptons in Supergravity

\[ + K^{(0)i}_{\beta\delta} y_i^* y_i y_{\beta}^* y_{\delta}^* + K^{(0)ia\beta} y_i^* y_i y_{a}^* y_{\beta}^* + \cdots, \]  

where \( K^{(0)} = \langle K \rangle \), \( K^{(0)}_{\alpha} = \langle K_{\alpha} \rangle \), etc., are some constants. Without loss of generality we can normalize

\[ K^{(0)i}_{\beta} = \delta_{i}^{\beta}, \quad K^{(0)ia}_{\beta} = \delta_{i}^{\beta}, \]  

so that the \( y \) fields have canonical kinetic terms.

Substituting Eq. (3) into Eq. (2), we find that the soft SUSY breaking mass terms for the non-singlet scalar bosons are

\[ (m^2)_{ij} = m_{3/2}^2 \delta_{ij} + \langle G_a \rangle (K^{(0)ia}_{\beta} K^{(0)\beta}_{i} - K^{(0)ia}_{\beta} K^{(0)la}_{i}) \langle G_a \rangle + \langle V \rangle \delta_{ij}, \]  

where \( \langle \cdots \rangle \) denotes the VEV of \( \cdots \) and

\[ m_{3/2} \equiv \langle \exp(G/2) \rangle \]  

is the gravitino mass. In deriving Eq. (5), we have taken supersymmetric mass terms vanishing, which is the case for squarks and sleptons.

For the minimal coupling, the Kähler potential in the observable sector (3) becomes

\[ K = \langle v_{a}^* + y_{a}^* \rangle (v_{a} + y_{a}) + y_{i}^* y_{i}. \]  

It follows that

\[ K^{(0)i}_{\beta} = K^{(0)i}_{\beta} = K^{(0)ia}_{\beta} = \cdots = 0. \]

Then Eq. (5) implies that the soft SUSY breaking masses for the non-singlets are degenerate. Notice that Eq. (5) also implies that, even in the case of non-minimal coupling, the mass degeneracy occurs if there is no singlet field in the observable sector. However, in the presence of singlet fields the mass degeneracy of squarks and sleptons is not always guaranteed.

It is instructive to illustrate an orbifold compactification of the \( E_8 \times E_8 \) heterotic string theory. For a \( Z_3 \) orbifold based on a torus \( K^6 / \Gamma \) where \( \Gamma \) is a root lattice of \( SU(3)^3 \), the Kähler potential in the observable sector is found to be

\[ K = -\log \prod_{m} (N_{m} + N_{m}^*) + \sum_{m} \frac{\langle N_{m} + N_{m}^* \rangle}{N_{m} + N_{m}^*} U_{m} U_{m}^* \]

\[ + \sum_{f} \frac{\langle \prod (N_{m} + N_{m}^*) \rangle}{\prod (N_{m} + N_{m}^*)} C_{f} C_{f}^* + \sum_{f} \frac{\langle \prod (N_{m} + N_{m}^*)^{2/3} \rangle}{\prod (N_{m} + N_{m}^*)^{2/3}} T_{f} T_{f}^* + \cdots, \]  

where \( U_{m} \) and \( T_{f} \) are untwisted and twisted fields, respectively. They are assigned to squarks and sleptons. \( N_{m} \) (\( m = 1, 2, 3 \)) are untwisted moduli fields. Real parts of their VEVs \( \langle N_{m} \rangle \) are associated with radii of the compactified space. \( C_{f} \) denote twisted moduli fields. We see that the untwisted moduli fields couple to the non-singlet fields \( U_{m} \) and \( T_{f} \) non-minimally. The coefficients of the last three terms are chosen so that the \( U_{m}, C_{f} \) and \( T_{f} \) have the correct normalization (4). Defining new variables \( n_{m} \) as

\[ N_{m} = \langle N_{m} \rangle + \langle N_{m} + N_{m}^* \rangle n_{m}, \]
the fields \( n_m \) also have the correct normalization (4). Equation (9) now reads

\[
K = -\log \prod_m \langle N_m + N_m^* \rangle - \log \prod_m (1 + n_m + n_m^*) + \sum_m \frac{1}{1 + n_m + n_m^*} U_m U_m^*
+ \sum_f \frac{1}{1 + n_m + n_m^*} C_f C_f^* + \sum_f \frac{1}{(1 + n_m + n_m^*)^{2/3}} T_f T_f^* + \cdots .
\]  

(11)

We should note that, in the case at hand, \( K_{c(n)} c_j, K_{d(n)} d_j \), etc., in Eq. (3) are of order one (in the Planck unit) and do not depend on the compactification size. Therefore the second term of Eq. (5) is not negligible and it seems to destroy the mass degeneracy at first sight.

3. Let us now present a scenario which naturally leads to the mass degeneracy. Concerning the SUSY breaking, we postulate the following two assumptions (a) and (b) which are widely accepted.4)
(a) In order for SUSY to resolve the hierarchy problem, the gravitino mass should be at the Fermi scale and consequently very small compared to the Planck mass. Equation (6) implies

\[
\langle |g + h| \rangle = O(m_{3/2}) .
\]  

(12)

Since the soft SUSY breaking masses must also be at the Fermi scale, we see, from Eq. (5), that \( \langle G_a \rangle \) is at most of order one, i.e.,

\[
\langle G_a \rangle \leq O(1) .
\]  

(13)

From the equation \( \langle G_a \rangle = K_a^{(0)} + \langle g_a \rangle / \langle g + h \rangle \), we obtain

\[
\langle g_a \rangle = O(m_{3/2}) ,
\]  

(14)

where \( K_a^{(0)} \) is supposed to be at most of order one (see the assumption (c) below).
(b) The hidden sector is responsible for (almost) the vanishing of the cosmological constant and therefore we have

\[
\langle G_x \rangle = O(1) .
\]  

(15)

Since \( G_x = H_x + h_x / (g + h) \), we must further require

\[
\langle h_x \rangle = O(m_{3/2}) .
\]  

(16)

Note that our assumptions (12), (14) and (16) have been indeed used in many phenomenologically successful supergravity models.

On the singlet fields in the observable sector, we impose:
(c) \( K_a^{(0)}, K_d^{(0)}, K_d^{(0)}, K_d^{(0)} \) and \( K_d^{(0)} \) are at most of order one,
(d) the singlets are superheavy and have large SUSY mass terms, that is,

\[
\langle g_a \rangle = O(1) .
\]  

(17)

The requirements (c) and (d) are characteristic to our discussion and will play an important role. It is easy to see that the assumption (c) holds in the orbifold example
we presented. The assumption (d) is very natural for moduli fields,*) as far as the
compactification size is near the Planck length.

Under the assumptions (a)~(d), it can be shown that the mass degeneracy occurs.
A proof is given by the following order estimation. Since the vacuum is a stationary
point of the scalar potential (2), the VEV of the first derivative of Eq. (2) with respect
to \( y^a \) vanishes

\[
0 = m_{3/2}^2 \left( \frac{\partial V}{\partial y^a} \right)
\]

\[
= \langle G_a \rangle \{ \langle G_4 \rangle \langle G^4 \rangle + \langle G_\bar{a} \rangle \langle G^\bar{a} \rangle - 3 \} + \langle G_{3a} \rangle \langle G^3 \rangle + \langle G_\bar{a} \rangle \langle G_\bar{a} \rangle \langle G^\bar{a} \rangle + \langle G_\bar{a} \rangle \langle G_{3a} \rangle \langle G^3 \rangle + \langle G_\bar{a} \rangle \langle G_{3a} \rangle \langle G^\bar{a} \rangle .
\]

Our assumptions imply

\[
\frac{\langle g_a \rangle}{\langle g + h \rangle} = O(1),
\]

\[
\frac{\langle h_\bar{a} \rangle}{\langle g + h \rangle} = O(1),
\]

\[
\frac{\langle g_\bar{a} \rangle}{\langle g + h \rangle} = O(m^{-1}_{3/2}),
\]

where the last equation is due to the assumption (d). The second derivatives of the
total Kähler potential are then

\[
\frac{\langle G_{3a} \rangle}{\langle g + h \rangle} = -\frac{\langle g_a \rangle}{\langle g + h \rangle} \frac{\langle h_\bar{a} \rangle}{\langle g + h \rangle} = O(1),
\]

\[
\frac{\langle G_\bar{a} \rangle}{\langle g + h \rangle} = K_{3a}^{(\bar{a})} + \frac{\langle g_\bar{a} \rangle}{\langle g + h \rangle} \frac{\langle g_a \rangle}{\langle g + h \rangle} \frac{\langle g_a \rangle}{\langle g + h \rangle} = O(m^{-1}_{3/2}).
\]

If \( \langle G_a \rangle \) were of order one, the last term of Eq. (18) were of order \( m_{3/2}^{-1} \), while the
other terms were of order one. This means the r.h.s. of Eq. (18) cannot vanish in this
case. We can see that Eq. (18) has a solution if and only if the first derivative of the
total Kähler potential is**)

\[
\langle G_a \rangle = O(m_{3/2}) .
\]

Therefore Eq. (5) reads

\[
(m^2)^i_j = (m_{3/2}^2 + \langle V \rangle) \delta^i_j + O(m_{3/2}^2).
\]

Since the second term is negligible, we can conclude that the soft SUSY breaking mass
terms are highly degenerate.

*) It is known that the superpotential for the moduli fields is flat to all orders in the non-linear sigma
model perturbations\(^1\) and consequently \( \langle g_{ab} \rangle = 0 \). We expect that the superpotential will be generated by a
world sheet instanton\(^2\) or something else so that the compactification scale is determined.

**) For the hidden sector, \( \langle h_{a\bar{a}} \rangle \) is assumed to be of order \( m_{3/2} \). Therefore the stationary condition
\( \langle V/\delta a \rangle = 0 \) is satisfied while keeping \( \langle G_a \rangle = O(1) \). This contrasts to the situation in the observable sector,
and one the assumption (d), \( \langle g_{ab} \rangle = O(1) \), is an essential point of deriving Eq. (21).
Here we should note that we have assumed the hidden sector \( z \) decouples to the observable sector. If the \( z \) couples to \( y \) in the Kähler potential, our proof is invalid.

4. To conclude this paper, we would like to discuss a validity and a limitation of our argument. For definiteness, we will assume again a heterotic string theory compactified to four dimensions.

We have considered the mass degeneracy of sfermions in the context of the Einstein supergravity. The first point we must consider is therefore its validity. In a heterotic string theory, it is known that the Einstein gravity is justified only at the lowest order of string tension (\( \alpha' \)) expansion (\( \alpha' \sim 1/m_{\text{Planck}}^2 \)). In order for the Einstein supergravity to be a good approximation, it seems necessary that the compactification scale is less than the Planck scale. In this case, it is likely that the higher order corrections are suppressed and our assumption of the Einstein supergravity will be justified.

The other point concerns radiative corrections. We will restrict ourselves to one-loop corrections for example. Let us first consider the radiative corrections due to the moduli fields. Since the interactions of the moduli fields in the Kähler potential are non-renormalizable, it is necessary to introduce a cutoff \( \Lambda \). In string theory, \( \Lambda \) is most likely taken at the compactification scale, which might be smaller than the Planck scale as explained above. If it is the case, the contributions of the moduli fields to the one-loop mass shifts of the squarks and sleptons will be at most of order \( m_{3/2} \Lambda^2 \) in the Planck unit. Therefore if the cutoff scale or the compactification scale is an order of magnitude or so smaller than the Planck scale, the one-loop corrections are sufficiently small and do not give rise to significant mass splittings. We must also consider the radiative corrections due to light particles. These effects can be taken into account by solving renormalization-group equations consisting of the light particles. Although gauge coupling do not distinguish the flavors, Yukawa couplings are different in different flavors and cause mass splittings. As long as the first and the second generations are concerned, however, the relevant Yukawa couplings are quite small** and do not induce large mass splittings which violate the phenomenological constraints derived from the observed suppression of the FCNCs.

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References


*) We should stress that we do not deny the possibility that the compactification scale is the same order of the Planck scale. In this case, however, more stringy argument is required, which unfortunately is beyond the scope of our present paper.

**) It is pointed out that superheavy fields such as colored Higgs in GUT may give dangerous corrections to the mass terms if their Yukawa couplings to the ordinary squarks and sleptons are large.
Mass Degeneracy of Squarks and Sleptons in Supergravity