Study of the Difference between the Spin-Orbit Potentials of \(^3\)He and \(t\) by the Resonating Group Method

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(Received August 5, 1985)

The difference between the spin-orbit potentials of \(^3\)He and \(t\) is studied by the resonating group method. Calculated spin-orbit potentials are compared with phenomenological ones. Remarks are made on the understanding of the phenomenological optical potential parameters.

Scattering experiments of \(^3\)He and triton \((t)\) have been extensively performed using the polarized beams and analyses of these data by the optical potentials have given us rich information on the spin-orbit \((l\cdot s)\) potentials of these projectiles. However, the obtained parameters of the \(l\cdot s\) potentials for \(^3\)He and those for \(t\) are found to be fairly different from each other. The \(l\cdot s\) potentials for \(^3\)He obtained by Birmingham group \(^1\) have very small diffuseness parameter \((a_{so}\approx 0.2 \text{ fm})\) and the radius parameter for the \(l\cdot s\) potential \(r_{so}\) is somewhat larger than that for the central potential. On the other hand, the \(l\cdot s\) potential for \(t\) obtained by Los Alamos group \(^2\) are deeper at least twice than are expected by the folding model. Typically \(V_{so}(t)/V_{so}(\text{He})=2\sim 3, \ r_{so}(t)/r_{so}(\text{He})=0.8\sim 0.9\) and \(a_{so}(t)/a_{so}(\text{He})=3\sim 4.\)

It is therefore quite interesting to study microscopically how difference can be derived between the \(l\cdot s\) potentials of \(^3\)He and \(t\). In this paper we report the results of the study of the difference between the \(l\cdot s\) potentials of \(^3\)He and \(t\); the equivalent local \(l\cdot s\) potentials are obtained from the non-local potentials of the resonating group method \((\text{RGM})\) by the use of the WKB approximation. The procedure to derive the \(l\cdot s\) potential for composite projectiles by the RGM is explained in detail in Ref. 3).

Since the nuclear force between two nucleons is isospin invariant, the difference between the \(l\cdot s\) potentials of \(^3\)He and \(t\) arises only from the difference in the Coulomb forces for these two projectiles. Stronger Coulomb force for \(^3\)He causes smaller local momentum for this projectile compared with that for \(t\) at the same incident energy. The smaller the local momentum is, the larger the exchange contributions in the Wigner transforms of the RGM non-local potentials are.

As is explained in Ref. 3), the \(l\cdot s\) potential comes not only from the two-nucleon spin-orbit interaction directly but also from the two-nucleon central interaction and the kinetic energy as a
renormalization effect; this renormalization arises because the local momentum for \( j = \frac{l+1}{2} \) is different from that for \( j = \frac{l-1}{2} \). Hence all the central interaction part, the kinetic energy part and the spin-orbit interaction part are responsible for the appearance of the difference between the \( l\cdot s \) potentials of \(^3\text{He}\) and \( t \).

We show in Fig. 1 the difference between the equivalent local \( l\cdot s \) form factors \( V_n^s(r) \) of \(^3\text{He}\) and \( t \); \( \Delta(r) = V_n^s(r)(^3\text{He}) - V_n^s(r)(t) \) for the target nucleus \(^{16}\text{O}\) at the incident energies per nucleon \( E = 0 \) and 10 MeV/\( u \). In Fig. 2 the same quantities for the target nucleus \(^{40}\text{Ca}\) are displayed. The oscillator parameter \( \nu \) of the internal wave functions of scattering nuclei is set commonly to 0.16 fm\(^{-2}\) for \(^{16}\text{O}\) case and 0.14 fm\(^{-2}\) for \(^{40}\text{Ca}\) case, respectively. The effective two-nucleon central force we adopt is Volkov No. 1\(^6\) with the Majorana mixture \( m = 0.60 \) for \(^{16}\text{O}\) and \( m = 0.668 \) for \(^{40}\text{Ca}\). As for the effective two-nucleon spin-orbit force, we adopt the following one.\(^5\)

\[
\nu_{l\cdot s}(\hat{S}) = \left( \sum_{i=1}^{3} u_i \cdot P(3O) \exp(-x_i r^2) \right) L \cdot (S_1 + S_2)
\]

with

\[
u_{l\cdot s} = \begin{cases} \begin{align*}
u_{l\cdot s} &= 900 \text{ MeV}, \quad x_1 = 5.0 \text{ fm}^{-2}, \\
u_{l\cdot s} &= -900 \text{ MeV}, \quad x_2 = 2.778 \text{ fm}^{-2},
\end{align*} \end{cases}
\]

where \( P(3O) \) is the projection operator onto the \(^3O\) (triplet odd) state. The Coulomb force is included by approximating the RGM Coulomb kernel as \( \sqrt{N} V_{nc}\sqrt{N} \) where \( V_{nc}(r) \) is the direct (or double folding) potential of the Coulomb interaction and \( N \) is the RGM norm kernel. From Figs. 1 and 2 we see that the \( l\cdot s \) potential for \(^3\text{He}\) is deeper than that for \( t \) in the tail region and is shallower in the inner spatial region when we compare them at the same incident energy. Since the Coulomb force which is repulsive is stronger for \(^3\text{He}\) than for \( t \), the local momentum (hence the local kinetic energy) for \(^3\text{He}\) is smaller than that for \( t \) when the incident energy is the same. The energy-dependence of \( V_n^s(r) \) is such that it becomes shallower (deeper) in the tail (inner spatial) region as energy gets higher. Thus we get the above-mentioned result for \( \Delta(r) \).

For the discussion of "strength" of the \( l\cdot s \) potential, the comparison by some scalar quantity which characterizes the \( l\cdot s \) potential is useful. Moreover, when we compare the calculated \( l\cdot s \) potential with the phenomenological one, since the radial shapes are quite different between them, the use of the scalar quantity is essential. For this purpose we use the \( r^2 \)-weighted radial integral \( J_4 \) of \( l\cdot s \) potential,\(^6\)

\[
J_4 = -\int_0^\infty r^2 V_{ns}(r) \cdot r^2 dr / A_t,
\]

where \( A_t \) is the mass number of the target nucleus. In Tables I and II we show the calculated value of \( J_4 \) of \( V_n^s \) for several incident energies; Table I is for \(^{16}\text{O}\) and Table II is for \(^{40}\text{Ca}\). When we compare \( J_4(\text{He}) \) with \( J_4(t) \) at the same incident energy, we find that \( J_4(\text{He}) \) is larger than \( J_4(t) \). This is due to the \( r^2 \)-factor in the integrand which plays the role to emphasize the behavior of the tail part of \( V_n^s(r) \). At present, however, the available data for \(^3\text{He}\) are near \( E = 10 \text{ MeV/}u \) and those for \( t \) are near \( E = 8 \text{ MeV/}u \). When we compare \( J_4(\text{He}) \) at \( E = 10 \text{ MeV/}u \) with \( J_4(t) \) at \( E = 5 \text{ MeV/}u \), we see that the former is smaller than the latter. This shows that, in comparing the \( l\cdot s \) potentials of \(^3\text{He}\) and \( t \), due consideration of the energy-dependence of the \( l\cdot s \) potential is quite important.

**Table I.** Calculated value of \( J_4 \) (\( r^2 \)-weighted radial integral of the \( l\cdot s \) potential, in units of MeV fm\(^s\)) for the incident energies per nucleon \( E = 5, 10, 15 \) and 20 MeV/\( u \). The upper column in for \( E = 5 \text{MeV} / u \) and lower one is for \(^3\text{He} + ^{16}\text{O}\).

<table>
<thead>
<tr>
<th>( E(\text{MeV/u}) )</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>( J_4(\text{He}) )</td>
<td>10.61</td>
<td>9.73</td>
<td>9.23</td>
<td>8.92</td>
</tr>
<tr>
<td>( J_4(t) )</td>
<td>10.87</td>
<td>9.86</td>
<td>9.31</td>
<td>8.97</td>
</tr>
</tbody>
</table>

**Table II.** The same quantities as in Table I for the target nucleus \(^{40}\text{Ca}\).

<table>
<thead>
<tr>
<th>( E(\text{MeV/u}) )</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>( J_4(\text{He}) )</td>
<td>9.73</td>
<td>8.65</td>
<td>8.14</td>
<td>7.84</td>
</tr>
<tr>
<td>( J_4(t) )</td>
<td>10.57</td>
<td>9.94</td>
<td>8.29</td>
<td>7.94</td>
</tr>
</tbody>
</table>

**Table III.** Empirical value of \( J_4 \) (in units of MeV fm\(^s\)) together with the value of \( J_4 \) (volume integral of the central potential per nucleon pair) for the systems of \(^3\text{He} + ^{16}\text{O}\), \(^{40}\text{Ca}\) (from Ref. 1) and \( t + ^{40}\text{Ca}\) (from Ref. 2).

<table>
<thead>
<tr>
<th>( \text{Exp} )</th>
<th>( ^{16}\text{O} )</th>
<th>( ^{40}\text{Ca} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( J_4(\text{MeV} \cdot \text{fm}^s) )</td>
<td>( \approx 420 )</td>
<td>( \approx 350 )</td>
</tr>
<tr>
<td>( J_4(t) ) ( (E_{lab} = 17 \text{ MeV}) )</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( J_4(\text{He}) ) ( (E_{lab} = 33 \text{ MeV}) )</td>
<td>12.74</td>
<td>24.74</td>
</tr>
</tbody>
</table>
We show in Table III the values of $J_\sigma$ for the phenomenological $t\cdot s$ potentials; they are taken from Ref. 1) for $^3$He and from Ref. 2) for $t$. The quantity $J_\sigma$ in Table III is the volume integral of the central potential per nucleon pair,

$$J_\sigma = -4\pi \int_0^\infty V_c(r) \cdot r^2 \, dr / {A_p A_T},$$

where $A_p$ is the mass number of the projectile. In the present study the value of $J_\sigma$ for the equivalent central potential which is of course energy-dependent is as follows; $J_\sigma \approx 450 \text{ MeV} \cdot \text{fm}^3$ for $^{16}$O case and $J_\sigma \approx 350 \text{ MeV} \cdot \text{fm}^3$ for $^{40}$Ca case in this energy region. The parameter set for $^3$He+$^{40}$Ca central optical potential which gives the value of $J_\sigma \approx 350 \text{ MeV} \cdot \text{fm}^3$ is believed to be the best choice from the analyses of the backward angle data and from the systematic behavior.\(^\text{7}\) Unfortunately, there is no parameter set for $t$+$^{40}$Ca which gives this value of $J_\sigma$ and no data for $t$+$^{16}$O. We see from Table III that the value of $J_\sigma$ increases according to the increasing value of $J_\sigma$. Moreover, by comparing the optical potential of $^3$He with that of $t$ both of which have the values of $J_\sigma \approx 500 \text{ MeV} \cdot \text{fm}^3$, we found that the calculated result is in qualitative accordance with experiments; $J(t, E=5 \text{ MeV}/u) > J(^3\text{He}, E=10 \text{ MeV}/u)$. These facts suggest that the consideration of the value of $J_\sigma$, the volume integral of the central potential per nucleon pair, is important in deriving the optical potential parameters of the $t\cdot s$ potentials.

The author thanks Professor H. Horiuchi for stimulating discussions. The computer calculations for this work were financially supported in part by the Research Center for Nuclear Physics, Osaka University.