On the “Optical Method” for the Scattering of High Energy Particles by Complex Nuclei

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The relation between the optical method \(^1\) and the quantum mechanical many body problem for the scattering of high energy particles by complex nuclei is investigated.

The total Hamiltonian of our problem is

\[ H = H_0 + V, \]

with

\[ H_0 = H_{0I} + H_{0II}, \]

and

\[ H_{0II} = K + U. \]

\( H_{0I} \) is the kinetic energy operator of the incident particle, \( K \) the total kinetic energy operator of nuclear constituents, \( U \) the nuclear potential energy, and \( V \) is the total interaction potential between the incident particle and the nucleons. We shall suppose \( V \) to be of the form

\[ V = \sum_{k=1}^{A} V_k, \]

where \( V_k \) is the interaction potential between the incident particle and the \( k \)th nucleon. We start from the exact solution of our problem

\[ \Phi(r; r_1 \ldots r_A) = \varphi(r) \Psi_1(r_1 \ldots r_A) \]

\[ + (E + i\eta - H_0 - V)^{-1} V_k = (\omega_k^{(4)} - 1) \]

\[ + (E + i\eta - H_0 - V)^{-1} \left\{ [U, \omega_k^{(4)}] \right\}, \]

the transition matrix \( T^{(4)} \) is represented, according to Chew-Goldberger\(^2\), as follows:

\[ T^{(4)} = \sum_{k=1}^{A} \left\{ t_k^{(4)} + V_k (E + i\eta - H_0 - V)^{-1} [U, \omega_k^{(4)}] \right\}, \]

\[ + [1 + V_k (E + i\eta - H_0 - V)^{-1} (V - V_k) (\omega_k^{(4)} - 1)], \]

where

\[ \omega_k^{(4)} = 1 + [E_k + i\eta - (H_0 + K) - V_k]^{-1} V_k, \]

and \( E_k \) is the eigenvalue of \( H_0 + K \).

Assuming the impulse approximation:

\[ \{U, \omega_k^{(4)}\} = 0, \]

we obtain

\[ (E + i\eta - H_0 - V)^{-1} V_k = (\omega_k^{(4)} - 1) \]

\[ + (E + i\eta - H_0 - V)^{-1} (V - V_k) (\omega_k^{(4)} - 1), \]

and by applying (5)' successively (6) becomes as follows:

\[ T^{(4)} = \sum_{k=1}^{A} \sum_{k'=1}^{A} t_k^{(4)} \omega_{k'}^{(4)} - 1 \]

\[ + \sum_{k=1}^{A} \sum_{k'=1}^{A} t_k^{(4)} (\omega_{k'}^{(4)} - 1) (\omega_k^{(4)} - 1) + \ldots \]

In order to obtain the correspondence of the above theory to the optical method, it is necessary to introduce the following approximation:

\[ \{K, V_A\} = 0. \]

With these approximations, (I) and (II), (4) is expressed by

\[ \Phi(r; r_1 \ldots r_A) \equiv \psi(r; r_1 \ldots r_A) \phi_1(r_1 \ldots r_A) \]

\[ = \psi(r) + \sum_{k=1}^{A} (E_k + i\eta - H_0) - V_k^{(4)} (r_k) \phi_k(r) \]

\[ \times \phi_1(r_1 \ldots r_A), \]

(4)' where

\[ \phi_k(r) = \psi(r) + \sum_{k'=1}^{A} (E_k + i\eta - H_0) - V_k^{(4)} \phi_k^{(4)}(r), \]

where \( \phi_k^{(4)}(r) \) is the eigenfunction of \( H_0^{(4)} \) belonging to \( E_k \), the energy of the incident particle, and \( \phi_1(r_1 \ldots r_A) \) is the eigenfunction of \( H_0^{(4)} \) belonging to \( E_1 \), the initial nuclear energy. There is the relation:

\[ E = E_1 + E_k. \]
and

\[ t_k^{(a)} = V_k \phi_k^{(a)} = V_k[1 + (E_k + i\eta - H_0)^{-1}V_k]. \]

Integral eq. (4') is the basic equation of the self-consistent procedure given by Lax\(^2\) for the multiple scattering problem, and is the starting point of the optical method. \((E_k + i\eta - H_0)^{-1} \phi_k^{(a)}\) represents the wave scattered by the \(k\)th nucleon, and \(\phi_k^{(a)}\) the wave incident on the scatterer \(k\).

The particle wave which has been used in the optical method is the absolute coherent wave, which can be represented by:

\[ \langle \phi(r') \rangle = \int \Phi_t^* (r_1 \ldots r_A) \phi(r; r_1 \ldots r_A) \times \Phi_t (r_1 \ldots r_A) dr_1 \ldots dr_A. \]  

It can be shown\(^3\) by solving the eq. (4') with some approximations that the coherent wave obeys the wave equation

\[ [p^2 + k^2 + 4\pi nq(0)] \langle \phi(r') \rangle = 0, \]

\[ f(0) = \frac{Z}{A} f_p(0) + \frac{(A-Z)}{A} f_n(0), \]

where \(n\) represents the average density of nucleons, \(f_p(0)\) and \(f_n(0)\) are the forward scattering amplitudes by the single proton and neutron, and \(c\) is a constant depending on the correlation between pairs of nucleons, and further that the effect of the incoherent scattering, the wave density of which is represented by \(\langle |\psi|^2 \rangle - |\langle \psi \rangle|^2\), gives the nuclear medium an turbidity, i.e., the absorption coefficient \(K\) of the coherent wave. Thus the absolute coherent wave travels in the nuclear medium with the index of refraction \((Rek')/k\), where \((k')^2 = k^2 + 4\pi nq(0)\), and the absorption coefficient \(K\), which agrees with that given by Fernbach, Serber and Taylor\(^1\) provided that the nuclear system can be described in Hartree approximation.

We have seen that the many body problem for the scattering of the high energy particles has been reduced to the optical method by using the two main approximations; (I) the impulse approximation, and (II) which may mean that the change of the nuclear state is neglected.

The details and the discussion of the validity of these approximations, i.e., the applicability of the optical method will be given in the later issue of this journal. The writer wishes to express his thanks to Mr. E. Yamada and Dr. O. Hara for their valuable discussions.