Weak Interactions as Determined by Permutation Symmetry in Iso-Space

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Fierz first discussed the behavior of the five irreducible Lorentz-invariants of four 4-spinors under permutation of the “unbarred” spinors.1) The eigenvectors of this “Fierz permutation” have been emphasized by Caianiello, Michel, Peaslee and others.2) As is well known, the $\alpha$-decay interaction is such an eigenvector, once the ratio of axial-vector to vector coupling constants is set equal to one by neglecting the effect of strong interactions, and it is then essentially equal to the $\alpha$-decay interaction.3)

The requirement of invariance, up to sign, under the Fierz permutation, of course, does not tell one which nucleons and leptons interact weakly with one another, and what the relative coupling strengths are. To make statements along these lines, the present author utilizes the hitherto unused notion of permutation symmetry in three dimensional iso-space.

There are two irreducible $R(3)$ invariants of four 2-spinors of the ordering $(1)(2)(3)(4)$

$$s=\left(\begin{array}{c}
(1);(2);
(3);
(4)
\end{array}\right)$$

and

$$v=\left(\begin{array}{c}
(1);r(2);
(3);r(4)
\end{array}\right).$$

There are also two eigenvectors of $P_{24}$ (or $P_{12}$)

$$v-s$$

and

$$v+3s. \quad (2)$$

With $(2)$ and $(4)$ (or $(1)$ and $(3)$) commuting, and for $V-A$ coupling in space-time, their eigenvalues are $1$ and $-1$, respectively.

Let nucleons and leptons be described

by the iso-doublets

$$N=\left(\begin{array}{c}
\phi
\mu
\end{array}\right), \quad U=\left(\begin{array}{c}
\nu_e
\mu
\end{array}\right), \quad E=\left(\begin{array}{c}
\nu_e
\nu_e
\end{array}\right). \quad (3)$$

Then one has, e.g.

$$(v-s)/2=\bar{N}t\bar{N}E/2-\bar{N}\bar{N}E\nu/2$$

$$=\bar{p}n\bar{v}+\bar{p}p\bar{e}-\bar{\eta}n\bar{\nu}+\bar{\eta}p\bar{\nu}.$$ \hspace{1cm} (4a)

Thus, the weak elastic scatterings $e+p\rightarrow p+e$ and $\nu+\nu\rightarrow\nu+\nu$ are predicted but not $\nu+p\rightarrow p+\nu$ and $e+n\rightarrow n+e$. The relation

$$(v+3s)/2=\bar{p}n\bar{v}+2(\bar{p}p\bar{v}+\bar{\eta}n\bar{\nu})$$

$$+(\bar{p}p\bar{e}+\bar{\eta}n\bar{e})$$

appears less attractive.4)

Replacing $N$ by $U$ in Eq. (4a) gives

$$(v-s)/2=\bar{\nu}_e\mu\bar{\nu}_e-\bar{\mu}\mu\bar{\nu}_e$$

$$-\bar{\mu}\nu_e\bar{e}+\bar{\mu}\nu_e\bar{e}, \quad (4b)$$

which predicts the elastic scatterings $\nu_\mu+e\rightarrow e+\nu_\mu$ and $\nu_\mu+\mu\rightarrow \mu+\nu_e$. But $v-s$ does not lead to $\nu_e+e\rightarrow e+\nu_e$ if $v-e$ and $\nu_\mu$ anti-commute:

$$(v-s)/2=\bar{\nu}_e\bar{e}\bar{\nu}_e+(\bar{\nu}_e\nu_\mu-\bar{\nu}_\mu\nu_\mu-\bar{\nu}_\mu\nu_\mu)/2$$

$$+\bar{e}\nu_e\bar{\nu}_e-(\bar{\nu}_\mu\nu_\mu+\bar{\nu}_\mu\nu_\mu)(\bar{\nu}_\mu\nu_\mu+\bar{\nu}_\mu\nu_\mu)/2$$

$$=0 \text{ if } \bar{\nu}_\mu\nu_\mu\bar{\nu}_e\bar{e}\nu_e, \text{ i.e. for } V-A.$$ \hspace{1cm} (4c)

For commuting $\nu_e$ and $e$,

$$(v-s)/2=4i\nu_e\bar{e}\bar{\nu}_e. \quad (4c')$$

For anti-commuting $e$ and $\nu_e$, $v+3s$ leads to the same amplitude for $\nu_\mu+e\rightarrow e+\nu_\mu$, as Eq. (4c'), but to none if $e$ and $\nu_e$ commute; however, it leads for both commutation rules to $\nu+\nu\rightarrow\nu+\nu$ and to weak $e+e\rightarrow e+e$.

The permutation symmetry in iso-space is an expression of the complete equivalence of the three doublets, (3), in the absence of strong interactions, and results in the selection of certain weakly interacting systems, (4) or (5), from among a greater variety. The equivalence postulated

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here goes, of course, beyond a nucleon lepton analogy, in which nucleon currents and lepton currents are considered to be substitutable for one another.

Of the two eigenvectors in three dimensional iso-space, $v-s$ is from an experimental standpoint preferable. No data indicating an absence of the charge-retention-current interactions predicted in (4a) and (4b) have been published.

The form $v-s$ is actually invariant under the two dimensional unimodular group, leading one to speculate on a generalization of iso-space to four dimensions with indefinite metric. The weak interaction of ordinary particles then assumes the same form in iso-space as in space-time.

Considering $\tau$ and $i\Omega$ as a 4-vector, $\tau_4$, gives rise to a further interesting notion: In the (current) $\times$ (current) picture $\tau_4$ causes non-invariance under time reversal; if its current is coupled to a current which is invariant under time reversal, e.g. a $\Delta I=1/2$ current. Since the same $\Delta I=1/2$ current will be coupled to a $\tau_3$ current, there will be interference.

The possibility of pinning down strangeness changing weak interactions by a permutation symmetry is being considered.

1) M. Fierz, Z. Phys. 104 (1937), 553. The behavior under permutation of the barred spinors is the same.


4) H. Faissner reports an upper limit of 3% for $\sigma(\nu+\rho\to\rho+\nu)$ in terms of $\sigma(\nu+n\to\rho +\nu)$ for $E>250$ Mev, Bull. Am. Phys. Soc. (October 23-24, 1964).

5) The fact that $v-s$ is the scalar product of two 4-vectors was immediately emphasized by Professor A. Coulter in a conversation with the author.

6) Going from 4-spinors $\psi$ to 2-spinors $\varphi=2^{1/2}$ $\times (1+\gamma_5)\psi$ turns $\bar{\psi}\gamma_\sigma (1+\gamma_5)\psi\bar{\psi}\gamma_\sigma (1+\gamma_5)\psi_4$ into $\varphi^\dagger \varphi_4 \varphi \varphi_4$ with $\sigma_4=ii$.

7) The imaginary representation of $\tau_4$ is used to avoid the otherwise necessary distinction of co- and contravariance.

8) T. Ahrens, C. P. Frahm and B. D. Quang, "On the Existence of Weakly Coupled Charge Retention Currents", submitted for publication (April 1965). The notion of permutation in iso-space is not mentioned in that work. However, various cross sections are given there, among which cross sections of the processes are mentioned here.