In a series of papers which appeared in this journal, Tomonaga and his collaborators have developed a new formalism of the quantum field theory in such a manner that it expresses the relativistic invariance of the theory as directly as possible. After giving the general idea of this formalism in the first paper, they have applied it to the quantum electrodynamics of the interacting electron and electromagnetic field, and then to the case of the interacting meson and electromagnetic field. In the latter case a slight generalization of the treatment given in I and II was necessary because of the non-scalar character of the interaction energy density and of the non-commutability of the energy densities at a pair of adjacent (space-like) world points. They have shown in III that one can obtain a characteristic operator to be used in the generalized Schrödinger equation by supplementing the energy density by a suitable term.

The aim of the present paper is to give the new formalism for the case of interacting meson and nucleon fields. Also here appears the same situation as in III. The interaction energy density is not a scalar, and the densities at two adjacent world points do not commute. But also here these difficulties can be settled by introducing a suitably chosen supplementary term into the generalized Schrödinger equation. Because the case of scalar meson field is so simple that it is not necessary to give the detailed description, we deal mainly with the case of vector meson field.
interacting with the nucleon-field, giving only the results for the scalar meson field at the end of the paper.

In § 1 of the present paper we summarize the method of Stiickelberg,\(^\text{(4)}\) who described the vector meson field using a four-potential and a scalar one. This way of describing the vector meson field is more general than the ordinary way, in which only a four-potential is used. In § 2 we introduce the four-dimensional commutation relations for the field quantites, and in § 3 we set up the generalized Schrödinger equation. As described in § 1, it is necessary to introduce an auxiliary condition in the Stiickelberg's method, so that § 4 is devoted to the discussions about this auxiliary condition and its elimination. In § 5 we show that one can go over from our formalism to the ordinary one by taking the variable surface \(C\) parallel to the \(xys\)-plane.

A. The Case of Vector Meson Field.

§ 1. Classical Theory.

To describe the charged vector meson field, we introduce, following Stüeckelberg,\(^\text{(4)}\) a four-potential \(A_i\), one scalar potential \(B\) and their complex conjugates. Further we use \(\varphi_s^*, \varphi_s\) to denote the spinors describing the nucleons. The fundamental tensor used throughout in this paper is

\[ g_{ij} = \delta_{ij} \quad (i, j = 1, 4). \]

For the easier understanding we shall first treat the classical unquantized theory briefly to show the necessity of the auxiliary conditions. According to Stüeckelberg, the Lagrange function is

\[ L = L' + L^2 + L^3 \]

with

\[ L' = -\frac{1}{4\pi} \left\{ (\partial_i A_j^* \partial_i A_j) + x^2 A_j^* A_j + (\partial_i B^* \partial_i B) + x^2 B^* B \right\}, \quad (1.1) \]

\[ L = \varphi^* \left\{ i\hbar \frac{\partial}{\partial T} - c\frac{\hbar}{i} (\vec{\text{grad}}) - (m^2 \tau_3 + m^2 \tau_5) \right\} \varphi, \quad (1.2) \]

and

\[ L^\mu = g_i \left\{ (A_i \cdot \frac{1}{x} \partial_i B, M_i) + (A_i + \frac{1}{x} \partial_i B, M_i^*) \right\} - \frac{g_i^2}{2x} \left\{ (\partial_i A_i^* - \partial_j A_j^*, S_{ij}) + (\partial_i A_i - \partial_j A_j, S_{ij}^*) \right\} - \frac{g_i^2}{2x} S_{ij}^* S_{ij}, \]  

(1.3)

where

\[ M_i = (\varphi^* \tau_{np} \varphi, i \varphi^* \tau_{np} \varphi), \quad M_i^* = (\varphi^* \tau_{np} \varphi, i \varphi^* \tau_{np} \varphi) \]

\[ S_{ij} = (\varphi^* (-i \beta \gamma_i \gamma_j \varphi), i \varphi^* (-i \beta \gamma_i \gamma_j \varphi)) \]

\[ S_{ij}^* = (\varphi^* (-i \beta \gamma_i \gamma_j \varphi), i \varphi^* (-i \beta \gamma_i \gamma_j \varphi)) \]  

(1.4)

\( \tau_{np} (\tau_{np}) \) being the operator changing the nucleon from the proton (neutron) state to the neutron (proton) state, and \( \tau_p (\tau_n) \) being the operator which is 1 in the proton (neutron) state and zero in the neutron (proton) state.

As is well known, ten wave equations for \( A_i, B \) and their conjugates can be derived from the variation principle \( \delta \int L dt = 0 \), where the Lagrangian \( L \) is the space integral \( \int L dt \). We obtain thus

\[ (\partial_i \partial_j - x^2) A_i = -4 \pi g_i M_i - 4 \pi \frac{g_i^2}{x} \partial_i S_{ij}, \]

\[ (\partial_i \partial_j - x^2) B = 4 \pi \frac{g_i^2}{x} \partial_i M_i. \]  

(1.5)

Now the Yukawa equations\(^{(5)}\) can be derived from these equations, introducing the tensor \( \chi_{ij} \) and vector \( \phi_i \) by mesons of

\[ \phi_i = A_i + \frac{1}{x} \partial_i B, \]  

(1.6a)

\[ \chi_{ij} = \partial_i A_j - \partial_j A_i, \]  

(1.6b)

and imposing the auxiliary condition

\[ \partial_i A_i + xB = 0. \]  

(1.7)

They are

\[ \partial_i \chi_{ij} - x^2 \phi_j = \partial_i (\partial_i A_j - \partial_j A_i) - x^2 (A_j + \frac{1}{x} \partial_j B) \]

\[ = (\partial_i \partial_i - x^2) A_j - \partial_j (\partial_i A_i + xB) = -4 \pi g_i M_j - 4 \pi \frac{g_i^2}{x} \partial_i S_{ij}. \]  

(1.8)

This condition $\partial_t A_t + \mu B = 0$ will hold at any time, if we assume that, at $t=0$, $\partial_t A_t + \mu B = 0$, and $\partial_t(\partial_t A_t + \mu B) = 0$. This is because $\Box - \mu^2(\partial_t A_t + \mu B) = 0$ follows from (1.5). The auxiliary conditions are thus introduced, in order that (1.5) and (1.7) be equivalent to (1.6b) and (1.8). But these conditions also guarantee that the Hamiltonian for free meson field becomes positive definite.


The variables which are canonically conjugate to $A_t, B$ and their complex conjugates, can be defined in the usual way by the relations:

$$
\Pi_t = \frac{\delta L}{\delta \partial_t A_t} = -\frac{i}{4\pi c} \partial_t A_t^* - i \frac{\xi}{c} S_{A_t}^* ,
$$

$$
\Pi_B = \frac{\delta L}{\delta \partial_t B} = \frac{i}{4\pi c} \partial_t B^* - i \frac{\xi}{c} M_{\mu}^* ,
$$

$$
\Pi_{\phi} = i \delta \phi^* ,
$$

(2.1)

The total Hamilton function becomes then in terms of these canonical variables

$$
H = (\Pi_t \frac{\partial A_t}{\partial T} + \Pi_B \frac{\partial B}{\partial T} + \text{conj}) + \Pi_{\phi} \frac{\partial \phi}{\partial T} - L
$$

$$
= H^I + H^\mu + H^\phi
$$

with

$$
H^I = 4\pi c^2 \Pi_t^* \Pi_t + 4\pi c^2 \Pi_t^* \Pi_B + \frac{1}{4\pi} \left( \sum_{\alpha=1}^3 \partial_\alpha A^*_\alpha \partial_\alpha A^*_4 + \mu^2 A^*_4 A^*_4 \right)
$$

$$
+ \frac{1}{4\pi} \left( \sum_{\alpha=1}^3 \partial_\alpha B^*_\alpha \partial_\alpha B^*_4 + \mu^2 B^*_4 B^*_4 \right) ,
$$

(2.2)

$$
H^\phi = e^{\phi} \left\{ \frac{\partial \phi}{\partial t} \sum_{j=1}^4 (\alpha_j \partial_\alpha + m^*_\beta \tau_\beta + m^*_\gamma \tau_\gamma) \phi \right\} ,
$$

(2.3)

and
The canonical variables $A_i \Pi_n$ and $B_{i\pi}$ satisfy the following relations

$$[\Pi_n(x'), A_i(x)] = \frac{\hbar}{i} \delta_{ij} \delta(x-x'),$$
$$[\Pi_n(x'), B(x)] = \frac{\hbar}{i} \delta_{ij} \delta(x-x'),$$
$$[\varphi_i^*(x') \varphi_i(x) + \varphi_i(x) \varphi_i^*(x')] = \delta_{ij} \delta(x-x'). \quad (2.5)$$

Now, in the Super-many-time theory, the field variables $A(x)$ etc. are to be transformed into $A(X) = U A(x) U^{-1}$ by the canonical transformation

$$U = \exp \left\{ \frac{i}{\hbar} (\overline{H'} + \overline{H^*}) t \right\}, \quad (2.6)$$

and they depend on the field time explicitly. Then there appear the following four-dimensional commutation relations in our theory, which can readily be derived from (2.5) and (2.6): 

$$[A_i^*(X'), A_j(X)] = \frac{4\pi}{i} \epsilon_{ijk} \partial_k D(X'-X),$$
$$[B_i^*(X'), B_j(X)] = \frac{4\pi}{i} \epsilon_{ijk} \partial_k D(X'-X),$$
$$[\varphi_i^*(X'), \varphi_j(X)] = \tau_N W_i^N(X-X') + \tau_N W_i^N(X-X') \quad (2.7)$$

with $W_i^N(X-X') = i \partial_i - u \partial_x - i x_N \partial_{x'} \delta^N(X-X')$, $D(X-X')$ and $D^N(X-X')$ representing the four dimensional $\delta$-function for the meson field and the nucleon field respectively.

(6) W. Pauli: Solvay Berichte (1938).
§ 3. Derivation of the generalized Schrödinger Equation.

Let the field time be denoted by the small letter \( t \), and the common time, by the large letter \( T \). To distinguish the differentiation with respect to common time and field time, we use \( \partial \) for the former and \( \partial / \partial x_i \) for the latter. Of course, \( \partial / \partial x_i = \partial_i \) for \( i = 1, 2, 3 \) (see (5.8)). In the preceding paragraph we see that the commutation relations have been represented in a relativistically invariant form. Our next task is to build up the relativistically generalized Schrödinger equation as has been put forward in I. But in the present case the situation is not so simple as in the case of interacting electromagnetic and electron fields, because our Lagrange function contains the time derivatives of the field variables in the interaction part \( L^I \) in (1.3). Similar circumstance has appeared in the case of the interaction between mesonic and electromagnetic fields treated by Kanesawa and Tomonaga.

It will be shown that, adopting the same procedure as in III, the generalized Schrödinger equation will be obtained also in our case. At the first step we put

\[
\Pi_i = \frac{i}{4\pi c} \frac{\partial A_i^*}{\partial x_i}, \quad \Pi_\mu = \frac{i}{4\pi c} \frac{\partial B^*}{\partial x_\mu}, \quad (3.1)
\]

as in III. Then the interaction energy \( H^I \) in (2.4) takes the following form:

\[
H^I = -g_i \left\{ (A_i^* + \frac{1}{x} \frac{\partial}{\partial x_i} B^*, M_i) + (A_i + \frac{1}{x} \frac{\partial}{\partial x_i} B, M_i^*) \right\} = G_1
\]

\[
+ \frac{g_2}{2x} \left\{ S_{ij} \chi_j^* + S_{ij} \chi_j \right\} = G_2 \quad (3.2)
\]

\[
+ \frac{4\pi}{x^2} g_3 \frac{S_{ij} S_{ij}^* + S_{ij} S_{ij}}{4} = G_3
\]

where, for simplicity, we have put \( \chi_j = (\partial A_j / \partial x_i - \partial A_i / \partial x_j) \), and have dropped the terms \( \sum_{a, \beta=1}^{3} S_{a\beta} S_{\alpha\beta} \) and \( M_i^* M_i \).

The consideration in III suggests that the generalized Hamilton function to be used in the generalized Schrödinger equation can be obtained by adding to \( H^I(P) \) a supplementary term, the functional \( \mathcal{A}[P, C] \) which
depends on the independent variable surface $C$ explicitly. The form of this term will be fixed in order that the generalized Schrödinger equation becomes integrable. This integrability condition requires

$$\left[H(P') + \mathcal{A}[P', C] - i\hbar \frac{\delta}{\delta C'}, H(P) + \mathcal{A}[P, C] - i\hbar \frac{\delta}{\delta C}\right] = 0 \quad (3.3)$$

to hold for any two points $P$ and $P'$ on the surface $C$. Now (3.3) will be satisfied, if $\mathcal{A}(P, C)$ can be fixed in such a manner that it satisfies the "rotational equations",

$$[H', H] - i\hbar \left(\frac{\delta \mathcal{A}[P, C]}{\delta C'} - \frac{\delta \mathcal{A}[P', C]}{\delta C}\right) = 0, \quad (3.4)$$

and

$$[\mathcal{A}[P, C], H'] + [\mathcal{A}[P', C], H] = 0 \quad (3.5)$$

simultaneously.

To begin with, we must compute $[H', H]$. From (3.2) we obtain

$$[H', H] = [G_1', G_1] + [G_1', G_2] + [G_1', G_3] + [G_2', G_1] + [G_2', G_2] + [G_2', G_3] + [G_3', G_1] + [G_3', G_2] + [G_3', G_3]. \quad (3.6)$$

We now calculate $[G_1', G_1]$:

$$[G_1', G_1] = \frac{e^2}{\pi} \frac{4\pi}{\hbar} \int d^3 \hat{x} \left\{ M_i' M_j' \frac{\partial^2}{\partial x_i' \partial x_j} D(X' - X) + M_i' M_j \frac{\partial^2}{\partial x_i \partial x_j} D(X - X') \right\}$$

$$+ \mathcal{G}_i' \phi_i' \phi_j' [M_i' M_j'] + \phi_i' \phi_j' [M_i' M_j'], \quad (3.7)$$

where we put $\phi_i = A_1 + (1/x)(\partial B/\partial x_i)$. In (3.7) we drop the terms with $D(X' - X)$, since $D$ function vanishes, so long as $P$ and $P'$ lie on a space-like surface. We shall show that in (3.7) the second row vanishes: According to (2.7), it is

$$\mathcal{G}_i' \phi_i' \phi_j' [\varphi'(u_{\gamma' \gamma} (W_{\gamma \gamma} W_{\gamma' \gamma}) \gamma_{\gamma' \gamma} \varphi') - \varphi' (u_{\gamma' \gamma} (W_{\gamma' \gamma} W_{\gamma' \gamma}) \gamma_{\gamma' \gamma} \varphi')$$

$$+ \phi_i' \phi_j' [\varphi'(u_{\gamma' \gamma} (W_{\gamma' \gamma} W_{\gamma' \gamma}) \gamma_{\gamma' \gamma} \varphi') - \varphi' (u_{\gamma' \gamma} (W_{\gamma' \gamma} W_{\gamma' \gamma}) \gamma_{\gamma' \gamma} \varphi')] \quad (3.8)$$

Here it is allowed to substitute $W_{\gamma' \gamma} + W_{\gamma' \gamma} \delta(x - x')$, when we refer
to the Lorentz-system tangent to $C$ at $P$ with its time axis parallel to the normal. Utilizing the property of $\delta(x-x')$:

$$f(x') \delta(x-x') = f(x) \delta(x-x'), \quad (3.9)$$

(3.8) becomes

$$f(x) \delta(x-x') \left( \partial^2/\partial x_i \partial x_j \right)D(x'-X) = 0.$$

For the later purpose, (3.7) can be written in a slightly different form:

$$[G_i, G_j] = \frac{\kappa^2}{4\pi} \frac{4\pi \hbar}{c} \left\{ \frac{M_i M_j^* + M_j M_i^*}{2} \partial^2 \partial x_i \partial x_j \right\} D(X-X').$$

We proceed to the calculation of the further terms in (3.6). As is evident from the proof in (3.8), we are allowed to treat $M_{\alpha}$, $S_{ij}$, and their conjugates as if they were $\epsilon$-numbers, so long as we are concerned with the calculation of $[H', H]$. Because, if there appear in $[H', H]$ terms with the $\delta$-function, they should be of the form $f(x', x) \delta(x'-x)$, $f(x', x)$ being antisymmetric with respect to $x'$ and $x$, since $[H', H]$ is also antisymmetric. Therefore such terms must vanish, because of $f(x', x) \delta(x'-x) = f(x, x) \delta(x'-x) = 0$. We shall thus drop off entirely such terms as $[M_i^* M_j]$ etc., and $[G_i', G_j]$ + $[G_i, G_j]$, $[G_i', G_j']$ + $[G_i, G_j']$, $[G_i', G_j']$, $[G_i, G_j']$ must vanish. Further, it can easily be verified that $[G_i', G_j] + [G_i', G_j] = 0$, so that there remains only the term $[G_i', G_j]$. Analogous to (3.7) we obtain for this term:

$$[G_i', G_j] = \frac{\kappa^2}{4\pi} \frac{4\pi \hbar}{c} \left\{ \frac{S_{i\alpha} S_{\alpha} + S_{\alpha} S_{i\alpha}}{2} \partial^2 \partial x_i \partial x_j \right\} D(X-X').$$

using the formula $[\chi_{i\alpha}', \chi_{\alpha}'] = \frac{4\pi \hbar}{i} \left( \frac{\partial^2}{\partial x_i \partial x_j} \delta_{i\alpha} + \frac{\partial^2}{\partial x_j \partial x_i} \delta_{i\alpha} - \frac{\partial^2}{\partial x_i \partial x_{\alpha}} \delta_{j\alpha} \right) D(X'-X).$
Thus from (3.7') and (3.10), the equation (3.4) becomes
\[
\frac{g^2}{\pi^2} \frac{4\pi\hbar}{i} \left\{ \frac{M_i M_i^* + M_j M_j^*}{2} \frac{\partial^2}{\partial x_i \partial x_j} D(X - X') - \frac{M_i^* M_i + M_j^* M_j}{2} \frac{\partial}{\partial x_i} D(X' - X) \right\}
+ \frac{g^2}{4} \frac{4\pi}{i} \left\{ \frac{S_{\alpha\beta} S_{\alpha\beta}}{2} \frac{\partial}{\partial x_i} D(X - X') - \frac{S_{\alpha\beta} S_{\alpha\beta}}{2} \frac{\partial}{\partial x_i} D(X' - X) \right\}
- \frac{i}{\hbar} \left( \frac{\partial A[P, C]}{\partial C'} - \frac{\partial A[P', C]}{\partial C} \right) = 0. \tag{3.11}
\]

Now (3.11) will be satisfied, if we put
\[
A[P, C] = \frac{4\pi}{\pi^2} \left\{ \frac{g^2}{2} \left[ (M_i^* N_i) (M_j N_j) + (M_i N_i) (M_j^* N_j) \right] + \frac{g^2}{2} \left[ (S_{\alpha\beta} S_{\alpha\beta} N_{\alpha\beta} N_{\alpha\beta}) \right] \right\}
\]
where \( N_i \) means the unit vector \( (N^2 = -1) \) parallel to the normal to the surface \( C \) at \( P \). It can readily be seen that \( A[P, C] \) also satisfies the condition (3.5) (see the discussions under the equation (3.1')). In this way we have succeeded in constructing the generalized Schrödinger equation:
\[
\left[ -\frac{\hbar^2}{2m} \left\{ \left( A_i^* + \frac{1}{x} \frac{\partial}{\partial x_i} B_i, M_i \right) + \left( A_i + \frac{1}{x} \frac{\partial}{\partial x_i} B_i, M_i^* \right) \right\}
+ \frac{4\pi}{\pi^2} \frac{g^2}{2} \left[ \left( \frac{\partial A_i^*}{\partial x_i} - \frac{\partial A_i}{\partial x_i}, S_{ij} \right) + \left( \frac{\partial A_i}{\partial x_i} - \frac{\partial A_i^*}{\partial x_i}, S_{ij} \right) \right]
+ \frac{4\pi}{\pi^2} \frac{g^2}{2} \left( S_{\alpha\beta} S_{\alpha\beta} + S_{\alpha\beta} S_{\alpha\beta} \right)
+ \frac{4\pi}{\pi^2} \left\{ \frac{g^2}{2} (M_i M_j N_{ij} + M_i M_j^* N_{ij}) + \frac{g^2}{2} (S_{\alpha\beta} S_{\alpha\beta} N_{\alpha\beta} + S_{\alpha\beta} S_{\alpha\beta} N_{\alpha\beta}) \right\}
- i\hbar \frac{\delta}{\delta C} \right] \psi = 0. \tag{3.13}
\]

It is to be noticed that we are allowed to remove the term \((4\pi/\pi^2) \cdot g^2 (S_{ij} S_{ij} + S_{ij} S_{ij})/4\) without contradicting with the integrability condition.

As is explained in the paragraph § 1, it is necessary in Stückelberg's formalism to put

\[ \frac{\partial A_i}{\partial x_i} + x B = 0 \quad \text{and} \quad \frac{\partial A_i^*}{\partial x_i} + x B^* = 0, \tag{4.1} \]

in order to describe the meson field correctly. But, as is well known, (4.1) cannot be considered as a relation between field variables in the quantum theory, for (4.1) is inconsistent with the commutation relations (2.7). Therefore, (4.1) should be regarded as the condition imposed on the Schrödinger functional \( \Psi \), analogous to the Lorentz condition in the quantum electromagnetic field:

\[ (\frac{\partial A_i}{\partial x_i} + x B)\Psi = 0 \quad \text{and} \quad (\frac{\partial A_i^*}{\partial x_i} + x B^*)\Psi = 0 \tag{4.2} \]

In order that there may exist \( \Psi \), which satisfies (4.2) and (3.13) simultaneously, the auxiliary conditions must commute with each other and with \( (H - i\hbar \frac{\partial}{\partial C}) \). These facts can be easily verified as follows:

\[ \left[ \frac{\partial}{\partial x_i} A_i^* + x B^*, \frac{\partial}{\partial x_j} A_j + x B \right] = \frac{4\pi}{i} \hbar \left( \frac{\partial^2}{\partial x_i \partial x_j} + x^2 \right) D(X' - X) = 0 \tag{4.3} \]

\[ \left[ \frac{\partial}{\partial x_i} A_i^* + x B^*, A_j + \frac{1}{x} \frac{\partial}{\partial x_j} B \right] = \frac{4\pi}{i} \hbar \left( \frac{\partial^2}{\partial x_i \partial x_j} + \frac{\partial}{\partial x_j} \right) D(X' - X) = 0 \tag{4.4} \]

\[ = \frac{4\pi}{i} \hbar \left[ \frac{\partial^2}{\partial x_i \partial x_j} - \frac{\partial}{\partial x_i} \frac{\partial}{\partial x_j} \right] D(X' - X) = 0 \tag{4.5} \]

It is obvious from (4.4) and (4.5) that the auxiliary conditions commute with \( (H - i\hbar \frac{\partial}{\partial C}) \).

Here we shall give a few remarks on the elimination of the auxiliary condition, which is in our case trivial in contrast to that of the Lorentz condition in the electromagnetic field.

Since \( A_j(X), A_i^*(X) \) etc. satisfy the second order wave equations

\[ \left( \frac{\partial^2}{\partial x_i \partial x_j} - x^2 \right) A_i(X) = 0, \tag{4.6} \]
they can be expanded in the Fourier series of the following form. (6)

\[ A_+^*(X) = \frac{1}{\sqrt{\nu}} \sum \sqrt{\frac{2\pi \hbar k}{k_0}} \left\{ A_+^*(k) e^{i(kx)} + A_-^*(k) e^{-i(kx)} \right\}, \]

\[ A_+^+(X) = \frac{1}{\sqrt{\nu}} \sum \sqrt{\frac{2\pi \hbar k}{k_0}} \left\{ A_+^+(k) e^{i(kx)} + A_-^-(k) e^{-i(kx)} \right\}. \]

(4.7)

With the help of (4.7), (4.2) can be expressed in the momentum space as follows:

\[ [i(KA_+) + xB_+] \Psi = 0 \]

\[ [i(KA_-) + xB_-] \Psi = 0, \]  

(4.8)

where \( A_+ \) etc. are substituted for \( A_+^+(k) \) for simplicity.

Since

\[ \left[ A_+(X) + \frac{1}{x} \frac{\partial}{\partial x_i} B(X) \right] \Psi = \frac{1}{\sqrt{\nu}} \sum \sqrt{\frac{2\pi \hbar c}{k_0}} \times \]

\[ \times \left[ \left( A_+ + \frac{iK_+}{x} B_+ \right) e^{i(kx)} + \text{conj} \right] \Psi, \]

and

\[ A_{+i} = -\frac{1}{x^2} (KA_+) K_i + (LA_+) L_i + (MA_+) M_i + (NA_+) N_i, \]

\( L, M, N \) being the space-like unit vectors perpendicular to one another, and to \( K \), we obtain

\[ (A_+ + \frac{iK_+}{x} B_+) \Psi = [(LA_+) L_i + (MA_+) M_i + (NA_+) N_i] \Psi \]

(4.9)

using (4.8). Thus it is to be noticed that, by eliminating the auxiliary condition, there appear no more the superfluous \( (KA_+) \) and \( B_+ \) etc., and remain the three space-like components \( (LA_+), (MA_+), \) and \( (NA_+) \), which is the characteristic feature of the field with spin 1. If we put \( L = \left( \frac{k}{k}, \frac{k_0}{x}, \frac{k}{x} \right), M = (e_i, 0), N = (e_i, 0), e_i, e_2 \) being perpendicular to each other and to \( k \), we call \( L, M, \) and \( N \) longitudinal and transversal respectively. Of
course the concept of longitudinal and transversal wave is not relativistically invariant. It is important to point out that the purpose of introducing the superfluous function $B(X)$ is merely the means of eliminating $(KA_+)$ and $(KA_-)$ which contribute negative to the energy for free meson field.

Further it is to be added that there is an alternative way in formulating the theory: that is, by putting

$$\phi_i = A_i + \frac{1}{x} \frac{\partial}{\partial x_i} B, \quad \chi_i = \frac{\partial \phi_i}{\partial x_i} - \frac{\partial \psi_i}{\partial x_i}, \quad (4.10)$$

by giving $\phi_i$ the commutation relations $[\phi_i^*, \phi_j] = (4\pi\mathcal{A}/i) (\partial_x + (1/x^2) \cdot (\partial^2/\partial x_i/\partial x_j)) D(X' - X)$ and by introducing the identity $\frac{\partial \phi_i}{\partial x_i} = \partial \phi_i^* / \partial x_i = 0$ in place of our auxiliary conditions imposed on the Schrödinger functional. We have refrained from doing so, because this scheme of formulation cannot explain the reason why $\phi_i$ is different from $\phi_i^*$ in (1.6a) or (5.6).

§5. Derivation of the Ordinary Formalism.*

It is now to be verified that the same equations as (1.6b) and (1.8) can be derived, when proceeding from the Schrödinger picture to the Heisenberg one. In the Heisenberg picture the dynamical variable $A$ can be expressed as the function of time in the following form,

$$\hat{A}(X) = T^{-1}[C, C_0] A(X) T[C, C_0], \quad (5.1)$$

$\hat{A}(X)$, $(A(X))$ being the operator in the Heisenberg (Schrödinger) picture. $T[C, C_0]$ is defined by

$$T = e^{\frac{i}{\hbar}(1 - \frac{i}{\hbar} H^m d\omega)} , \quad (5.2)$$

which is constructed by piling up the infinitesimal unitary transformation $(1 - (i/\hbar) H^{m} d\omega)$ step by step from $C_0$ to $C$ without losing the space-like shape of the surface. Let $C$ be the arbitrary space-like surface crossing the world point $P$, and $C_0$ the fixed space-like surface on which the observations have been made before.

It must be remembered that, though $C$ is arbitrary, $\hat{A}(X)$ is completely determined as the function of $X$, since in $T[C, C_0]$ only the domain lying

---

* The author is much indebted to Mr. Z. Koba for completing this section.
within the past side of the light cone with vertex at $P$, contributes to $\hat{A}(X)$.

Let us consider the differential with respect to the common time. Since

$$\Delta A(\omega, x_4) = T^{-1} \left( 1 + \frac{i}{\hbar} H d\omega \right) A(\omega, x_4 + \Delta x_4) \left( 1 - \frac{i}{\hbar} H d\omega \right) T^{-1} A(\omega, x_4) T$$

$$= T^{-1} [A(\omega, x_4 + \Delta x_4) - A(\omega, x_4)] T + \frac{i}{\hbar} T^{-1} [H, A(\omega, x_4)] T,$$

we obtain

$$\partial_4 \hat{A}(X) = T^{-1} \frac{\partial A(X)}{\partial x_4} T + \frac{1}{\hbar c} [\hat{H}, \hat{A}(X)]$$

$$= \frac{\partial A(X)}{\partial x_4} + \frac{1}{\hbar c} [\hat{H}, \hat{A}(X)], \quad (5.3)$$

$$(\partial A(X)/\partial x_4)$$ standing for $(T^{-1}(\partial A(X)/\partial x_4)) T$. It is to be noticed that, if we require the time-like infinitesimal translation, we must choose such a surface whose normal at $P$ lies in the direction of the required time-like one, and must calculate according to (5.3) by piling up $d\omega$ on $C$. In (5.3) $\partial_4$ means the so called differential with respect to common time, $\partial/\partial x_4$, the differential with respect to the field time, and the second term on the right-hand side is equal to the sum of the local time differentials near $P$.

Using (5.3) and (3.13), we obtain

$$\partial_4 \hat{A}_t = \frac{\partial \hat{A}_t}{\partial x_4} + \frac{1}{\hbar c} \frac{g_2}{2x} \frac{4 \pi \hbar}{i} \int d\omega' \, S_{\omega\omega'} \left( \frac{\partial}{\partial x_4} \frac{\partial}{\partial x_4'} - \frac{\partial}{\partial x_4} \frac{\partial}{\partial x_4'} \right) D(X' - X)$$

$$= \frac{\partial \hat{A}_t}{\partial x_4} + \frac{4 \pi \hbar g_2}{x} S_{\omega\omega'} \quad (5.4)$$

since $\partial/\partial x_4' D(X' - X) = 1/i \partial (x' - x)$, $\partial/\partial x_4' D(X' - X) = 0$ for $i = 1, 2, 3$. In (5.4) it is to be remembered that the commutation relations will not change by the unitary transformation $T$; that is, between $\hat{A}$ etc., the same commutation relations will hold as between $A$. Next, we obtain similarly

$$\partial_4 \hat{B} = \frac{\partial \hat{B}}{\partial x_4} + \frac{4 \pi \hbar g_3}{x} \hat{M}, \quad (5.4)$$
and

$$\partial_i \phi_i = -4\pi g \frac{x_i}{x} \partial_j S_{ij} - 4\pi g_i M_i$$

Therefore,

$$(\partial_\eta - x^i) \phi_i = -4\pi g_i M_i - \frac{4\pi}{x} g_\eta \partial_j S_{ij}$$

(5.5)

using $$(\partial^2/\partial x_i^2) \phi_i = (x^2 - \sum_{j=1}^3 \partial_j \partial_j) \phi_i$$. Also

$$(\partial_\eta - x^i) B = \frac{4\pi}{x} g_i \partial_i M_i$$

(5.5'')

According (1.6a) and (1.6b), we shall introduce $\phi_i$ by the following relations:

$$\phi_i = A_i + \frac{1}{x} \partial_i B_i$$

(5.6)

$$\chi_{ij} = \partial_i A_j - \partial_j A_i = \partial_i \phi_j - \partial_j \phi_i$$

(5.7)

Then, with the help of (5.5)

$$(\partial_\eta - x^i) \phi_i \mathcal{T}_0 = \left\{ \partial_i (\partial_i A_j - \partial_i A_j) - x^i (A_j + \frac{1}{x} \partial_j B_j) \right\} \mathcal{T}_0$$

$$= \left\{ (\partial_\eta - x^i) A_j - \partial_j (\partial_i A_i + xB) \right\} \mathcal{T}_0$$

$$= (-4\pi g_i M_i - \frac{4\pi}{x} g_\eta \partial_j S_{ij}) \mathcal{T}_0$$

$$- \partial_j (\partial_i A_i + xB) \mathcal{T}_0$$

$\mathcal{T}_0$ being defined by $\mathcal{T} = T \mathcal{T}_0$. It will be easily verified that $\partial_j (\partial_i A_i + xB) \mathcal{T}_0$ will vanish. In the first place we can put

$$\partial_i A_i = \frac{\partial A_i}{\partial x_i}$$

for $i = 1, 2, 3$ (5.8)

because $A_i = T^{-1}(x_i + \Delta x_i, x_i) T - T^{-1}(x_i, x_i) T$, or $\partial_i A_i = T^{-1}(\partial A(x_i, x_i)/\partial x_i) T = \partial A_i/\partial x_i$, since, by the space-like infinitesimal transformation, $T$ will not suffer any change in contrast to the time-like one (see (5.3)). Therefore,
\[ \partial_t A_i = \sum_{i=1}^{3} \frac{\partial A_i^0}{\partial x_i} + \frac{4\pi}{x} g_s S_{ij} \frac{\partial A_j^0}{\partial x_i}, \quad (5.9) \]

and also
\[ \partial_j (\partial_A A_i + x B^0) = \frac{\partial}{\partial x_j} \left( \frac{\partial A_i^0}{\partial x_i} + x B^0 \right) \quad \text{for } j = 1, 2, 3 \]
\[ \partial_i (\partial_A A_i^0 + x B^0) = \partial_i \left( \frac{\partial A_i^0}{\partial x_i} + x B^0 \right) \]
\[ = \frac{\partial}{\partial x_i} \left( \frac{\partial A_i^0}{\partial x_i} + x B^0 \right) + \frac{1}{\hbar c} \int df \left[ \frac{\partial}{\partial x_i} \left( \frac{\partial A_i^0}{\partial x_i} + x B^0 \right) \right] \]
\[ = \frac{\partial}{\partial x_i} \left( \frac{\partial A_i^0}{\partial x_i} + x B^0 \right) \]

since auxiliary conditions commute with \( H \): Now,
\[ \partial_j (\partial_A A_i + x B) = \partial_j \left( \frac{\partial A_i^0}{\partial x_i} + x B^0 \right) \]
\[ = T^{-1} \left( \frac{\partial}{\partial x_i} \left( \frac{\partial A_i^0}{\partial x_i} + x B^0 \right) \right) \]
\[ \Psi = 0, \]

using (4.2). Then we obtain
\[ (\partial_k q_j - x_k q_j)^0 \Psi = \left( -4\pi g_1^2 M_j - \frac{4\pi}{x} g_s \partial_j S_{ij} \right) \Psi. \quad (5.10) \]

(5.7) and (5.10) are the well known Yukawa equations.\(^6\)

It is to be added that \( \hat{\phi} \) (in (5.6)) = \( T^{-1} \hat{\phi} T \) (in (4.10)) + \( (4\pi/x) g_1 M_j \)
(see the discussions at the end of § 4).

Next we shall derive the equations for nucleons:
\[ -\hbar c \partial \varphi^0 = -\hbar c \frac{\partial \varphi^0}{\partial x_4} - \int df'[H', \varphi] \]
\[ = \left[ c \frac{\hbar}{\imath} (\vec{\alpha} \text{ grad}) + \beta (m_e^2 \tau_w + m_e^2 \tau_p) \right. \]
\[ - g_1 \left\{ \left( A_i^0 + \frac{1}{x} \frac{\partial B^0}{\partial x_i}, a_i \right) \tau_{lw} + \text{conj} \right\} \]
\[ + \frac{g_2}{2\pi} \left\{ \left( \frac{\partial A_j^0}{\partial x_j}, \gamma_{ij} \right) \tau_{lw} + \text{conj} \right\} ] \]
On the Interaction of the Meson and Nucleon

As has been pointed out in (3.13), it is allowed to drop off

$$B.$$ 

The Case of Scalar Meson Field. This case is very simple. We shall give the results only. The generalized Schrödinger equation is as follows:

$$[\frac{\hbar}{i}(\phi, \partial_t) + \beta (m u^2 \tau_N + m u^2 \tau_p) - \delta_1 \phi, \phi^0 \tau_{pN} + \text{conj}]$$

$$+ \frac{4\pi}{2\hbar} \delta_2^z \{ (S_{ij} \phi^0 \tau_{pN} + (S_{ij} \phi^0 \tau_{Np}) \phi = 0 \ (5.11)$$

As has been pointed out in (3.13), it is allowed to drop off

$$\frac{4\pi}{2\hbar} \delta_2^z \{ (S_{ij} \phi^0 \tau_{pN} + (S_{ij} \phi^0 \tau_{Np})$$

B. The Case of Scalar Meson Field.

This case is very simple. We shall give the results only. The generalized Schrödinger equation is as follows:

$$[\frac{\hbar}{i}(W^* \phi + \phi^* W) + \frac{f_2}{x} \left\{ \left( \frac{\partial \phi^*}{\partial x_i} M_i \right) + \left( M_i^* \frac{\partial \phi^*}{\partial x_i} \right) \right\}$$

$$+ \frac{4\pi}{x^2 \hbar} \delta_2^z \left( M_i^* M_j^* N_i N_j + M_i^* M_j N_i^* N_j \right) - i\hbar \delta \left[ \phi^* \phi \right] \phi = 0$$

with the commutation relation

$$[\phi^* (x'), \phi (x)] = \frac{4\pi \hbar}{i} D(x' - x).$$

$W$ is defined by $W = \phi^* \rho_3 \tau_{pN} \phi.$
Acknowledgements.

This paper was accomplished under the guidance of Professor S. Tomonaga. The author wishes to express his cordial thanks to Professor S. Tomonaga for his kind guidance and encouragements. He is also much indebted to Mr. Z. Koba for his valuable discussions.