The $\tau^+$-decay and the Boson Isobar with $I=2$ of the Sakata Model

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Some analyses of the single pion production in $\pi^-N$ collision\(^1\) have been made to find the evidence of the boson isobars\(^2\) of the Sakata model. From these analyses it has been suggested that the boson isobar with $I=2$, i.e. $B_4^1(0,2)$ of the Sakata model, exists and has the $2\pi$-decay mode.

The $B_4^1(0,2)$ isobar is, then, expected to have the effect to the pionic decay of kaon. Especially the $3\pi$ decay process of kaon is very interesting since the present experimental data of pion energy spectra in $\tau^+$-decay process\(^3\) show a small deviation from the theoretical spectrum for spinless kaon given by Dalitz and Fabri.\(^4\) (See Figs. 1 and 2.)

Some studies have already been made to understand the deviation as the effect due to the possible strong interaction among the final pions.\(^5\)

In this short note we also study the problem by taking the effect of $B_4^1(0,2)$ which can be corresponded to the strong $\pi\pi$ interaction, but our approach is a little different from these analyses.

Now let us assume the following for the moment.

1) The $\tau^+$ and $\tau^{+'}$ decay mainly consist of the following cascade decay.

\[
K^+ \rightarrow B_4^1(0,2) + \pi \text{ (recoil pion)},
\]

\[\rightarrow 2\pi \text{ (decay pions).} \]

2) $B_4^1(0,2)$ is $0^+$ and can be treated in the calculation as though it is a metastable particle, except that it has not a definite mass, but mass distribution $\rho(m_B)$.

3) The decay interaction for
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$K^+ \rightarrow B_1^0(0, 2) + \pi$ and $K_1^0 \rightarrow 2\pi$ are effectively of the type

$g\phi_n\phi_\pi \phi_K$ and $g'\phi_n\phi_\pi\phi_K$ (2)

with $g \sim g'$ (Universality of weak interaction).

The theoretical energy spectra of the final pions in $\tau^+$-decay and $\tau^+\tau^-$-decay through the decay scheme (1) will be given by the following expressions when the $dI=1/2$ rule holds for these processes:

$\pi^-$-spectrum in $\tau^+$-decay

$I^-(E_\pi) dE_\pi \propto [1/30 I_1(E_\pi)
+ 18/30 I_2(E_\pi)] dE_\pi$, (3a)

$\pi^+$-spectrum in $\tau^+$-decay

$I^+(E_\pi) dE_\pi \propto 2[18/30 I_1(E_\pi)
+ 1/30 I_2(E_\pi)] dE_\pi$, (3b)

$\pi^+$-spectrum in $\tau^+\tau^-$-decay

$I^+(E_\pi) dE_\pi \propto [9/30 I_1(E_\pi)
+ 2/30 I_2(E_\pi)] dE_\pi$, (3c)

$I_1(E_\pi)$ and $I_2(E_\pi)$ are the energy distribution function of the decay pions and the recoil pions respectively and expressed by

$I_1(E_\pi) dE_\pi \equiv m_\pi^{\text{max}}$

$= N_1 \int_{m_\pi^{\text{min}}}^{m_\pi^{\text{max}}} dM_B \rho(m_B) m_B / p_{\pi B} dE_\pi$, (4)

$I_2(E_\pi) dE_\pi = N_2 \rho(m_B) m_K p_B / m_B dE_\pi$,

where $m_K$ and $m_B$ are the mass of the kaon and $B_1^0(0, 2)$ respectively and $p_{\pi B} = \sqrt{(m_B/2)^2 - m_\pi^2}$ (for further notations, see reference 1)).

The dominant factor for the pion energy spectra is the mass distribution of $B_1^0(0, 2); \rho(m_B)$. However, we know little about it now. Some information will be obtained from the consideration of the decay rate of kaons. From the decay interactions (2) we shall obtain

$R = \frac{W(K^+ \rightarrow B_1^0(0, 2) + \pi)}{W(K_1^0 \rightarrow 2\pi)}$

$\frac{m_B^2 - m_\pi^2}{g^2 p'}$

where $\rho(m_B) (= p_B)$ and $p'$ is the pion momentum in $K^+ \rightarrow B_1^0(0, 2) + \pi$ and $K_1^0 \rightarrow 2\pi$ process respectively. If we take $280 \text{ MeV} < m_B^2 < 354 \text{ MeV} (= m_K^2 - m_\pi^2)$, where $m_B^2$ is the parameter characterizing the maximum of the mass distribution, and put $g \sim g'$, we shall obtain

$R \approx 0.1 \sim 0.6$

which is to be compared with the experimental value $R < 10^{-3}$.* This fact indicates that the maximum of the $\rho(m_B)$ will probably be at $> 354 \text{ MeV}$. The tendency of the $\rho(m_B)$ may manifest itself in the pion energy spectrum. Neglecting the $\cdot$ term $I_1(E_\pi)$, in $\pi^-$-spectrum, we obtain

$\rho(m_B) \propto I_1(E_\pi) \cdot (p_B / m_B)^{-1}$

$\simeq I^-(E_\pi) (p_B / m_B)^{-1}$.

Then assuming that the $\tau^+$-decay occurs entirely through (1), we shall obtain the experimental $\rho(m_B)$. It is noted that such a $\rho(m_B)$ can be safely represented by the monotonic in-

* The mass distribution $\rho(m_B)$ with the parameters $m_B^2=400 \text{ MeV}$ and $r_B=30 \text{ MeV}$ which was used in reference 1) gives this ratio $\sim 10^{-8}$. 

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creasing curve (the solid curve in Fig. 3).

![Graph](image)

Fig. 3. The mass distribution $\rho(m_B)$ fitted to the experimental data of $\pi^-$ spectrum.

Using thus obtained $\rho(m_B)$, we have calculated $\pi^+$-energy spectra which is given in Fig. 2. The agreement of the calculated spectrum with the experiment is fairly satisfactory.

As for the direct $3\pi$ decay process we shall only remark that the consistent interpretation of the $\pi$-spectrum is also possible even if this process is comparable to the $B_1(0, 2)$ channel, by taking the slightly changed $\rho(m_B)$. The branching ratio $\tau^+/\tau^{+\prime}$ is worthwhile to notice. With no direct process the ratio is $19/11$ for $\Delta I=1/2$. This value seems to be a little smaller than the present experimental value $2.9\pm0.5$. In order to increase the branching ratio from $19/11$ it is necessary to introduce either the $\Delta I=3/2$ decay amplitude or the direct $3\pi$ decay process.

The possible existence of $B_1(0, 2)$ may bring the effect in $K\rightarrow 2\pi$, but the problem essentially needs information about the large mass part of $\rho(m_B)$ and the strength of the reaction with $\Delta I=3/2$, and will require further investigation. It is noted that the process $K\rightarrow B_1(0, 2)+e+\nu$ is forbidden in the Sakata model.