Remarks on the Ambiguity in Quantum Field Theory

Y. Katayama and S. Hori
Dept. of Physics, Kyoto University
August 20, 1949

The so-called ambiguities raised by the photon self-energy problem have been seemed to be removed for a time by the ingenious method of Pauli and Villars. However, if we want to rewrite this method in the customary form, many difficulties arise.

Here, we intend to examine an alternative method to remove the ambiguities. The first problem concerning above mentioned ambiguities is whether the quadratic divergences exist or not. As is well known, we get them by the usual momentum space integration, but do not by Schwinger's calculation method. This difference is due to the definition of $\tilde{A}(l)(x)$ at $x=0$. The latter calculation takes its principal value as follows:

$$\lim_{x_{\mu}>0,0} + \lim_{x_{\mu}<0,0} A(l)(x) \quad (1)$$

and then removes the first quadratic divergence contrary to the former.

The second problem is how to treat the integral

$$\int g_\mu g_\nu e^{q_0} dq. \quad (1)$$

According to Miyazima, it is evaluated as

$$-\delta_{\mu\nu} \varepsilon(a) \delta(a) \quad (3)$$

and an ambiguity arises from the last $e$-term. If we determine this term uniquely by the suitable definition of the integral (2), a part of ambiguities will be removed. It is likely to define it so as to reduce

$$\delta(q) \frac{\partial A(l)}{\partial x_\mu} = (-\Box + m^2)\tilde{A}, \frac{\partial A(l)}{\partial x_\mu} \quad (4)$$

to zero. In fact, Miyazima has indicated that taking $e=2$, contrary to Wentzel, the photon self-energy vanishes.

However, we notice that this result does not give unique answer. This arises from the fact that one can reduce

$$\tilde{A}(-\Box + m^2) = \theta(l) \quad (5)$$

to zero by taking $e=1$, but not $e=2$. Therefore, if we put $(q^2 + m^2)\delta(q^2 + m^2) = 0$ before calculations, we get different result, the difference being the value of (4) with $e=2$. When we take $e=1$, the photon self-energies do not vanish, but become to the finite values $-\frac{a}{\pi} \frac{m^2}{2}, \frac{a}{\pi} \frac{4}{1}$, and $-\frac{a}{\pi} \frac{3l^2}{4}$ for the spinor, scalar and vector fields respectively, contradicting to the conditions of gauge invariance. (Notice that the results are half of Wentzel’s one). This discrepancy is due to the fact that the left-hand side of (4) vanishes conditionally while the right-hand side does not.

In order to remove this discrepancy, we are obliged to rely on the mixed theory, but there arises a new difficulty which is caused by the vector field introduced in order to compensate the divergences of charge-renormalizations with the condition $4m + m - 3l = 0$, where $n, m$ and $l$ are the numbers of particles of spinor, scalar and vector fields. In spite of them, there remain other difficulties which are not removed by the mixed theory, for instance, the problems of Dyson theorem.

In these circumstances, we can hardly say that we have any consistent method of calculations and so, at the present stage, to get a temporary way of escape, we must utilise the following three methods: i) Pauli’s regulator.
ii) the definition $c=2$ and iii) the definition $c=0$ after using (4) at first in the calculation (by which the p. s. e. vanishes). The last two methods, however, make us resign ourselves to prove the identity (5) by direct calculations.

1) W. Pauli and Villars, Rev. Mod. Phys.
2) K. Sawada, Prog. Theor. Phys. 4 (1949) in
3) T. Miyazima, Lecture at the Annual Meeting of J. P. S.
4) G. Wentzel, Phys. Rev. 74 (1948), 1070.

We calculated by Schwinger's calculation methods and so the conclusions are a few different with them, but essentially the same.