Note on the Interaction Representation in case of Meson Field interacting with Electromagnetic Field.

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In the ordinary meson theory, the tensor equation of meson is used, but in this note the following Duffin-Kemmer equation\(^{(1)}\) is taken instead of the tensor
one in order to give a unified formulation of the interaction representation in case of mesonic field interacting with electromagnetic field:

\[
(\beta_\lambda \frac{\partial}{\partial x_\lambda} + x)u = 0, \quad (\lambda = 1, 2, 3, 4) \tag{1}
\]

where \(x\) is connected to the rest mass \(m\) of the meson by the relation \(x = \frac{mc}{\hbar}\), and \(\beta\)'s satisfy the commutation relation

\[
\beta_\lambda \beta_\mu + \beta_\mu \beta_\lambda = \beta_\lambda \delta_{\mu\nu} + \beta_\nu \delta_{\lambda\mu}. \tag{2}
\]

As is well known, there exist two inequivalent irreducible representations of \(\beta\)-algebra(2) except a trivial one, the one is given by five-row and five-column matrix, the other ten-row and ten-column one. The former representation leads to the scalar (pseudoscalar) meson, the latter the vector (pseudovector) meson. This is the very fact which enables the formalism described in this note to be possible.

Now the interaction representation of the system considered here is given by

\[
\left\{ L_{\mu}[C] - \frac{i}{\hbar} \frac{\delta}{\delta U_{\mu}} \right\} \Psi[C] = 0 \tag{1'}
\]

with

\[
L_{\mu}[C] = \frac{1}{4\pi} \left( \frac{e}{\hbar c} \right) A_\lambda u^\dagger \beta_\lambda u + \frac{i}{x} \left( \frac{e}{\hbar c} \right)^2 A_\lambda A_\mu u^\dagger \{ \beta_\lambda \beta_\mu \right.+ \beta_\lambda (\beta_\nu N_\lambda)^2 \beta_\mu \} u, \tag{1'}
\]

where \(u^\dagger\) is defined by the equation

\[
\begin{align*}
u^\dagger = & i u^\dagger \gamma_4, \\
\gamma_4 = & 2 \gamma_4^2 - 1.
\end{align*} \tag{3}
\]

The field variables contained in (1') are the solutions of the free field equations and satisfy the following four dimensional commutation relations:

\[
\begin{align*}
[A_\lambda(x), A_\mu(x')] = & -4\pi i \hbar \delta_{\lambda\mu} D_\mu(x - x') \\
[u^\dagger(x), u_m(x')] = & -4\pi \hbar \left( \beta_\lambda \frac{\partial}{\partial x_\lambda} - \frac{1}{x} \beta_\lambda \beta_\mu \frac{\partial^2}{\partial x_\lambda \partial x_\mu} \right) u_m(x - x') \\
[u\mu(x), u_m(x')] = & [u\mu^\dagger(x), u_m^\dagger(x')] = 0, \tag{II}
\end{align*}
\]

where the suffices \(l\) and \(m\) of the mesonic field variables \(u\) and \(u^\dagger\) run respectively from one to five and one to ten according to the scalar (pseudoscalar) or the vector (pseudovector) mesons. The auxiliary condition in this case has the form

\[
\begin{align*}
\mathcal{E}[C] \Psi[C] = & 0, \\
\mathcal{E}[C] = & \frac{\partial A_\lambda}{\partial x_\lambda} + \left( \frac{e}{\hbar c} \right) \int [u^\dagger(P') \beta_\lambda u] \\
& \times \langle P' \rangle N_\lambda(P') D_\mu(P' - x) dF_{P'}. \tag{III}
\end{align*}
\]

That the equation (I) satisfies the integrability condition and the auxiliary condition (III) is compatible, is examined directly on account of the relation (2) and (II). Moreover the formulation of Kanesawa and Tomonaga(3) which treated the interaction representation in case of the scalar and vector meson fields coexisting with the electromagnetic field separately by means of the tensorial type equations of mesons, is derived from the formalism stated above by the following substitution:

\[
\begin{align*}
u^\dagger = & \left( \frac{1}{\sqrt{x}} \left( \frac{\partial \phi}{\partial x_0}, -\frac{\partial \psi}{\partial x_1}, -\frac{\partial \phi}{\partial x_2}, \right. \right. \\
& \left. \left. -\frac{\partial \psi}{\partial x_3} \right) \sqrt{x} \phi \right), \tag{4}
\end{align*}
\]

(for scalar meson)

\[
\begin{align*}
u^\dagger = & \left( -\frac{i}{\sqrt{x}} (x_{14}, x_{24}, x_{34}), \\
& -\frac{1}{\sqrt{x}} (x_{25}, x_{31}, x_{12}), \sqrt{x} (\phi_1, \phi_2, \phi_3, \phi_4) \right), \quad (4')
\end{align*}
\]

(for vector meson)
Letters to the Editor

Detailed accounts of this and related problems will appear in a later issue of this journal.

(1) R. J. Duffin: Phys. Rev. 54 (1938) 1114.

(2) N. Kemmer: ibid.