Conservation Law or Violation Law?*)

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(Received June 24, 1968)

An attempt is given, which aims at representing the fundamental laws of weak interactions as conservation laws instead of conventional violation laws. We assume that the conserved quantities characteristic to weak interactions are anti-commutate with those characteristic to strong interactions. All weak interactions are assumed to occur via intermediate boson, which is assumed to belong to an eigenstate of the conserved quantities for weak interactions. $V-A$ law is derived in this way by adopting the chirality as one of the conserved quantities. We adopt also as an conserved quantity the iso-chirality which is defined to be an operator anti-commutate with the iso-parity. Then isospin and hypercharge are not conserved, and $|\Delta I|=1/2$ rule is derived using weak current composed of bilinear forms of quarks. The Cabibbo angle appears naturally in the general definition of the iso-chirality operator. Also briefly discussed is a possible scheme of leptonic interaction, which satisfies the three conditions; $|\Delta I|=1/2$ rule for hadronic processes, non-appearance of neutral lepton currents and universality of coupling constants in the primary interaction between intermediate bosons and weak currents.

§1. Introduction

We know the existence of three kinds of interactions which act among elementary particles, strong, electromagnetic and weak, aside from the gravitational interaction. It is usually regarded that weaker the interaction is, it has less symmetrical properties. In opposition to the conservation of $P$ (parity), $C$ (charge conjugation), $Y$ (hypercharge) and $I$ (isospin) for strong interactions, $I$ is not conserved for electromagnetic interactions, and all the above-stated quantities are not conserved for weak interactions. Laws of weak interactions are expressed usually as the violation laws which govern the ways of violating the conservation laws for strong interactions, such as $|\Delta I|=1/2$, $\Delta Y/\Delta Q=+1$ and $|\Delta Y|\leq 1$.

This conventional viewpoint regards the strong and the weak interactions as the allowed and the forbidden transitions, respectively. However, it is by no means the only possibility to grasp the laws of weak interactions as the violation laws. Rather it might be strange if the fundamental laws of the weak interactions were the violation laws because the weak interaction is more universal than the strong interaction in the sense that the weak

*) Preliminary version of this paper was published in Soryushiron Kenkyu 29 (1964), 215 (mimeographed circular in Japanese).
interaction acts among all elementary particles except photon whereas the strong interaction acts only among hadrons. Further we do not know whether the weak interactions are actually weak or not at extremely high energies.

If the weak interaction is more fundamental than or of the same rank as the strong interaction, it would be inappropriate to express the fundamental laws of the weak interactions in terms of the conserved quantities of the strong interactions.

It is the aim of this paper to report on an attempt to express the laws of the weak interactions as conservation laws. Although both the conventional violation law and the conservation law discussed in this paper are phenomenological in nature, one may regard the latter as a deeper understanding of the nature of the weak interactions.

We assume in this paper that the weak interactions occur via intermediate bosons with spin 1. There are many possibilities on the role which the intermediate boson, if it really existed, should play in the weak interactions. We assume here as one of the most attractive possibilities that the intermediate boson plays an essential role not only for the occurrence of the weak interactions but also for the determination of their fundamental properties.

First, we assume that all weak interactions, both hadronic and leptonic, occur as results of the exchange of the intermediate boson of the same kind. The intermediate boson must, then, be not Tanikawa-type, but Yukawa-type with zero baryon and lepton numbers.

Next, we assume that the fundamental laws which govern the weak interactions may be expressed as conservation laws. To search for the conserved quantities for the weak interaction we look at the phenomenological laws of weak interactions; i.e. $V-A$ law, $|\Delta l| = 1/2$, $\Delta Y/\Delta Q = +1$, $|\Delta Y| \leq 1$ and so on. Rather simple forms of these phenomenological violation laws suggest that there are direct and simple relations between the conserved quantities for weak interactions, now being looked for, and those for strong interactions. As a first step to look for such quantities we assume that the conserved quantities which are characteristic of the weak interaction anticommute with those of the strong interaction.

This assumption enables us to derive the conserved quantities for weak interactions from those for strong interactions. Conservation of chirality, which anticommutes with parity, is one of the consequences of this assumption.

We assume moreover that the intermediate boson is an eigenstate of the all conserved quantities for weak interactions.

We discuss on the law of conservation of chirality in §2. We define iso-chirality in §3, and show that the non-conservation of isospin and
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hypercharge follows from the conservation of iso-chirality. \(|\Delta I| = 1/2\) law is derived in §4 from the conservation of iso-chirality using the two body urbaryon current. In §5 iso-chirality is re-discussed and the Cabibbo angle is shown to appear in the natural definition of the iso-chirality. Leptonic processes are discussed briefly in §6.

§2. Chirality

Conservation of chirality has been discussed in many papers. Especially Tanikawa and Watanabe\(^9\) investigated the chiral intermediate boson theory where the intermediate boson was assumed to be an eigenstate of chirality. For convenience of the investigation of iso-chirality we review very briefly the properties of chirality. We may take as the chirality operator the quantities which are more general than that adopted by Tanikawa and Watanabe. This generalization allows the introduction of the Cabibbo angle in the case of iso-chirality.

We define the parity of a quantity to be plus when the quantity transforms in the same way as the coordinates under proper Lorentz transformation and space reflection, to be minus when its sign changed further under space reflection. Chirality is defined to be an operator which anticommutes with parity.\(^9\)

Since parity is a dichotomic variable, there exist following dichotomic variables which satisfy

\[
xy = iz, \quad yz = ix, \quad zx = iy,
\]

\[
x^2 = y^2 = z^2 = 1, \quad (2.1)
\]

where \(z\) is the parity operator and \(x\) and \(y\) are chirality operators.

When the intermediate boson is assumed to be an eigenstate of \(x\) or \(y\) and the Lagrangian for the free intermediate boson field is assumed to be \(CP\)-invariant, we must adopt \(y\) as the chirality operator. Then the weak current, which couples with the intermediate boson, must be of the form

\[
\bar{\psi} T_\mu(1 \pm \gamma_5) \psi
\]

to guarantee the interaction Lagrangian to be commutable with \(y\). \(\pm\) sign in the above expression is determined by the eigenvalue of \(y\) which the intermediate boson belongs to, and all weak currents must be of \(V-A\) type if the intermediate boson is the \(y = +1\) eigenstate.

§3. Iso-chirality. I

For hadronic decay, \(I\) and \(Y\) do not conserve, and
\[ |\Delta I| = 1/2, \quad |\Delta Y| \leq 1 \]

hold in good accuracy. To search for a conserved quantity for hadronic weak interactions, we first consider an inversion in the isotopic space for hadrons. This is given by

\[ \phi \rightarrow \phi' = R_\phi \phi = \exp\left(i \frac{\pi}{2} J_3\right) \cdot C \phi = \exp\left(-i \frac{\pi}{2} Y\right) \phi, \quad (3.1) \]

for a state composed of hadrons.\(^6\) Though \(R_\phi\) has four eigenvalues \(\pm 1, \pm i\), it is simpler and also sufficient for our purpose to use the double inversion only:

\[ \phi \rightarrow \phi'' = J_3 \phi = R_3^2 \phi = (-1)^Y \phi. \quad (3.2) \]

\(J_3\) has eigenvalue \(+1\) \((-1)\), which corresponds to \(Y = \text{even (odd)}\). Denoting the corresponding eigenstate as \(\phi_+ (\phi_-)\), we have

\[ J_3 \phi_+ = +\phi_+ \cdots \quad Y = \text{even, } I = \text{integer}, \]
\[ J_3 \phi_- = -\phi_- \cdots \quad Y = \text{odd, } I = \text{half integer}, \quad (3.3) \]

since

\[ Q = J_3 + \frac{Y}{2} \]

must be an integer. As in the case of chirality there exist \(J_1\) and \(J_2\) which satisfy

\[ J_1 J_2 = i J_3, \text{ etc.} \]
\[ J_i^2 = 1, \quad i = 1, 2, 3. \quad (3.4) \]

Let the eigenfunctions of \(J_1\) and \(J_2\) be \(\chi_\pm\) and \(\omega_\pm\), respectively;

\[ J_1 \chi_\pm = \pm \chi_\pm, \]
\[ J_2 \omega_\pm = \pm \omega_\pm, \quad (3.5) \]
\[ J_3 \phi_\pm = \pm \phi_\pm, \]

and determine the phase relation between \(\phi_-\) and \(\phi_+\) by

\[ \frac{1}{2} (J_1 + i J_2) \phi_- = \phi_+, \]

then we have

\[ \chi_\pm = \frac{1}{\sqrt{2}} (\phi_+ \pm \phi_-), \]
\[ \omega_\pm = \frac{1}{\sqrt{2}} (\phi_+ \pm i \phi_-) \quad (3.6) \]
and
\begin{align}
J_1 \phi_\pm &= \phi_\pm, & J_1 \omega_\pm &= \pm i \omega_\mp, \\
J_2 \chi_\pm &= \mp i \chi_\mp, & J_2 \phi_\pm &= \pm i \phi_\mp, \\
J_3 \omega_\pm &= \omega_\mp, & J_3 \chi_\pm &= \chi_\mp.
\end{align}
(3.7)

We may call \( J_z \), which is the conserved quantity for strong interactions, as iso-parity, then arbitrary linear combinations of \( J_1 \) and \( J_2 \) are iso-chiralities by definition. We adopt \( J_1 \) or \( J_2 \) as chirality in this section, and discuss more general case in §5. As in the case of space-chirality where the intermediate boson belongs to the \(+1\) eigenvalue of the chirality operator \( y \), we assume that the intermediate boson corresponds to one of four eigenstates \( \chi_\pm \) and \( \omega_\pm \) of \( J_1 \) or \( J_2 \). Then we must take \( J_3 \) as the iso-chirality operator to assure the \( CP \)-invariance of the free Lagrangian for the intermediate boson, since \( \chi_\pm \) and \( \omega_\pm \) transform under \( CP \)-transformation in the following way,
\begin{align*}
\chi_\pm &\to \chi_\mp^\dagger, \\
\omega_\pm &\to \omega_\mp^\dagger.
\end{align*}

Therefore the field of the intermediate boson, \( W \), may be expressed as superposition of eigenstates of \( J_z \), \( Y \) or \( I \) as follows:
\begin{align}
W &= \frac{1}{\sqrt{2}} \left[ W(\bar{J}_z=1) \pm W(\bar{J}_z=-1) \right] \\
&= \frac{1}{\sqrt{2}} \left[ W(Y=\text{even}) \pm W(Y=\text{odd}) \right] \\
&= \frac{1}{\sqrt{2}} \left[ W(I=\text{integer}) \pm W(I=\text{half-integer}) \right].
\end{align}
(3.8)

Since the interaction Lagrangian
\[ L_{\text{int}} = g \left[ J_3^* W_\lambda + J_1 W_\lambda^* \right], \]
(3.9)
where \( W_\lambda^* = \eta_\lambda W_\lambda^* \), \( \eta_\lambda = 1 \) (\( \lambda = 1, 2, 3 \)), \( -1 \) (\( \lambda = 4 \)), and \( W_\lambda^* \) is the hermitian conjugate of \( W_\lambda \), should be commutative with iso-chirality, the current must also be an eigenstate of the iso-chirality with the same eigenvalue;
\[ J = \frac{1}{\sqrt{2}} \left[ J(I=\text{integer}) \pm J(I=\text{half-integer}) \right]. \]
(3.10)

\section{§4. \( \Delta I = 1/2 \) rule}

To construct the interaction described in the previous section, we must assume at least four kinds of intermediate bosons; charged \( W, W^\dagger \) and neutral \( W^0, W^{0\dagger} \). We investigate in this paper the most simple case where
just these four kinds of intermediate bosons exist, all of them being assumed to belong to the eigenvalue +1 of the space-chirality and to one of the two eigenvalues of the iso-chirality, $J$. The last assumption assures the non-
occurrence of transitions with $|\Delta Y| \geq 2$ in the first order of the weak interaction.

We consider in the following the weak current to be composed of bilinear forms of quarks ($p_0$, $n_0$, $A_0$). Then, weak currents are sums of $I=0$, $1/2$ and $1$ components, and the interaction between weak currents and intermediate bosons should be of the following form

$$L_{\text{int}} = \frac{g}{\sqrt{2}} \left[ (J^* + s^*) W^0 + (J + s) W^* + (j^{0*}) W^0 + (j^0 + s^0) W^{0*} \right],$$  \hspace{1cm} (4.1)

because of Eqs. (3.9) and (3.10). $J^*$ and $J$ are charged current with $I=1$, $s$ and $s^0$ are currents with $I=1/2$, and $j^0$ and $j^{0*}$ are neutral currents with $I=0, 1$. Possible forms of some of these currents are

$$J = (\bar{n}_0 p_0),$$

$$s = (\bar{A}_0 p_0),$$

$$s^0 = (\bar{A}_0 n_0),$$  \hspace{1cm} (4.2)

where

$$(ab) = i\bar{\psi}_u \gamma_\lambda (1 + \gamma_5) \psi_b.$$  \hspace{1cm} (4.3)

Equation (4.1) may also be expressed as

$$L_{\text{int}} = \frac{g}{\sqrt{2}} \left[ L_{WJ} + L_{WS} \right],$$  \hspace{1cm} (4.4)

$$L_{WJ} = J^* W + JW^* + J^*_0 W^0 + J^*_0 W^0,$$  \hspace{1cm} (4.5)

$$L_{WS} = s^* W + s W^* + s^{0*} W^0 + s^0 W^{0*},$$  \hspace{1cm} (4.6)

where

$$J^*_0 = \frac{1}{\sqrt{2}} (-j^0 - j^{0*}),$$

$$J^*_0 = \frac{1}{\sqrt{2}} (j^0 - j^{0*}),$$  \hspace{1cm} (4.7)

and

$$W^0 = \frac{1}{\sqrt{2}} (-W^0 - W^{0*}),$$

$$W^0 = \frac{i}{\sqrt{2}} (W^0 - W^{0*}).$$  \hspace{1cm} (4.8)

Equations (4.4)\(~(4.6)\) coincide with the schizion theory of Lee and Yang.7
Weak interactions derived from these equations satisfy \(|\Delta I| = 1/2\) and \(|\Delta Y| \leq 1\). There is, however, one difference between the schizon theory and our results. In the schizon theory, \((W, W^*_u, W^*)\) are regarded to form a triplet with \(I = 1\) and \((W, W^*)\) a doublet with \(I = 1/2\) in Eq. (4·5) and in Eq. (4·6), respectively. Though this dual property with respect to \(I\) is common to the schizon theory and our theory, intermediate bosons are not assumed to be an eigenstate of the iso-chirality in the schizon theory. As a consequence of this difference, coupling constants may be different for \(L_{\phi J}\) and \(L_{\phi S}\) in Eq. (4·4) in the schizon theory whereas they must be same in our theory.

Although form of \(J^\circ\) may be given by

\[
J^\circ = \frac{1}{\sqrt{2}}[(\vec{p}_6 n_6) - (\vec{n}_6 n_6)],
\]

(4·9)

some remarks are needed on \(J^\circ\), as have already been noted by Lee and Yang.\(^a\) Writing the transformation of \(W\) under time reversal as follows,

\[
TW_\lambda T^{-1} = -\eta_\lambda W^*_\lambda,
\]

(4·10)

\[
\eta_\lambda = \begin{cases} +1 & \text{for } \lambda = 1, 2, 3, \\ -1 & \text{for } \lambda = 4, \end{cases}
\]

we have

\[
TW^a_\lambda T^{-1} = -\eta_\lambda W^a_\lambda,
\]

\[
TW^b_\lambda T^{-1} = +\eta_\lambda W^b_\lambda.
\]

(4·11)

Therefore, if the invariance under time reversal of \(L_{\phi J}\), Eq. (4·5), is demanded, \(J\)'s should satisfy the following equations,

\[
TJ T^{-1} = -\eta_\lambda J^*,
\]

\[
TJ^* T^{-1} = -\eta_\lambda J,
\]

\[
TJ^a T^{-1} = -\eta_\lambda J^a,
\]

\[
TJ^b T^{-1} = +\eta_\lambda J^b.
\]

(4·12)

\(J\) and \(J^\circ\) given in Eqs. (4·2) and (4·9) satisfy Eq. (4·12). We cannot, however, construct the two body current, \(J_0\), from \(p_0, n_0\) and \(A_0\). Neutral two body current with \(I = 0\)

\[
\begin{equation}
(\vec{p}_0 p_0) + (\vec{n}_0 n_0), \\
2(\vec{A}_0 A_0) - (\vec{p}_0 p_0) - (\vec{n}_0 n_0)
\end{equation}
\]

(4·13)

do not obey Eq. (4·12). If these currents existed, \(T\)-invariance is violated for processes where neutral hadronic current contributes, such as \(K^0 \rightarrow 2\pi, 3\pi\), while \(T\) is conserved for leptonic and semi-leptonic processes.
If $T$-invariance is assumed to hold for all weak interactions we must put $J_y=0$.

§5. Iso-chirality. II

It is known from experimental data on leptonic decays of hadrons that

$$|\Delta Y|=1 \text{ current} \sim \frac{1}{5} \times [|\Delta Y|=0 \text{ current}].$$

This ratio is, however, exactly 1 in Eq. (4.4), instead of $\sim 1/5$. We show in this section, that this result is not the necessary result of our theory but is due to undue restriction imposed on the iso-chirality operator.

We restricted the iso-chirality operator to be either $J_1$ or $J_5$ in §3 for simplicity. However, the most general form of the iso-chirality operator is a linear combination of $J_1$ and $J_5$. Upon imposing a condition that the free Lagrangian of the intermediate boson field, which belongs to an eigenstate of the iso-chirality, be invariant under $CP$-transformation, the most general form of the iso-chirality operator may be written as

$$X=\sqrt{1+\gamma^2} J_1 + i \gamma J_5,$$  \hspace{1cm} (5.1)

where $\gamma$ is an arbitrary real constant and $\gamma=0$ corresponds to the case of §3. Denoting the eigenstates of $X$ as $\chi'_\pm$, we have

$$X \chi'_\pm = \pm \chi'_\pm,$$  \hspace{1cm} (5.2)

where

$$\chi'_\pm = \cos \theta \cdot \Phi_+ \pm \sin \theta \cdot \Phi_-,$$  \hspace{1cm} (5.3)

$\Phi_\pm$ being eigenstates of iso-parity, $J_5$, defined by (3.3). Relation between $\theta$ and $\gamma$ is given by

$$\cos 2\theta = \frac{\gamma}{\sqrt{1+\gamma^2}},$$  \hspace{1cm} (5.4)

and $X$ may be expressed in terms of $\theta$ by

$$X = \csc 2\theta \cdot J_1 + i \cot 2\theta \cdot J_5.$$  \hspace{1cm} (5.1')

$\chi'_+$ and $\chi'_-$ are not orthogonal to each other, since $X$ is not hermite. When the intermediate boson is in the state $\chi'_+$ or $\chi'_-$, the weak current which couple with it should be given by not Eq. (3.10) but

$$J = \cos \theta \cdot J(I=\text{integer}) \pm \sin \theta \cdot J(I=\text{half-integer}).$$  \hspace{1cm} (5.5)

Therefore, the primary interaction, (4.4), should be replaced by

$$L_{\text{int}} = g [\cos \theta L_{W1} + \sin \theta L_{W3}],$$  \hspace{1cm} (5.6)
§6. Leptonic decay

We discuss on a possible form of interaction of leptons with intermediate bosons very briefly. Since leptons and intermediate bosons do not undergo strong interactions, it will be natural to consider that leptonic currents are also eigenstates of the conserved quantities which characterize weak interactions, such as chirality and iso-chirality.

In an unified treatment of hadronic and leptonic weak interactions we encounter with a problem how to reconcile the following three points: (i) \(|M|=1/2\) rule, (ii) non-existence of neutral leptonic current, (iii) universality of coupling constants. (i) and (iii) are satisfied by our theory described in §§3~5. Though (ii) is derived naturally by assuming that neutral intermediate bosons do not couple with leptons, (iii) is destroyed in this case.

We give in the following an example of interaction between intermediate bosons and leptons which satisfy (i)~(iii).

We may group four leptons into two doublets in the following two ways:

(A) leptonic isospin space

\[
L = \left( \begin{array}{c} L_1 \\ L_2 \end{array} \right), \quad L_1 = \left( \begin{array}{c} \nu_e \\ e^- \end{array} \right), \quad L_2 = \left( \begin{array}{c} \nu_\mu \\ \mu^- \end{array} \right).
\] (6·1)

We denote the Pauli matrices which operate on \(L_1\) and \(L_2\) as \(\tau_i\), and write

\[
T_i = \left( \begin{array}{cc} \tau_i & 0 \\ 0 & \tau_i \end{array} \right).
\]

(B) leptonic spin space

\[
l = \left( \begin{array}{c} l_1 \\ l_2 \end{array} \right), \quad l_1 = \left( \begin{array}{c} \nu_e \\ \nu_\mu \end{array} \right), \quad l_2 = \left( \begin{array}{c} e^- \\ \mu^- \end{array} \right).
\] (6·2)

We denote the Pauli matrices which operate on \(l\) as \(\rho_i\).

Interactions between intermediate bosons and leptonic currents are assumed to be given by

\[
L^\alpha = \frac{g}{2\sqrt{2}} [\bar{L} T_i L W_i + \bar{\rho} l W_i],
\] (6·3)
where $W_i$'s correspond to vector components in leptonic isospin space,

$$W_1 = \frac{(W^+ + W^+)}{\sqrt{2}},$$

$$W_2 = i\frac{(W^- - W'^-)}{\sqrt{2}},$$

$$W_3 = \frac{(-W^0 - W'^0)}{\sqrt{2}},$$

and $W_i'$s correspond to those in leptonic spin space. $W_i$ is transformed into $W_i'$ under switching over of the leptonic isospin space to the leptonic spin space, and the transformation property is assumed to be

$$W_i \rightarrow W_i' = CPT_1(\pi) W_i = (W_1, W_2, -W_3),$$

i.e. the combined charge symmetry transformation including space reflection, where $T_1(\pi)$ is $180^\circ$ rotation around the first axis in the isospin space. Space reflection is included in Eq. (6.5) so as to assure that $W_i$ and $W_i'$ belong to the same eigenvalue of chirality. Using Eq. (6.5), Eq. (6.3) may be written as

$$L_{\text{int}}^0 = g \left\{ \left[ (\bar{\nu}_e e^-) + (\bar{\nu}_\mu \mu^-) \right] W \\
+ \left[ (e^- \nu_e) + (\bar{\nu}_e \nu_e) \right] W^*, \right\},$$

where neutral current does not appear.

§7. Summary and conclusions

We have attempted to substitute the violation laws, which are used conventionally to characterize weak interactions, by conservation laws. We have shown that it is possible to reproduce the known phenomenological violation laws by adopting as conserved quantities the chirality and the iso-chirality which are anti-commutative with the parity and the iso-parity, respectively, and assuming intermediate boson to belong to an eigenstate of them. $|\Delta I| = 1/2$ and $|\Delta Y| \leq 1$ have been derived by assuming these properties and weak currents composed of bilinear forms of quarks.

The Cabibbo angle appears naturally as a constant which specify the general form of the iso-chirality operator.

A possible form of couplings of intermediate bosons with leptons has been discussed briefly, which satisfies $|\Delta I| = 1/2$ for hadronic decays, non-appearance of neutral lepton currents and the universality of the coupling constants.

Although we did not discuss the interpretation of the $|\Delta I| = 1/2$ law as the octet dominance within the scheme of unitary symmetry, similar discussions as ours should also be possible when one works within the scheme of unitary symmetry instead of charge independence."
References


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