Letters to the Editor

The opinions expressed in these columns do not necessarily reflect those of the Editors.


Veneziano-Like Model with $2q2\bar{q}$
Meson Trajectory and pp
Scattering at $90^\circ$e.m.

Taketoshi INO

Department of Physics, Shimane University
Matsue 690

February 13, 1978

The pole-pole duality, introduced by Veneziano in his model for scattering amplitude and supplied with the quark-diagram interpretation by several authors, may be one of the fundamental characteristics of hadron interactions. A possible framework for the duality is the Veneziano-type model in terms of the $q\bar{q}$- and $2q2\bar{q}$-meson and $3q$-baryon trajectories. In this short note, we make an application of the model to pp scattering, for which reaction exchanged states in a channel should be $2q2\bar{q}$-mesons.

Some indications of $2q2\bar{q}$-meson exchange have been obtained in NN scattering\(^{11}\) and other reactions.\(^{33}\) Theoretical models\(^{3}\) suggest that there exist various types of $2q2\bar{q}$-mesons. It is interesting to examine what $2q2\bar{q}$-mesons dominate in low-energy NN scattering. In this work, an effective $2q2\bar{q}$ trajectory, denoted by $\alpha_4(u)$ hereafter, is assumed. Its slope and intercept are motivated from the phenomenological $\sigma$\(^{11}\) and $\rho'$\(^{40}\) trajectories. The $\rho'$ trajectory $\alpha_{\rho'}(u) = -0.13 + 0.3u$, introduced to understand $\pi N$ charge-exchange scattering, may be a $2q2\bar{q}$ trajectory.\(^{31}\) The $\sigma$ meson, needed in models of low-energy $NN$ scattering (the OBE models), is assumed to be related to the $\sigma$ trajectory ($\alpha_\sigma(u)$

\[ = -0.4 + 0.9u \].\(^{11}\)

It is in general not so easy to compare a Veneziano model for $NN$ scattering with experiments, because the strong absorptive effects from $s$-channel unitarity must be considered. We discuss the $pp$ elastic scattering at $90^\circ$e.m. in the $P_L$ range of $1.5\sim 11$ GeV/$c$, from the viewpoint that the scattering is non-diffractive at least at low energies. The suppression due to the absorption is taken into account by a simple parameter modification. Our interest is in the slope of energy-dependence of $(d\sigma/dt)_{pp}$ and breaks in it.

Now, we assume the following amplitude:

\[
A_i(s, t, u) = \mu_e \left[ \frac{\Gamma(1 - \alpha_\rho(t))}{\Gamma(1 - \alpha_\rho(t) - \alpha_4(u))} + (t \leftrightarrow u) \right] \\
+ \nu_e \left[ \frac{\Gamma(1 - \alpha_\rho(t))}{\Gamma(1 - \alpha_\rho(t) - \alpha_4(u))} + (t \leftrightarrow u) \right].
\]

(1)

Here, $\alpha_{\rho}(t)$ represents the exchange-degenerate $\rho - A_2 - \omega - f$ trajectory. The amplitudes $A_i$ are related, by our definition, to the invariant amplitudes $G_i^{(5)}$ as $A_i = m_q^2 G_i (i = 1, 3, 5)$. (The $G_2$ and $G_4$ for $pp$ scattering are antisymmetric under $t \leftrightarrow u$ and vanish at $90^\circ$.) The two terms in Eq. (1) are needed to maintain two kinds of the $\rho NN$ couplings and the interference between them. And, for the $\omega NN$ vertex, the tensor coupling is set equal to zero. Equating the residues at $\rho$ and $\omega$ poles of Eq. (1) to those of the one-$\rho$ and $\omega$-exchange amplitudes in models of low-energy $NN$ scattering respectively, we have
\[ \nu_1 = \frac{f_p^2}{16} \left( 1 + 2 \frac{g_{p^0}}{f_p^2} \right) \alpha_\pi', \]

\[ \mu_1 = \nu_1 \alpha_\pi(0) + m_p^2 \alpha_\rho \left[ \frac{5r+4}{32} f_p^2 \right. \]

\[ \left. + \frac{r-2}{8} f_p^2 g_{p^0} - 2 \left( \frac{g_{p^0}^2}{4} + g_{\rho^0}^2 \right) \right], \]

\[ \nu_3 = -\frac{f_p^2 \alpha_\pi'}{16 \alpha_\pi}, \]

\[ \mu_3 = \nu_3 \alpha_\pi(0) + m_p^2 \alpha_\rho \left[ \frac{r-4}{32} f_p^2 \right. \]

\[ \left. + \frac{f_p g_{p^0}}{4} + \left( \frac{g_{p^0}^2}{4} + g_{\rho^0}^2 \right) \right], \] (2)

\[ \nu_3 = \nu_1, \]

\[ \mu_5 = \nu_5 \alpha_\pi(0) + m_p^2 \alpha_\rho \left[ \frac{5r+4}{32} f_p^2 \right. \]

\[ \left. + \frac{r+6}{8} f_p g_{p^0} + 2 \left( \frac{g_{p^0}^2}{4} + g_{\rho^0}^2 \right) \right], \]

where \( r = m_p^2 / m_\rho^2 \). Here, it is assumed that the \( \rho \) and \( \omega \) have not satellites. We assume also \( \alpha_\pi(u) = 1 \) from the isovector nucleon electromagnetic from factor. \( \alpha_\pi(0) = 0.6u - 0.3(\alpha_\pi(u) + \alpha_\rho(u))/2 \), and \( \alpha_\rho(0) = 0.9t + 0.5 \).

With the values of \( g_{\rho^0}^2 \) and \( g_{\omega^0}^2 \) consistent with those obtained in models of low-energy NN scattering,\(^\text{1,7}\) we find that

\[ \frac{d\sigma}{dt} \bigg|_{90^\circ \text{c.m.}} = \frac{\pi}{10} \frac{E_p}{m_p^2} \left[ (E^2 A_1 + m_p^2 A_2) + (p^2 A_3) + (p^2 A_4) \right] \] (3)

shows an exponential-like behaviour with \( p^2 \) and has two concave curvature structures (called breaks hereafter for simplicity) in its slope around \( p^2 \simeq 0.5 \) and 1.5 (GeV/c)^2. Here, \( p^2 \) is the squared c.m. momentum of proton, and \( E^2 = p^2 + m_p^2 \).

In Fig. 1, the present model is compared with experiments\(^8\) with \( g_{\rho^0}^2 = 2.5 \) and \( g_{\omega^0}^2 = 6 \), and considering the absorptive effect by multiplying \( A_4 \) by a factor \( c = 0.13 \). The values of \( \mu_1 \) and \( \nu_1 \), corresponding to these coupling constants, are \( \mu_1 = -8.6 \), \( \mu_3 = 4.9 \), \( \mu_5 = 19.7 \), \( \nu_1 = 4.9 \), \( \nu_3 = -3.2 \) and \( \nu_5 = 4.9 \). And so, the interference between the two terms in Eq. (1) is destructive for \( A_1 \) and \( A_3 \); while, for \( A_3 \) it is constructive. The dot-dashed curve in Fig. 1 denotes the contribution from the first plus second terms in Eq. (3) multiplied by the factor \( c^2 \). The exponential by dotted line is for convenience's sake. In this model, the break at \( p^2 \simeq 0.5 \) (GeV/c)^2 is due to the spin effects and the interference between the two terms in Eq. (1), and the slope change around \( p^2 \simeq 1.5 \) (GeV/c)^2 is owing not only to these spin and interference effects but also to the fact that in the \( A_3 \) amplitude the \( \mu \)-term dominates at momenta \( p^2 < 1.5 \) (GeV/c)^2, while at \( p^2 \simeq 1.5 \) (GeV/c)^2 the \( \nu \)-term becomes influential more and more with momentum and overcomes the \( \mu \)-term soon. The main cause responsible for the slope change around \( p^2 \simeq 1.5 \) (GeV/c)^2 is,
of course, the latter fact.

As seen in Fig. 1, the $pp$ elastic scattering at 90° t.m. in the range $0.4 \leq p^2 \leq 3$ (GeV/c)^2 is understood fairly well by the amplitude (1) with an effective $2q2\bar{q}$ trajectory $\alpha_4(u) \approx (\alpha_\sigma(u) + \alpha_\rho(u))/2$. Thus, it is suggested that the $\sigma$ and $\rho'$ trajectories (and some $2q2\bar{q}$ trajectories being exchange-degenerate with the $\sigma$ and $\rho'$ ones) dominate in the $u$-channel (or $t$-channel) of low-energy $NN$ scattering.

If the $\rho'$ trajectory couples with the $\rho$-$A_2$ trajectory (into a dual pair) more strongly than with the $\omega$-$f$ one, then we may have a larger slope change around $p^2 \approx 1.5$ (GeV/c)^2 than one obtained here by Eq. (1).

The break at $p^2 \approx 3.4$ (GeV/c)^2, seen in experiments, cannot be understood by the present model.

3) See models cited in Ref. 2).
5) B. Nicolescu, in Ref. 2).