The Breakdown of Pomeranchuk’s Conjecture and the Composite Structure of Hadrons

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Quark model is applied to high energy asymptotic limit scattering processes. It is shown that the Pomeranchuk's conjecture may break down for hadron-hadron scattering even if it holds good for quark-quark scattering.

§ 1. Introduction

Recently IHEP-CERN Collaboration reported the experimental data on the total cross-sections of \( \pi^-\), \( K^-\) and \( \bar{p} \) on protons and deuterons in the momentum range \( 20 \sim 65 \text{ GeV}/c \). Their main results were: (1) The total cross-sections of \( \pi^-p \), \( \pi^-n(=\pi^+\bar{p}) \), \( K^-p \) and \( K^-n \) have all become energy independent in the region above \( 30 \text{ GeV}/c \), where as the \( \bar{p}-p \) and \( \bar{p}-n \) cross-sections are still decreasing. (2) There exists a significant difference of about 3 mb, which is apparently energy independent, between the newly measured \( K^-p \) cross-sections and the extrapolated values of the previously measured \( K^+\bar{p} \) cross-sections. (3) There are small energy independent difference of about 1.3 mb between \( \pi^-p \) cross-sections and \( \pi^-n(=\pi^+\bar{p}) \) cross-sections. These results pose a rather puzzling situation, since according to Pomeranchuk the total cross-sections of particle and anti-particle on a given hadron must be equal in the high energy asymptotic limit region.

It may be too hasty to conclude at this stage that the Pomeranchuk’s conjecture is broken, since IHEP-CERN Collaboration concluded that the difference between \( \pi^-p \) cross-sections and \( \pi^-n(=\pi^+\bar{p}) \) cross-sections is partly due to the uncertainty in the hydrogen and deuterium density, and is partly due to the uncertainty in the Glauber correction. But we cannot convince their conclusion literally, because beyond \( 30 \text{ GeV}/c \) they obtained energy independent values of \( \pi^-p \) total cross-section, \( \sigma_{\pi^-p}=24.6 \pm 0.3 \) mb, which is of about 1.2 mb larger than the values of \( \pi^+\bar{p} \) total cross-section measured at \( 22.1 \text{ GeV}/c \). At all rates it seems to be meaningful to examine the possibility of the breakdown of Pomeranchuk’s conjecture and to search what kind of a change must be brought into the concept of elementary particles by it, since the Pomeranchuk’s conjecture seems to be so tightly connected with the local field theoretical point of view of elementary particles.

In this paper we conclude that the breakdown of Pomeranchuk’s conjecture
is due to the composite structure of hadrons. Two years ago we proposed a model of high energy hadron-hadron scattering,\(^5\) in which hadrons were assumed to be composed of quarks and anti-quarks and large angle hadron-hadron scattering processes were explained through the multiple quark-quark scattering processes, using Glauber's approximation method. Here we apply the same model to the high energy asymptotic limit case and show that the breakdown of Pomeranchuk's conjecture is due to the hadron's composite structure even when we admit that Pomeranchuk's conjecture holds good for the quark-quark and quark-antiquark scattering amplitudes. Recently Shiga and Uehara\(^6\) proposed an idea on the hypothesis of the quark model to interpret the breakdown of the Pomeranchuk's conjecture. There they assumed an existence of a new nonet-like Pomeranchon and therefore Pomeranchuk's conjecture breaks down at the quark-quark and quark-antiquark scattering process in contrast to our model.

In § 2, we describe the general formulae for hadron-hadron scattering and their relation to Pomeranchuk's conjecture. The calculated results for particular processes are shown in § 3. In the last section some general remarks are made.

§ 2. Composite structure of hadrons and Pomeranchuk's conjecture

I. Pomeranchuk had conjectured\(^3\) the equality of the total cross sections of a particle and antiparticle on a given hadron under the following three assumptions:

1. The forward scattering amplitude \(F(E, 0)\) satisfies the ordinary dispersion relation, i.e. \(F(E, 0)\) is analytic in \(E\), and regular in the \(E\)-plane, cut along \((-\infty, -m)\) and \((m, \infty)\), where \(m\) is the mass of the incident particle.

2. The total cross-sections approach the constant values as the energy \(E\) approaches infinity.

3. The real part of the forward amplitude is not too large compared with the imaginary part.

These three assumptions seem to stand on the well-established experimental and theoretical conclusions, since the second and the third assumptions may be considered as the experimental results, and the first assumption is a well-established result from quantum field theory especially for \(\pi-N\) scattering and is generally believed as a consequence of causality. Thus here we encounter a rather puzzling situation.

One way out of the difficulty is to abandon any one of these assumptions. Another way is to assume that Pomeranchuk's conjecture holds good only in the subhadronic world such as a quark world. We wish to analyze this latter possibility in the present paper and discuss the relation between these two different approaches in the last section.

In a previous paper\(^4\) the multiple quark scattering formalism has been described with the treatment of the large angle hadron-hadron scattering. Here
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we summarize the formalism in order to establish notations and terminologies.

There are four types of scattering amplitudes for the elastic quark-quark scatterings in the $SU(3)$ symmetric limit, i.e.

- **type 1:** $f_{uu} = f_{dd} = f_{ss} = f_1,$  \(1\)
- **type 2:** $f_{uu} = f_{dd} = f_{ss} = f_2,$  \(2\)
- **type 3:** $f_{ud} = f_{ud} = f_{ds} = f_3,$  \(3\)
- **type 4:** $f_{ud} = f_{ud} = f_{ds} = f_{ds} = f_4,$  \(4\)

where $f_{ab}$ represents the elastic scattering amplitude for quarks $a$ and $b$.

Now we can construct the elastic scattering amplitude $F_{AB}$ for hadrons $A$ and $B$ from their constituent quark-quark scattering amplitude using the Glauber's method of multiple scattering:\(^6\)

\[
F_{AB}(s, k_L) = \sum_{j=1}^{N_{j}} \sum_{j_1=0}^{N_{j_1}} \sum_{j_2=0}^{N_{j_2}} \sum_{j_3=0}^{N_{j_3}} \left(-\frac{2}{i\pi s}\right)^{j_1+j_2+j_3+j_4-1} (N_j)_{j_1} (N_j)_{j_2} 
\]

\[
\times \frac{1}{j_1! j_2! j_3! j_4!} \prod_{k=1}^{j_1} \prod_{l=1}^{j_2} \prod_{m=1}^{j_3} \prod_{n=1}^{j_4} f_1(s, k_{j1}) \times f_2(s, k_{j2}) \times f_3(s, k_{j3}) \times f_4(s, k_{j4}) \times \delta(k_{j1} + \cdots + k_{j4} - k_L) \times d^4k_{j1} d^4k_{j2} d^4k_{j3} d^4k_{j4}, \]  \(5\)

where $s$ and $k_L$'s are the total energy of the system squared and the transverse components of the transferred momentum in the center-of-mass system of the particles participating to each collision, and $N_j$ is the number of pairs of scattered quarks whose scattering amplitude belongs to the type $j$ in Eqs. (1) \sim (4).

A more specified form of $F_{AB}(s, k_L)$ can be obtained if we further assume

\[
f_1(s, k_L) = \frac{i s}{16\pi} b_1 \exp\left[ -\frac{C_1}{2} k_L^2 \right], \]  \(6\)

\[
f_2(s, k_L) = \frac{i s}{16\pi} b_2 \exp\left[ -\frac{C_2}{2} k_L^2 \right], \]  \(7\)

\[
f_3(s, k_L) = \frac{i s}{16\pi} b_3 \exp\left[ -\frac{C_3}{2} k_L^2 \right], \]  \(8\)

\[
f_4(s, k_L) = \frac{i s}{16\pi} b_4 \exp\left[ -\frac{C_4}{2} k_L^2 \right], \]  \(9\)

where the $b_j$'s and $C_j$'s are constants.

Substituting Eqs. (4) \sim (9) into Eq. (5), we obtain

\[
F_{AB}(s, k_L) = \frac{i s}{16\pi} \sum_{j_1=0}^{N_{j_1}} \sum_{j_2=0}^{N_{j_2}} \sum_{j_3=0}^{N_{j_3}} \sum_{j_4=0}^{N_{j_4}} \left(-\frac{1}{4\pi}\right)^{j_1+j_2+j_3+j_4-1} \]
Now let us assume that Pomeranchuk's conjecture holds good for quark-quark scattering amplitudes, i.e. the forward scattering amplitudes for quark-quark elastic scatterings satisfy the aforementioned three assumptions, then the constants $b_j$'s are not independent of each other and have the following interrelations:

$$b_1 = b_2^*,$$
$$b_3 = b_4^*,$$  \(11\)

where the asterisk $^*$ denotes the complex conjugate. It must be noticed that for constants $\alpha_j$ we do not have any such relations, since the assumed form of $f_j$'s have the essential singularities in $s$ for nonforward scattering amplitude and we cannot apply the so-called generalized Pomeranchuk conjecture \(^7\) which requires the relations:

$$\alpha_1 = \alpha_2,$$  \(13\)
$$\alpha_3 = \alpha_4.$$  \(14\)

It is easy to see that if Eqs. (13) and (14) are satisfied in addition to Eqs. (11) and (12) Pomeranchuk's conjecture holds good also for hadron-hadron scattering. In our case, however, only Eqs. (11) and (12) are required and the breakdown of the Pomeranchuk's conjecture may occur for hadron-hadron scattering. This is the effects of multiple scattering of the hadron-constituent quarks, i.e. the forward scattering amplitude for hadron-hadron scattering consists not only of the forward quark-quark scattering amplitudes but of the nonforward quark-quark scattering amplitudes also.

\(\S\) 3. Applications

In our model various hadron-hadron and hadron-antihadron scatterings are characterized by the set of the quark-pair number $(N_j)$, which are given in Table I.

It has been observed that there is little difference between $p$-$p$ scattering and $p$-$n$ scattering at high energy, which may suggest that there exists no difference between the type 1 quark-quark scattering and the type 3 quark-quark scattering in our model. Thus we may choose the parameters of the quark-quark scattering amplitudes as follows:
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Table I. The relations between the hadron-hadron scattering process and the set of the quark-pair numbers \((N_1, N_2, N_3, N_4)\).

<table>
<thead>
<tr>
<th>process</th>
<th>(N_1)</th>
<th>(N_2)</th>
<th>(N_3)</th>
<th>(N_4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(pp \rightarrow pp)</td>
<td>5</td>
<td>0</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>(pn \rightarrow pn)</td>
<td>4</td>
<td>0</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>(\bar{p}p \rightarrow \bar{p}p)</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>(pn \rightarrow \bar{p}n)</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>(\pi^- p \rightarrow \pi^- p)</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>(\pi^+ p \rightarrow \pi^+ p)</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>(K^- p \rightarrow K^- p)</td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>(K^+ p \rightarrow K^+ p)</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

\[ b_1 = b_2, \quad (15) \]
\[ a_3 = a_5. \quad (16) \]

As for quark-antiquark scattering, the experimental evidence that there is a difference of the total cross-sections between \(\pi^+ p\) scattering and \(\pi^- p\) scattering may suggest the following result,

\[ a_4 \neq a_4. \quad (17) \]

Furthermore the existence of considerable differences of the total and the differential cross sections between \(p-p\) and \(\bar{p}-\bar{p}\) scattering suggests

\[ a_1 \neq a_5. \quad (18) \]

Now we have five parameters for the quark-quark and the quark-antiquark scattering amplitudes, i.e. \(a_1 (= a_5)\), \(a_3\), \(a_4\), \(\text{Re} b_1 (= \text{Re} b_3 = \text{Re} b_5 = \text{Re} b_4)\) and \(\text{Im} b_1 (= - \text{Im} b_3 = \text{Im} b_5 = - \text{Im} b_4)\), and with these parameters the total cross-sections, the differential cross-sections and the real-imaginary ratio of the scattering amplitudes for various hadron-hadron and hadron-antihadron scatterings can be determined if one neglects the effects of spin dependence which seems to be quite small in a high energy limit.

In Table II the results of the numerical calculations are shown.

§ 4. Discussion of results

It has been shown that Pomeranchuk's conjecture may break down for hadron-hadron scattering even if it holds good for quark-quark scattering. Our model, however, deviates from the experimental data at the following two points:

1. The ratio of the total cross-sections between baryon-baryon scattering and meson-baryon scattering, \(\sigma(MB)/\sigma(BB)\), is too large compared to the experimental data.
2. The real-imaginary ratio of the forward scattering amplitude of proton-antiproton scattering, \(R(\bar{p}p)\), takes a quite large positive value, whereas
Table II. Calculated results for $\bar{p}p$, $p\bar{p}$, $\pi^\pm p$, $K^\pm p$ and $K^{*+}p$ scattering. Parameters used are $\alpha(=\alpha_1=\alpha_3=\alpha_4)$, $\bar{\alpha}(=\alpha_2)$, $\sigma(=Re\,b_1=Re\,b_2=Re\,b_3=Re\,b_4)$ and $R(=Im\,b_1/Re\,b_1=Im\,b_2/Re\,b_2=Im\,b_3/Re\,b_3=Im\,b_4/Re\,b_4)$. Calculated results for hadron-hadron scattering are listed in terms of $A$, $\sigma$ and $R$, where $A$ is the slope of the forward diffractive peak of the differential cross-section, $\sigma$ is the total cross-section and $R$ is the real-imaginary ratio ($=Re\,F_{AB}(s,0)/Im\,F_{AB}(s,0)$) of the forward scattering amplitude.

<table>
<thead>
<tr>
<th>Quark-Quark</th>
<th>$\bar{p}p$</th>
<th>$p\bar{p}$</th>
<th>$\pi^\pm p$</th>
<th>$\pi^\pm p$</th>
<th>$K^\pm p$</th>
<th>$K^{*+}p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$ (GeV/c)$^2$</td>
<td>$\bar{\alpha}$ (GeV/c)$^2$ (mb)</td>
<td>$A$ (GeV/c)$^2$</td>
<td>$\sigma$ (mb)</td>
<td>$R$</td>
<td>$A$</td>
<td>$\sigma$</td>
</tr>
<tr>
<td>6.8</td>
<td>9.8</td>
<td>5.76</td>
<td>0.5</td>
<td>9.9</td>
<td>41.1</td>
<td>0.351</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>$\Theta$</td>
<td>5.40</td>
<td>$\Theta$</td>
<td>9.8</td>
<td>39.1</td>
<td>0.358</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>10.8</td>
<td>5.76</td>
<td>$\Theta$</td>
<td>10.4</td>
<td>41.6</td>
<td>0.357</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>$\Theta$</td>
<td>5.40</td>
<td>$\Theta$</td>
<td>10.4</td>
<td>39.6</td>
<td>0.364</td>
</tr>
</tbody>
</table>

Table III. Calculated results for $\pi^\pm p$ and $K^{*+}p$ scattering with form factors, where the effects of form factors are included in the slope parameters of the quark-quark and quark-antiquark scattering amplitudes.

<table>
<thead>
<tr>
<th>Quark-Quark</th>
<th>$\pi^\pm p$</th>
<th>$\pi^\pm p$</th>
<th>$K^\pm p$</th>
<th>$K^{*+}p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$ (GeV/c)$^2$</td>
<td>$\bar{\alpha}$ (GeV/c)$^2$ (mb)</td>
<td>$A$ (GeV/c)$^2$</td>
<td>$\sigma$ (mb)</td>
<td>$R$</td>
</tr>
<tr>
<td>5.0</td>
<td>7.0</td>
<td>5.40</td>
<td>0.2</td>
<td>6.5</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>$\Theta$</td>
<td>5.04</td>
<td>$\Theta$</td>
<td>6.4</td>
</tr>
<tr>
<td>4.5</td>
<td>6.5</td>
<td>5.40</td>
<td>$\Theta$</td>
<td>6.0</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>$\Theta$</td>
<td>5.04</td>
<td>$\Theta$</td>
<td>6.0</td>
</tr>
</tbody>
</table>
the experiments show the small negative value.

As to the ratio \( \sigma(MB)/\sigma(BB) \), the easiest way to explain the experimental data is to introduce form factors for hadrons; as a results of these form factors the effective \( \alpha' \)'s may vary their values for meson-baryon scattering from the values for baryon-baryon scattering. Some numerical calculations are shown in Table III.

On the other hand it is very difficult to obtain the real-imaginary ratio \( R(\bar{p}p) \) whose value well fits the experimental data, since Eqs. (11) and (12) require the relation \( R(\bar{p}p) \equiv -R(\bar{p}p) \). Therefore to get rid of this troublesome puzzling situation one has to introduce an amplitude with a universally negative real part, which contradicts Eqs. (11) and (12), i.e. the analytic property of the forward scattering amplitude of the quark-quark scattering must be violated. The existence of an amplitude with a universally negative real part, however, may also be useful for better understanding of \( \pi^\pm p \) scattering experiments, since in our model the real-imaginary ratios of the forward \( \pi^\pm p \) scattering amplitudes take small positive values whereas the experiments show the small negative values. The physical meaning of the scattering amplitude with the universally negative real part may be considered as a proof of the existence of a repulsive core.

Since the breakdown of Pomeranchuk’s conjecture in our model comes from the assumption that the diffraction slope for quark and its antiquark scattering process is larger than that for other processes, our results show the same tendency of the degree of the breakdown for several hadron-hadron scatterings as the model proposed by Shiga and Uehara, i.e.

\[
\sigma(\bar{p}p) - \sigma(pp) > \sigma(K^-p) - \sigma(K^+p) > \sigma(\pi^-p) - \sigma(\pi^+p).
\]

This choice of parameters of the diffraction slope, i.e. \( \alpha_2 > \alpha_1 \), also explains the “cross-over” phenomena of the differential cross-sections, together with the following results,

\[
\begin{align*}
\sigma(\bar{p}p) &> \sigma(pp), \\
\sigma(K^-p) &> \sigma(K^+p), \\
\sigma(\pi^-p) &> \sigma(\pi^+p).
\end{align*}
\]

References


5) K. Shiga and M. Uehara, Kyushu University Preprint, KYUSHU-70-E-1.

