Effective Description of a Composite Quark-Lepton Symmetry

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We demonstrate the possibility that a basic \([SU(2)]^N\) symmetry of \(N\) subconstituents, which describes particle and antiparticle transitions, turns out to be at most an "effective" \(SO(2N)\) symmetry and at least an "effective" \(SU(N) \times U(1)\) symmetry of composite quarks and leptons whose states are specified by the \(N\) different kinds of subconstituents. The generators of the "effective" symmetry are identified by the correct algebraic properties specific to \(SO(2N)\) of composite operators constructed from the \([SU(2)]^N\)-operators acting on the composite quark-lepton states. The composite quarks and leptons are found to respect \(SO(4) \times SO(6)\) or \(SU(2)_c \times SU(3)_c \times U(1)_B\), according to a new selection rule, which is generated by the bilinear products of the raising and lowering operators of \([SU(2)]^N\). This construction of the \(SO(4) \times SO(6)\) generators allows us to uniquely define the five quantum numbers of this symmetry even at the subconstituent level. The full \(SO(10)\) generators can be also constructed; however, one needs a newly arranged \("[SU(2)]^N\) symmetry only defined at the composite level, the generators of which turn out to be at most five body operators of the original \([SU(2)]^N\).

§ 1. Introduction

There have been various investigations of subconstituent models of quarks and leptons\(^1\) to understand mysterious properties of quarks and leptons: the existence of three or more generations,\(^2\) weak mixing,\(^3\) etc., and to overcome some difficulties associated with the point-like structures of quarks and leptons and their masses: forming light composite fermions,\(^4\) the smallness of anomalous magnetic moments,\(^5\) etc. Within each generation, their interactions are well described by the broken \(SU(2)_c \times U(1)\) gauge theory for the electroweak interactions and the unbroken \(SU(3)_c\) gauge theory for the strong interactions. The subconstituents are usually introduced in such a way that each one of subconstituents respects only one of these symmetries (more precisely, \(SU(2)_c \times U(1)_{B-L}\) and \(SU(3)_c \times U(1)_B\)).\(^6\) Therefore, the symmetries are a priori adjusted to reproduce the correct symmetries at the quark-lepton level. However, it will be of great interest if one can compose those symmetries instead of imposing them. A quark-lepton symmetry of \(SU(2)_c \times U(1) \times SU(3)_c\), in which

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quarks and leptons belong to the simplest representations, will be composed by a postulated symmetry of the subconstituents. Furthermore, additive quantum numbers of quarks and leptons must be at least expressed in terms of those defined at the subconstituent level.\textsuperscript{7}

This paper presents one of the possibilities for composing the symmetries and for defining operators describing quark-lepton quantum numbers as well as subconstituent quantum numbers in a consistent way. To explain the emergence of the $SU(3)_c$ symmetry was first attempted by Harari and Shupe\textsuperscript{8} in the rishon model as Harari calls it. The present author has suggested a composite model of quarks and leptons related to composite structures of $SO(10)$, where the $SO(10)$ symmetry shows up in the same way as the $SU(3)_c$ symmetry,\textsuperscript{9} and has claimed that there is a systematic way of involving Harari and Shupe's method.\textsuperscript{10}

The important thing is to recognize that the spinor of $SO(2N)$ can be specified by $N$ kinds of subconstituents. It is closely related to the spinor constructed from an $SU(N)$ singlet state successively operated upon by $N$ fermionic creation operators which belong to the $N$-plet of $SU(N)$.\textsuperscript{11} The $SU(N)$ singlet state is regarded as the state occupied by the $N$ fermionic subconstituents, which will be denoted by $s_j$ ($j=1, 2, \ldots, N$). Multiplying the creation operators is replaced by introducing the $N$ basic transitions between $s_j$ and $\bar{s}_j$, particle and antiparticle transitions.\textsuperscript{12} Quarks and leptons are composite states of $(s_1 s_2 \cdots s_N), (\bar{s}_1 \bar{s}_2 \cdots \bar{s}_N), \ldots$, $(s_1 \bar{s}_2 \cdots \bar{s}_N), (\bar{s}_1 s_2 \cdots s_N)$ with odd $N$.\textsuperscript{13} Except for the trivial case with $N=1$, the minimal value of $N$ is 3. The composite model of $N=3$ is nothing but a "modified" version of the rishon model.\textsuperscript{10} The next case with $N=5$ corresponds to the composite model based on $SO(10)$, where the electroweak interactions are treated on the same footing as the strong interactions of $SU(3)_c$.

The $s-\bar{s}$ transition leads to an $SU(2)$ charge algebra. Accepting the five $(N=5)$ $SU(2)$ charges as building blocks of the $SO(10)$ symmetry, we obtain expressions for the generators of $SO(10)$. The five diagonal charges are associated with the third components of the $SU(2)$'s. Other operators are assumed to be in the form of bilinear products of the raising and lowering operators of the $SU(2)$'s. A maximal symmetry of the composite quarks and leptons described by the bilinear operators is found to be $SU(2)_L \times SU(2)_R \times SU(4)$ ($\sim SO(4) \times SO(6)$). This "effective" symmetry is identified through the relevant commutation relations of the bilinear operators which are coincident with those of $SU(2)_L \times SU(2)_R \times SU(4)$ in the restricted internal space involving only the composite quarks and leptons. As a result of the proper identification of the

\textsuperscript{4} It is also implied by the weight diagram of $SO(2N)$. See, for example, B. G. Wybourne, Classical Groups for Physicists (A Wiley-Interscience Publication, 1974), p. 78 & p. 122. The author thanks J. L. Rosner for informing him of this implication.
bilinear operators, we can obtain the quantum numbers of the five subconstituents expressed in terms of the quark-lepton quantum numbers because these are well defined at the subconstituent level.

It is adequate for the present purpose to compose $SO(4) \times SO(6)$ at the composite level. However, if one wishes to obtain the full $SO(10)$ generators at the composite quark-lepton level, one needs to define "modified" raising and lowering operators which form another $\left[ SU(2) \right]^6$ showing up only at the composite level and which include more than the bilinear products of the original $\left[ SU(2) \right]^6$.

§ 2. A particle and antiparticle transition

The importance of the particle and antiparticle transition can be seen in the confining $SU(2)_l$ gauge model. There are the doublets of fermionic subconstituent $\psi$ and bosonic subconstituent $\phi$. The $SU(2)_l$ confines these subconstituents by means of the gluons of $SU(2)_l$, $G^{(a)}_{\mu}$ ($a=1,2,3$). After the confinement is completed, the physical fermions are $(\psi^c, \phi^c)$ and $(\psi^*, \phi^*)$ where $\phi^c$ is defined by $\phi^c = i \sigma^2 \phi^*$. These can be made to form a new doublet of the $\left[ SU(2) \right]$ symmetry followed by the three vector bosons: $(\phi^c - D_{\rho} \phi^c), (\phi^c \gamma_5 D_{\rho} \phi^c)$ and $(\phi^c \gamma_5 D_{\rho} \phi^c)$. It indicates that the $\psi-\psi^c$ transition plays an important role to produce the $\left[ SU(2) \right]$ symmetry at the composite level. It is the symmetry with $(\phi, \phi^c)$ as a doublet. Apparently, we have another possibility of choosing a doublet. Another doublet is given by $(\psi, \gamma_5 \psi^c)$ where $\psi^c$ is defined by $\psi^c = i \sigma^2 C[\tilde{\phi}]^T$. The composite fermions are $(\psi^c \phi^c)$ and $(\psi^* \phi^c)$ which turn out to form a doublet of the $\left[ SU(2) \right]$ with the three vector bosons: $(\bar{\psi} \gamma_\mu \gamma_5 \psi^c), (\bar{\psi} \gamma_5 \gamma_\mu \psi^c)$ and $(\bar{\psi} \gamma_\mu \psi^c)$. It is readily recognized that altogether these four composite fermions, the configurations of which are typically given by $(\psi \phi^c, \phi^c \psi, \psi^c \phi, \phi \psi^c)$, form a vector representation of a new $SO(4)$ symmetry whose generators are given by $T^{(a)}_\rho$ of $SU(2)_\rho$ and $T^{(a)}_\psi$ of $SU(2)_\psi$ ($a=1,2,3$). This is not surprising because the Lagrangian of the $SU(2)_l$ gauge model exhibits a global $SU(2)_\rho \times SU(2)_\psi$ symmetry and there is no direct $\psi-\phi$ coupling.

In what follows, we generalize the idea of employing the particle-antiparticle transition to describe a composite quark-lepton symmetry. However, the relevant vector bosons at least come in four-body composites of the subconstituents. The generators of the "effective" symmetry are constructed at the composite level.

§ 3. Basic ingredients

The five $SU(2)$ charges ($Q_j : j=1 \sim 5$) are given by
if $s_j$ is two-component spinor and then $S_j(x)$ is

$$S_j(x) = \begin{pmatrix} s_j(x) \\ s_j^c(x) \end{pmatrix}.$$  \hspace{1cm} (2)

with

$$s_j^c(x) = C[s_j(x)]^T, \quad C^{-1}\gamma^\mu C = -\gamma^\mu^T \quad \text{and} \quad C^T = -C.$$  \hspace{1cm} (3)

All the quark-lepton states are specified by the five different subconstituents; $s_1(s_1^c), \ldots, s_5(s_5^c)$ as follows:

$$Q_j^a = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}, \quad Q_j^{a\dagger} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}, \quad Q_j^R, Q_j^L,$$

$$Q_{kR}, Q_{kL}, Q_{kR}^- Q_{kL}^-, Q_{kL}^- Q_{kR}^-, \ldots$$  \hspace{1cm} (4)

where $|j\rangle (\downarrow\rangle)$ represents $s_j(s_j^c)$. It is convenient to express these composite states in terms of

$$w_{1L} = (\uparrow \downarrow), \quad w_{1R} = (\uparrow \downarrow),$$

$$w_{2L} = (\downarrow \uparrow), \quad w_{2R} = (\downarrow \uparrow),$$

$$C_0 = (\uparrow \uparrow \uparrow), \quad C_1 = (\uparrow \downarrow \downarrow)$$  \hspace{1cm} (5)

and

$$C_2 = (\downarrow \uparrow \downarrow), \quad C_3 = (\downarrow \downarrow \uparrow),$$  \hspace{1cm} (6)

where we have implicitly used the fact to be stated in (14). Quarks and leptons are described by

$$u_0 = (w_{1L}C_0), \quad \bar{u}_0 = (w_{1R}C_0)$$

and

$$d_0 = (w_{2L}C_0), \quad \bar{d}_0 = (w_{2R}C_0),$$  \hspace{1cm} (7)

where the lepton number is regarded as the fourth color ($a = 0$), which are taken as left-handed states in the $SO(10)$ grand unified model. Right-handed states are given by $L\rightarrow R$. These two sets are distinguished by the sign of $P : P = \Pi_{\pm}^0, (2Q^0)$ which is + for the $R$-handed states and - for the $L$-handed states. These two different states cannot be linked together by the bilinear products of $Q$'s. We now confine ourselves to the space spanned by (7), which means that
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\[ \sum_{a=0,1,2,3} (|w_{il}C_a\rangle \langle w_{il}C_a| + |w_{il}\bar{C}_a\rangle \langle w_{il}\bar{C}_a| + L \leftrightarrow R) \approx I, \quad (8) \]

because we are interested in an "effective" symmetry emerging at the composite quark-lepton level.

§ 4. An effective quark-lepton symmetry 1

The "effective" quark-lepton symmetry is assumed to be generated by the bilinear products of Q's: \( Q^{+}Q^{+}, \ Q^{+}Q^{-} \) and \( Q^{+}Q^{-} \)\( * \) which imply in some sense the four body composites of the subconstituents. We denote these operators by

\[ N_{jk} = Q_{j}^{+}Q_{k}^{-}, \quad (= N_{kj}) \quad (9a) \]
\[ M_{jk} = Q_{j}^{+}Q_{k}^{-}, \quad (= M_{kj}) \quad (9b) \]
\[ \bar{M}_{jk} = Q_{j}^{-}Q_{k}^{-} \quad (= \bar{M}_{kj}) \quad (9c) \]

and

\[ N_{j} = [Q_{j}^{+}, Q_{j}^{-}]. \quad (9d) \]

The commutator relations formed by \( N_{jk}, M_{jk}, \bar{M}_{jk} \) and \( N_{j} \) are

\[ [N_{jk}, N_{kl}] = N_{jl}N_{k}, \quad [M_{jk}, N_{kl}] = M_{jl}N_{k}, \quad (10a, b) \]
\[ [N_{jk}, \bar{M}_{kl}] = M_{jl}N_{k}, \quad [M_{jk}, \bar{M}_{kl}] = N_{jl}N_{k}, \quad (10c, d) \]
\[ [N_{jk}, N_{kj}] = (N_{j}^{TOT}N_{k} - N_{k}^{TOT}N_{j})/2, \quad (11a) \]
\[ [M_{jk}, \bar{M}_{kj}] = (N_{j}^{TOT}N_{k} + N_{k}^{TOT}N_{j})/2, \quad (11b) \]
\[ [N_{j}, N_{kl}] = 2(\delta_{jl}N_{k} - \delta_{jk}N_{l}), \quad (12a) \]
\[ [N_{j}, M_{jk}] = 2M_{jk}, \quad [N_{j}, \bar{M}_{jk}] = -2\bar{M}_{jk}, \quad (12b, c) \]

where \( j \neq k \neq l \) and

\[ N_{j}^{TOT} = \{ Q_{j}^{+}, Q_{j}^{-} \}. \quad (13) \]

We have used the fact that \( Q_{j}^{+}Q_{j}^{+} = Q_{j}^{+}Q_{j}^{-} = 0 \) if the space is limited to satisfy (8) because

\[ Q_{j}^{+} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad \text{and} \quad Q_{j}^{-} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \quad (14) \]

for the spinor of \( j \)-th \( SU(2) \). Furthermore, \( N_{j}^{TOT} \) and \( N_{j} \) are simply given by

\[ ) \quad \text{The} \ U(2) \text{ symmetric construction of the operators for} \ N = 3 \text{ involves} \ Q, \ QQ, \ QQQ \text{ as discussed by Adler.}^{15} \]
and others vanish.

Let us proceed to discuss how the effective symmetry for the composite quarks and leptons arises. We are expecting that this symmetry is associated with the commutation relations of \( (10) \sim (12) \). Our result is that \( W_L \) and \( W_R \) respect the \( \text{SO}(4) \) symmetry \( (N=2) \) and \( C_a \) and \( \tilde{C}_a \) respect the \( \text{SO}(6) \) symmetry \( (N=3) \).

1. \( \text{SO}(4) \)

The relevant operators are \( N_{13}, N_{23}, M_{12}, \tilde{M}_{12}, N_1 \) and \( N_2 \). The commutation relations are given by \( (11a), (11b), (12a \sim c) \). It is readily noticed that an “effective” \( \text{SU}(2)_L \times \text{SU}(2)_R \) (\( \approx \text{SO}(4) \)) symmetry is in fact generated by these operators, which must be identified with the six generators of \( \text{SO}(4) \), \( T_6^{(a)} \) and \( T_6^{(a)} \) \( (a=1 \sim 3) \):

\[
T_6^{(+)} = Q_2^{(+)} Q_1^{(+)} , \quad T_6^{(-)} = Q_1^{(-)} Q_2^{(-)} , \\
T_6^{(a)} = (N_1 - N_2)/4
\]

and

\[
T_6^{(+)} = Q_2^{(+)}, \quad T_6^{(-)} = Q_1^{(-)} Q_2^{(-)}, \\
T_6^{(a)} = (N_1 + N_2)/4.
\]

We have used the fact that \( N_{ij}^{\text{TOT}} = N_j = n_j(\bar{n}_j) = 1 \) for the composite quarks and leptons \( 7 \). Furthermore, the property of \( (14) \) ensures \( [T_L, T_R] = 0 \) over the space specified by \( 8 \).

2. \( \text{SO}(6) \)

The operators are found to form the \( \text{SO}(6) \) generators \( L_6^{(a)} \) \( (a=1 \sim 15) \). These are identified with

\[
L_6^{(1)} + iL_6^{(2)} = Q_4^{(-)} Q_5^{(+)} , \quad L_6^{(4)} + iL_6^{(5)} = Q_6^{(-)} Q_8^{(+)} , \\
L_6^{(6)} + iL_6^{(7)} = Q_5^{(-)} Q_4^{(+)} , \quad L_6^{(9)} + iL_6^{(10)} = Q_6^{(-)} Q_4^{(-)} , \\
L_6^{(11)} + iL_6^{(12)} = Q_5^{(-)} Q_8^{(+)} , \quad L_6^{(13)} + iL_6^{(14)} = Q_4^{(-)} Q_5^{(-)}
\]

and
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\[ L^{(3)} = \frac{(N_3 - N_4)}{4}, \quad L^{(8)} = \frac{(N_3 + N_4 - 2N_5)}{4\sqrt{3}}, \]
\[ L^{(15)} = -\left(\frac{N_3 + N_4 + N_5}{2}\right) \sqrt{6}. \]

(20)

This set is applied to the quartet \(C_a\) while for the charge conjugated \(\bar{C}_a\), all but \(L^{(3), (8), (15)}\) must be replaced by \(-L^{(a)*}\). Notice that because \(s_j\) is transformed into \(s_{j*}\) by the charge conjugation: \(s_{j*} = -C^{-1}g_0s_j^c, -L^{(a)*} (a = 3, 8, 15)\) turns out to be \(L^{(a)}\). \(N_j\) has a configuration of \(s_j^c \rightarrow s_{j*}^c\).

To understand that this identification yields the correct commutator relations, we demonstrate three examples.

(a) \([N_{34}, N_{45}] = N_{35}N_4\): The RHS is coincident with \(-N_{35}\) because the non-vanishing element is only \(\langle C_3|N_{35}N_4|C_4\rangle\) and then (8) allows us to set \(N_{35} = -N_{35}\). The commutator relation of \(L^{(a)}\), \([L^{(1)} - iL^{(2)}, L^{(8)} - iL^{(15)}] = -(L^{(4)} - iL^{(5)})\), is consistent with our identification of \(L^{(a)}\)s. Note that \(I, V\) and \(U\)-spins of \(SU(3)\) are defined by \(N_5 = -1, N_4 = -1\) and \(N_3 = -1\), respectively.

(b) \([M_{48}, M_{53}] = -M_{43}N_5\): The non-vanishing element is \(\langle C_1|N_{43}N_5|C_2\rangle\) \(= -\langle C_1|N_{43}C_2\rangle\), which is consistent with the corresponding commutator relation of \(L^{(a)}\). There exist three \(SU(2)\) subgroups specified by the sign of \(N_j\): \(N_3 = +1\) for \((C_a, C_4)\), \(N_4 = +1\) for \((C_4, C_2)\) and \(N_5 = +1\) for \((C_0, C_5)\).

(c) \([N_{34}, M_{38}] = -N_{34}M_{36}\): The non-vanishing element is given by \(\langle C_0|N_{34}M_{36}|C_1\rangle = \langle C_0|M_{36}|C_1\rangle\), which yields a correct commutation relation; \([L^{(1)} - iL^{(2)}, L^{(11)} - iL^{(12)}] = -(L^{(9)} - iL^{(10)})\).

All other commutation relations can be checked similarly and the correct ones required by \([L^{(a)}, L^{(b)}] = if_{abc}L^{(c)}\) are obtained if the space is restricted to satisfy (8). Unfortunately, the commutation relations (10a~d) with \(j = (1, 2)\) and \(l = (3, 4, 5)\) or vice versa cannot be uniquely defined even if we confine ourselves to the space (8) because the sign of \(N_j\) on the RHS varies depending on the states. Therefore, the full \(SO(10)\) symmetry of the composite quarks and leptons cannot be composed by the bilinear products of the operators.

§ 5. An effective quark-lepton symmetry

To extend our discussion into the case of \(N\) larger than 3, one has to define new operators such that \(Q_{f^+}^{(j+)} = N_1 \cdot N_2 \cdots N_{j-1} Q_{f^+}^{(j)}\)

(21)

instead of \(Q_{f^+}^{(j)}\), which turns out to satisfy

\[ \{\tilde{Q}_{f^+}^{(j)}, \tilde{Q}_{f^+}^{(k)}\} = \delta_{jk} \]

(22)
acting on the space spanned by (7). It is the generalization of (13) \((N_j^{\text{tot}} = 1)\).

This relation corresponds to the anticommutation relation of the fermionic creation and annihilation operators.\(^{11}\) Therefore, it is guaranteed that the \(SO(10)\) generators are in fact constructed from \(Q_j^{(2)}\). So as for the \(SO(2N)\) generators with any \(N\). However, the operators (21) are well defined only at the composite level except for \(j = 1\). The commutation relations of \(Q_j^{(5)}\)'s are

\[
\{Q_j^{(5)}, Q_k^{(5)}\} = 2Q_j^{(3)},
\]

by using the fact that \((N_j)^2 = 1\) over the space defined by (8), which means that \(Q_j^{(5)}\) cannot change \(|s_i^{\gamma}\rangle\) into \(|s_j\rangle\) at the subconstituent level.

The new operators (21) describe the changes of \(j\) subconstituents. The corresponding generators of the "effective" quark-lepton symmetry such as \(N_{jk} = Q_j^{(5)} N_j N_{j+1} \cdots N_{k-1} Q_k^{(5)}\) if \(k > j\). One may regard the relevant vector bosons as \(2(|j-k|+2)\)-body composites of the subconstituents instead of the four body composites as in the previous two cases.

\section{Quantum numbers}

It is worthwhile to note that if there exists a kind of a selection rule as \(\sum_j \Delta Q_j^{(3)} = 0\) for all \(j\) at the level of composites, the "effective" symmetries are \(SU(2)_L \times U(1)_R\) instead of \(SU(2)_L \times SU(2)_R\) and \(SU(3)_C \times U(1)_{\theta-L}\) instead of \(SU(4)\) (\(\cong SO(6)\)). In the \(SO(10)\) case, it leads to \(SU(5) \times U(1)\). According to the value of \(|\Delta Q_j^{(3)}|\), the interactions are classified as

a. The basic \(s - s^c\) transition\(\cdots |\Delta Q_j^{(3)}| = 1\).

b. The effective interactions\(\cdots |\sum_j \Delta Q_j^{(3)}| = 0\) for the \(SU\)-symmetry and \(|\sum_j \Delta Q_j^{(3)}| = 0, 2\) for the \(SO\)-symmetry.

Therefore, the \(SU\)-singlet vacuum is given by

\[
|\uparrow \uparrow \uparrow \uparrow \uparrow \rangle \text{ and } |\downarrow \downarrow \downarrow \downarrow \downarrow \rangle.
\]

The five quantum numbers of the "effective" symmetry group are then given by

\[
T_L^{(3)} = (N_1 - N_2)/4, \quad T_R^{(3)} = (N_1 + N_2)/4,
\]

\[
Y_e^{(3)} = (L^{(3)}) = (N_3 + N_4)/4,
\]

\[
Y_e^{(8)} = 2L^{(8)}/\sqrt{3} = (N_3 + N_4 - 2N_5)/3
\]

and

\[\) It is due to the discussions with K. I. Aoki and M. Bando and especially with T. Kugo. The author also thanks T. Maskawa for suggesting such a possibility.
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\[ B - L = -2\sqrt{2}L^{(15)}/\sqrt{3} = -(N_3 + N_4 + N_5)/3. \]

The electric charge \( Q' \) is defined as

\[ Q' = T^{(p)}_3 + T^{(p)}_2 + \frac{B - L}{2}, \tag{26} \]

according to the usual definition in the \( SO(10) \) grand unified model, leading to

\[ Q' = [3N_1 - (N_3 + N_4 + N_5)]/6. \tag{27} \]

Furthermore, because all the diagonal operators \( N_j \) are well defined at the subconstituent level as well as the composite level, one can find the corresponding quantum numbers of the five subconstituents. These are listed in Table I.

<table>
<thead>
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<th>subconstituent</th>
<th>( T^{(p)}_3 )</th>
<th>( T^{(p)}_2 )</th>
<th>( Y^{(p)}_3 )</th>
<th>( Y^{(p)}_2 )</th>
<th>( B-L )</th>
<th>( Q' )</th>
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<td>1/4</td>
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<td>0</td>
<td>0</td>
<td>1/2</td>
</tr>
<tr>
<td>( s_2 )</td>
<td>-1/4</td>
<td>1/4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( s_3 )</td>
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<td>0</td>
<td>1/4</td>
<td>1/6</td>
<td>-1/3</td>
<td>-1/6</td>
</tr>
<tr>
<td>( s_4 )</td>
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<td>0</td>
<td>-1/4</td>
<td>1/6</td>
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<td>-1/6</td>
</tr>
<tr>
<td>( s_5 )</td>
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<td>0</td>
<td>0</td>
<td>-2/6</td>
<td>-1/3</td>
<td>-1/6</td>
</tr>
</tbody>
</table>

§ 7. Epilogue

The composite structure of (7) suggests that quarks and leptons can be regarded as composite states of the bosonic \( w \)'s and the fermionic \( C \)'s. Their symmetries are \( SO(4) \) for \( w \)'s and \( SO(6) \) for \( C \)'s. However, these symmetries are not a priori imposed but are generated due to the internal arrangement of the subconstituents carrying \([SU(2)]^N \) \((N = 2 \text{ or } 3)\). An intuitive discussion based on such a composite structure has been made to give a clue of understanding possible origins of the number of generations and the weak mixing. An explicit example is given by J. L. Rosner in the case \( N = 3 \) in Ref. 17.

We can have in principle the \( SO(10) \) operators constructed from the raising and lowering operators of the “modified” \([SU(2)]^p \) defined at the composite level, which still describe the \( s \cdot s^c \) transitions. However, if we insist that the (gauge) bosons are composites of definite numbers of subconstituents \((= 4 \text{ in the present formulation by using } Q^{25}, (1))\), the “effective” symmetry of the composite quarks and leptons will be \( SO(4) \times SO(6) \). The composite particles we have introduced are listed in Table II.
Table II. Classification of composites: The terminologies in the parentheses are the nicknames of those composites. *Kona* means a top in Japanese. The other two are introduced by Terazawa.3)

<table>
<thead>
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<th>#</th>
<th>subconstituent</th>
<th>composites</th>
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<tbody>
<tr>
<td>1</td>
<td>$\nu_L$ &amp; $\nu_R$</td>
<td>(subkomas)</td>
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<tr>
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<td>$C_6$ &amp; $\bar{C}_6$</td>
<td>(wakens)</td>
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<td>3</td>
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<td>(chroms)</td>
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<td>4</td>
<td>$\gamma$</td>
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</tr>
<tr>
<td>5</td>
<td>quarks and leptons</td>
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To summarize our discussion, we have explicitly shown that the five quantum numbers of the subconstituents defined by the third components of the $[SU(2)]^5$ turn out to be the five diagonal generators of $SO(4) \times SO(6)$ realized at the composite quark-lepton level. Therefore, these are uniquely expressed in terms of quark-lepton quantum numbers even at the subconstituent level. The 16 states could come in the 16-plet of $SU(16)$ as a maximal symmetry. Gauging the whole $SU(16)$ symmetry is not allowed owing to the anomaly-free condition. The maximal anomaly-free symmetry is $SO(10)$.19) According to the present discussion, there have to exist some dynamical reasons why the specific bilinear operators $(9a-\sim c)$ are effective at the composite level to produce $SO(4) \times SO(6)$ (or the specific channel corresponding to the operators $(21)$ to produce $SO(10)$). There could show up an $SO(4) \times SO(6)$ gauge theory if those vector bosons become massless and (almost) massless quarks and leptons arise.* We at least have massless gauge bosons associated with $T^{(5)}_L$, $T^{(5)}_R$, $L^{(3)}$, $L^{(8)}$ and $L^{(15)}$ if the basic interactions are gauged.

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* The speculative discussion is also made by H. Harari and N. Seiberg in Ref. 1). Various constraints on massless particles are discussed by Weinberg and Witten, and Kugo and Uehara.8)
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