Hoyle–Lyttleton Accretion onto Superdisks

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Abstract

We examine Hoyle–Lyttleton accretion onto superdisks, where the mass-accretion rate greatly exceeds the critical one and the luminosity is of the order of the Eddington one. Hoyle–Lyttleton accretion onto a luminous accretion disk is drastically changed from the spherical case, due to the non-spherical nature of disk radiation fields. In edge-on accretion, for example, mass accretion does not cease at $\Gamma = 1$, but continues in the regime of $\Gamma \geq 1$, where $\Gamma$ is the disk luminosity normalized by the Eddington one. We found that Hoyle–Lyttleton accretion onto superdisks is not a simple extension of that onto standard accretion disks, where the mass-accretion rate gradually decreases with increasing $\Gamma$. Instead, in the case of a superdisk, the mass-accretion rate increases with increasing $\Gamma$. This is due to a geometrical shadowing effect. That is, in contrast to the infinitesimally-thin standard disk, the superdisk has finite thickness. As $\Gamma$ increases, the relative thickness increases and the shadowing region in the equatorial direction increases. We also briefly discuss the canonical luminosity in the steady state and the system evolution.

Key words: accretion, accretion disks — radiation mechanisms — superdisks — X-rays: stars

1. Introduction

Recently, Hoyle–Lyttleton accretion onto a luminous accretion disk was first examined, and shown that it is drastically different from that onto a spherical source (Fukue, Ioroi 1999; cf. Fukue 1999).

In classical Hoyle–Lyttleton accretion onto a spherical source of mass $M$ (Hoyle, Lyttleton 1939; Bondi, Hoyle 1944), accretion takes place in an axially symmetric manner around the so-called accretion axis, and the accretion rate $\dot{M}$ is given by

$$\dot{M} = \dot{M}_{HL} = \pi R_{HL}^2 \rho_\infty v_\infty = \frac{4 \pi G^2 M^2 \rho_\infty}{v_\infty^3},$$

where $\rho_\infty$ and $v_\infty$ are the gas density and velocity at infinity, and $R_{HL}$ the classical Hoyle–Lyttleton accretion radius:

$$R_{HL} = \frac{2GM}{v_\infty^2}.$$  

When the central object is a luminous source with luminosity $L$ (e.g., Taam et al. 1991), the accretion rate reduces to

$$\dot{M} = \dot{M}_{HL}(1 - \Gamma)^2. \quad (3)$$

Here, $\Gamma$ is the central luminosity normalized by the Eddington one:

$$\Gamma \equiv \frac{L}{L_E}, \quad (4)$$

where $L_E (= 1.25 \times 10^{38} M_\odot \text{erg s}^{-1})$ is the Eddington luminosity.

On the other hand, if the central object is a compact star with a luminous accretion disk, the radiation field becomes “non-spherical” (Fukue, Ioroi 1999; Fukue 1999). That is, the radiative flux $F$ at distance $R$ depends on the polar angle $\theta$ (see figure 1) as

$$F = \frac{L}{2\pi R^2 \cos \theta}. \quad (5)$$

In such a case the axial symmetry around the accretion axis breaks down and the accretion rate $\dot{M}$ depends on an inclination angle $i$ between the accretion axis and the symmetry axis of the disk. In the case of pole-on accretion ($i = 0$), the accretion rate, which becomes smaller than that of the spherical case, is approximately expressed as

$$\dot{M} \sim \dot{M}_{HL}(1 - \Gamma)(1 - 2\Gamma). \quad (6)$$

In the case of edge-on accretion ($i = 90^\circ$), the shape of the accretion cross-section varies from a circle ($\Gamma = 0$), an ellipse, a hollow ellipse ($\Gamma \sim 0.5$), and a twin lobe ($\Gamma \gtrsim 0.64$). The accretion rate, which is larger than that of the spherical case, is approximately expressed as

$$\dot{M} = \dot{M}_{HL}(1 - \Gamma) \quad (7)$$
for $\Gamma \leq 2/\pi = 0.6366$, and

$$M = \dot{M}_{\text{HL}} \left(1 - \Gamma - \frac{2}{\pi} \phi_0 + \Gamma \sin \phi_0\right)$$

(8)

for $\Gamma \geq 2/\pi$, where $\cos \phi_0 = 2/(\pi \Gamma)$ (Fukue 1999).

It was emphasized that in Hoyle–Lyttleton accretion onto a luminous disk mass accretion does not cease at $\Gamma = 1$, but may continue in the regime of $\Gamma > 1$. Hence, the anisotropic radiation field of accretion disks drastically changes the accretion nature of a Hoyle–Lyttleton type.

In that study (Fukue, Iorio 1999), they supposed the traditional geometrically thin disk, where the disk luminosity is less than $L_\text{E}$. Hence, the accretion nature in the super-Eddington regime is still unknown. In this paper we thus examine Hoyle–Lyttleton accretion onto a disk source in the super-Eddington regime, using an appropriate model. In such a supercritical disk, where the mass-accretion rate greatly exceeds the critical rate, the disk has a finite thickness. As a result, equatorial radiation is suppressed and the accretion nature again drastically changes.

In the next section we summarize the superdisk model and its radiation fields. In section 3 we derive the mass-accretion rate of Hoyle–Lyttleton accretion onto the superdisk. In sections 4 and 5 we examine the canonical luminosity and evolution of the present system, respectively. The final section is devoted to concluding remarks.

2. Superdisk Model

Superaccretion disks (or shortly superdisks) around the central object of mass $M$ are such disks that the mass-accretion rate $\dot{M}$ greatly exceeds the critical rate $M_{\text{crit}}$ (see, e.g., Kato et al. 1998). That is,

$$\dot{M} \gg \dot{M}_{\text{crit}} \equiv \frac{L_\text{E}}{c^2} = 1.4 \times 10^{17} \frac{M}{M_\odot} \text{ g s}^{-1}$$

(9)

or

$$\dot{m} \equiv \frac{\dot{M}}{M_{\text{crit}}} \gg \frac{1}{\eta},$$

(10)

where $\eta$ is the efficiency of the release of the accretion energy.

Such superdisks were first proposed by Abramowicz et al. (1988), and extensively studied by several researchers (Szuszkiewicz et al. 1996; Beloborodov 1998; Watarai, Fukue 1999; Watarai et al. 2000; Mineshige et al. 2000; Fukue 2000). In this study we used a self-similar model (Watarai, Fukue 1999) with the help of numerical results (Watarai et al. 2000).

In the self-similar model of superdisks (Watarai, Fukue 1999; Fukue 2000), the velocity components and sound speed are assumed to be proportional to the Keplerian velocity. As a result, the relative thickness becomes constant. From the continuity equation, the surface density and disk optical depth are obtained. Assuming that the radiation pressure is dominant, the disk flux and disk central temperature is obtained. Finally, using the optical depth, we obtain the disk surface temperature and total luminosity. Comparing the numerical results (e.g., Watarai et al. 2000), the self-similar model cannot reproduce the innermost region, which forms high-energy spectra, but well reproduces the overall structures.

2.1. Configuration and Luminosity

The self-similar solution (Watarai, Fukue 1999; cf. Narayan, Yi 1994) has a conical surface with a declination angle of $\delta$. The relative thickness becomes

$$\frac{H}{r} = \tan \delta = \text{const},$$

(11)

where $H$ is the disk half-thickness and $r$ the radial distance. The disk luminosity $L$ is evaluated as

$$\Gamma \equiv \frac{L}{L_\text{E}} = \frac{3}{4} \tan \delta \ln \frac{r_{\text{out}}}{r_{\text{in}}},$$

(12)

where $r_{\text{in}}$ and $r_{\text{out}}$ are the inner and outer radii, respectively. In this paper we set $r_{\text{in}} = 3R_g$ and $r_{\text{out}} = 3 \times 10^5 R_g$ (i.e., $r_{\text{out}}/r_{\text{in}} = 10^5$), where $R_g = G M/c^2$ is the Schwarzschild radius. This yields $L/L_\text{E} \approx 8.6347 \tan \delta$. This relation is consistent with the results of the numerical model (Watarai et al. 2000), as shown below.

According to a numerical calculation by Watarai et al. (2000), the relative thickness and disk luminosity are roughly approximated in terms of the normalized accretion rate $\dot{m}$ as

$$\frac{H}{r} \sim \frac{1}{4} \ln \left(1 + \frac{\dot{m}}{20}\right),$$

(13)

$$\frac{L}{L_\text{E}} \sim 2 \ln \left(1 + \frac{\dot{m}}{20}\right),$$

(14)

for $\alpha = 0.1$. These equations are fitting formulae using the numerical results obtained by Watarai et al. (2000). Since $H/r = \tan \delta$, these relations yield $L/L_\text{E} \approx 8 \tan \delta$, which is consistent with the result of a self-similar model.

In table 1 several values of configuration and luminosity are summarized for the convenience of readers. In table 1 $f_{\text{adv}}$ is an advection parameter.

2.2. Radiation Fields

The radiation fields produced by the luminous disk are not spherical, but anisotropic, in the sense that they are enhanced in the poleward direction. This anisotropy of disk radiation fields is important in the present Hoyle–Lyttleton accretion onto the disk.

For a disk source with luminosity $L$ and inclination angle $i$ the radiative flux $f$ at distance $R$ is expressed as $f = (L/4\pi R^2) \cos i$ ($R$ is assumed to be sufficiently larger than the disk size). In the present case of supercritical accretion disks, the radiative flux at infinity would be modified, due to the geometrical thickness of the disk (Fukue 2000).

In figure 2 the radiative flux $f$ normalized by the the spherical case $L/(4\pi R^2)$ is shown as a function of $i$ for several declination angles $\delta$. As shown in figure 2, the radiative flux at

<table>
<thead>
<tr>
<th>$m$</th>
<th>$L/L_\text{E}$</th>
<th>$H/r = \tan \delta$</th>
<th>$\delta$</th>
<th>$f_{\text{adv}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.13</td>
<td>1</td>
<td>0.116</td>
<td>6°62</td>
<td>0.074</td>
</tr>
<tr>
<td>0.34</td>
<td>2</td>
<td>0.232</td>
<td>13°06</td>
<td>0.146</td>
</tr>
<tr>
<td>0.70</td>
<td>3</td>
<td>0.348</td>
<td>19°19</td>
<td>0.214</td>
</tr>
<tr>
<td>1.28</td>
<td>4</td>
<td>0.463</td>
<td>24°8</td>
<td>0.277</td>
</tr>
<tr>
<td>2.24</td>
<td>5</td>
<td>0.579</td>
<td>30°1</td>
<td>0.336</td>
</tr>
</tbody>
</table>
in the super-Eddington regime (\(\Gamma \leq 1.65\)) the accretion rate increases as \(\Gamma\) increases (a thick solid curve). This can be understood as follows. In the super-Eddington regime the relative thickness \(H/r\) (or \(\delta\)) of the superdisk increases as \(\Gamma\) increases (cf. table 1). As a result, the self-occluding region in the equatorial direction enlarges. Hence, mass accretion from the equatorial direction increases.

Indeed, if the flux vanishes at \(i > \delta\), the normalized accretion area becomes roughly

\[
\frac{A_{\text{acc}}}{\pi R_{\text{HL}}^2} = \frac{\delta}{\pi/2} = \frac{2}{\pi} \tan^{-1} \left( \frac{\Gamma}{0.75 \ln 10^5} \right),
\]

which is just the rightward part of the curve in figure 3. This is because the gas is assumed to accrete onto the central object only in the shadow of the disk.

Together with the previous results (Fukue, Ioroi 1999; Fukue 1999), we finally have the following expression for Hoyle–Lyttleton accretion onto the disk under the edge-on case and with general \(\Gamma\):

\[
\frac{\dot{M}}{M_{\text{HL}}} \equiv f (\Gamma, i = 90^\circ) = \begin{cases} 
(1 - \Gamma) \\
\left( 1 - \Gamma - \frac{2}{\pi} \varphi_0 + \Gamma \sin \varphi_0 \right) \\
\frac{2}{\pi} \tan^{-1} \left( \frac{\Gamma}{0.75 \ln 10^5} \right)
\end{cases}
\]

for \(0 \leq \Gamma \leq 0.64\), \(0.64 \leq \Gamma \leq 1.65\) (16),

where \(\cos \varphi_0 = 2/(\pi \Gamma)\).

### 4. Canonical Luminosity

In this section we examine the condition for steady-state accretion, in which the mass-accretion rate and disk luminosity are determined consistently within the present model. In the present model the mass-accretion rate is given by equation (16). On the other hand, the accretion luminosity \(L\) is related to the mass-accretion rate \(\dot{M}\) via \(L = \eta \dot{M} c^2\) (and therefore, \(\Gamma = \eta M c^2/L_E\)).

Introducing the Eddington accretion rate \(\dot{M}_E\) by \(\dot{M}_E = M_E/(\eta c^2)\), we have

\[
\frac{\dot{M}}{M_{\text{HL}}} = \frac{\dot{M}_E}{M_{\text{HL}}} \Gamma = \frac{1}{m_{\text{HL}}} \Gamma,
\]

(17)
5. System Evolution

We now examine the long-term evolution of the Hoyle–Lyttleton accretor onto a superdisk (Fukue 1999). We first consider the edge-on case.

The basic equations for the evolution of the superdisk accretor in the present case are expressed as

\[
\frac{dM}{dt} = \frac{4\pi G^2 M^2 \rho}{v^3} f (\Gamma, i),
\]

\[
\frac{dv}{dt} = -\frac{1}{M} \frac{dM}{dt} v - \frac{4\pi (\ln \Lambda) G^2 M \rho}{v^3} f (\Gamma, i) v,
\]

\[
\Gamma = 2 \ln \left( 1 + \frac{\eta c \sigma_T \dot{M}}{2 \frac{4\pi G m_p M}{} \right),
\]

where \(\dot{m}_{\text{HL}}\) is a dimensionless parameter of the system, defined by

\[
\dot{m}_{\text{HL}} \equiv \frac{M_{\text{HL}}}{M_E} = \frac{\eta c G M \sigma_T \rho_\infty}{m_p v_\infty^3} = 0.2648 \frac{\eta M}{0.1 \times 10 M_\odot} \frac{n_\infty}{10^3 \text{ cm}^{-3}} \left( \frac{v_\infty}{10 \text{ km s}^{-1}} \right)^{-3}.
\]

For a given parameter \(\dot{m}_{\text{HL}}\), these two relations (16) and (17) have an intersection point on the \((\Gamma, M)\)-plane (cf. figure 3). This intersection (solution) is the canonical luminosity \(L_{\text{can}}\) (or \(\Gamma_{\text{can}}\)) and the canonical accretion rate \(M_{\text{can}}\) (Fukue, Ioroi 1999). In figure 4 the canonical luminosity \(\Gamma_{\text{can}}\) is shown as a function of \(\dot{m}_{\text{HL}}\). The canonical accretion rate is obtained by \(M_{\text{can}}/M_E = L_{\text{can}}/L_E = \Gamma_{\text{can}}\) or by \(M_{\text{can}}/M_{\text{HL}} = \Gamma_{\text{can}}/\dot{m}_{\text{HL}}\).

As shown in figure 4, the canonical luminosity \(\Gamma_{\text{can}}\) increases with \(\dot{m}_{\text{HL}}\) in the super-Eddington regime of \(\Gamma \gtrsim 1.65\), similar to that in the sub-Eddington region but more rapidly.

\[
\frac{\dot{M}}{\dot{m}_{\text{HL}0} \dot{M}^{5+3\ln \Lambda}} = f (\Gamma, i),
\]

\[
\Gamma = 2 \ln \left[ 1 + \frac{1}{2} \dot{m}_{\text{HL}0} \dot{M}^{4+3\ln \Lambda} f (\Gamma, i) \right].
\]

where \(f (\Gamma, i)\) is given by equation (16) and \(\ln \Lambda \sim 10\) is the so-called Coulomb logarithm (see Fukue 1999).

Measuring the mass, velocity, and time in units of the initial mass \(M_0\), initial velocity \(v_0\), and the Eddington time \(t_E\) defined by \(t_E \equiv M/E = 4.53 \times 10^7 (\eta/0.1) \text{ yr}\), we obtain the basic equations in a dimensionless form. Moreover, eliminating \(\dot{v}\) from the equations, where the hat means dimensionless quantities, we finally obtain the equations for evolution:

\[
\frac{d\hat{M}}{dt} = \dot{m}_{\text{HL}0} \dot{M}_{\text{HL}0} \frac{\dot{M}^{5+3\ln \Lambda}}{\dot{M}_{\text{HL}0}^{5+3\ln \Lambda}} f (\Gamma, i),
\]

\[
\Gamma = 2 \ln \left[ 1 + \frac{1}{2} \dot{m}_{\text{HL}0} \dot{M}_{\text{HL}0} \dot{M}^{4+3\ln \Lambda} f (\Gamma, i) \right].
\]

The parameters are the normalized accretion rate \(\dot{m}_{\text{HL}0}\) and the Coulomb logarithm \(\ln \Lambda\) (= 10).

The numerical solutions of equations (22) and (23) are shown in figure 5 for several parameters. In figure 5 the abscissa is the dimensionless time \(\hat{t}\), whereas the ordinates are the dimensionless mass \(\hat{M}\) (dashed curves), the dimensionless velocity \(\hat{v}\) (dotted curves), and the normalized luminosity \(\Gamma\) (solid curves). The parameters are \(\dot{m}_{\text{HL}0} = 10\) (a thin curve) and 20 (a thick curve).
6. Concluding Remarks

We have examined Hoyle–Lyttleton accretion onto a superdisk, where the mass-accretion rate highly exceeds the Eddington one. Because the radiation field of accretion disks is anisotropic, the accretion nature is quite different from that of the spherical case, qualitatively as well as quantitatively. In the sub-Eddington regime, as the disk luminosity increases, the mass-accretion rate decreases, but does not cease at $\Gamma = 1$ like the spherical case. In the super-Eddington regime examined in the present paper, as the disk luminosity increases, the mass-accretion rate increases for $\Gamma > 1.65$, because of self-occultation due to the finite thickness of the disk.

In the present superdisk case, similar to the disk case, since accretion mainly takes place in the disk plane, the anisotropic radiation fields of the central accretion disks automatically act to maintain the disk plane (the direction of jets associated with the disk is also maintained).

Such Hoyle–Lyttleton accretion onto the superdisk may take place in giant molecular clouds or in Thorne–Zytkow objects (Fukue 1999). For example, in the case of giant molecular clouds, the typical velocity of the Hoyle–Lyttleton accretor is $10 \text{ km s}^{-1}$; this is a slow mover. The crossing time of this slow mover is about $10^4 \text{ yr}$. Hence, in order to maintain Hoyle–Lyttleton accretion for a long time, the slow mover should revolve in giant molecular clouds in an orbital motion. Similarly, the fast runner in Thorne–Zytkow objects (Fukue 1999) would be a Hoyle–Lyttleton accretor, if it maintains orbital motion.

In the present situation we implicitly assumed that the disk sky is clear; we ignored the effect of disk corona and/or a gaseous environment. If a disk corona exists, the disk radiation would be reprocessed by the corona gas. Some part of the reprocessed light turns onto the disk and heats it up, while some part reaches the occulted region in the equatorial direction. This reprocessed light would modify the present results. Furthermore, the intense disk radiation may ionize the surrounding (interstellar) gas and melt the dust. These effects are beyond the scope of this paper.

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