A Quadratic Theory of Gravitation with Einstein Limit and Dynamical Torsion

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We discuss a particular $R + R^2$ gravitational theory in the Riemann-Cartan space which reduces to the usual Einstein theory when the torsion vanishes, in view of the Bach-Lanczos identity. The theory allows propagation of torsion in vacuum.

The well-known difficulties concerning the construction of a correct quantum version for General Relativity (GR) have led to the investigation of alternative theories of gravitation. In particular it has been argued that theories of the type $R + R^2$ would be renormalizable.\textsuperscript{1,2} Recently, theories of this type in the context of the Riemann-Cartan space $U_4$ have attracted considerable attention.\textsuperscript{3-8} Even though the gravitational Lagrangians usually proposed generate equations with propagation of torsion, those equations do not have the Einstein limit in the torsionless case, not even in the macroscopic domain. The aim of this article is to present a quadratic model with not only torsion propagation but also the restitution of Einstein gravity when the torsion tensor vanishes. This is accomplished with the aid of the Bach-Lanczos identity,\textsuperscript{9} adapted to the $U_4$ space.

Unorthodox variational principles have been used in the literature to study alternative theories of gravitation\textsuperscript{10-15} within the contexts of $U_4$ and $V_4$ (Riemann space). Besides the Palatini variation, which in general gives results that are not equivalent to those of the Hilbert (orthodox) variation, there has been also used in $V_4$ a modified method characterized by the introduction of a Lagrange multiplier which imposes a constraint. The variations in this last case are carried out with respect to: the metric $g_{\alpha\beta}$, the symmetric affine connection $\Gamma^\alpha_{\beta\gamma}$ and the Lagrange multiplier $\Lambda_{\alpha}\,^\beta$ independently.\textsuperscript{12} This procedure is extendible to the $U_4$ space.\textsuperscript{14} A particular form of this extension will be used here.

Let us first study the general action

$$I = \int \left[ L + L_M + \sqrt{-g} \Lambda_{\alpha}^{\beta\gamma} \left( \Gamma^\alpha_{\beta\gamma} - \frac{1}{2} \Gamma^\alpha_{\beta\gamma} - 2S_{\beta\gamma} \right) \right] d^4 x ,$$

where $L = L(g_{\mu\nu}, \Gamma^\alpha_{\mu\nu}, \partial_\mu \Gamma^\alpha_{\nu\lambda}, S_{\mu\nu}, \partial_\lambda S_{\mu\nu})$ is the gravitational Lagrangian, $L_M = L_M (g_{\mu\nu}, S_{\mu\nu}, \psi_\alpha, \partial_\lambda \psi_\alpha)$ is the matter Lagrangian, $\Gamma^\alpha_{\beta\gamma}$ and $\Gamma^\alpha_{\mu\nu} = S_{\mu\nu}$ are, respectively, the symmetric and the antisymmetric parts of the connection, and $\psi_\alpha$ is a matter field. (We follow the notation of Ref. 16.)

By varying independently $g_{\mu\nu}, \Gamma^\alpha_{\mu\nu}, S_{\mu\nu}$ and $\Lambda_{\alpha}^{\beta\gamma}$ we get

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\[
\frac{\delta L}{\delta g_{\mu\nu}} + \sqrt{-g} M^\mu{}^\nu = -\frac{\delta L_M}{\delta g_{\mu\nu}},
\]
(2)

\[
\frac{\delta L}{\delta \Gamma^\lambda_{(\mu\nu)}} + \sqrt{-g} \Lambda_{(\mu\nu)} = 0,
\]
(3)

\[
\frac{\delta L}{\delta S_{\mu\nu}^\lambda} - 2\sqrt{-g} g_{\lambda\rho} A^{(\mu\nu)\rho} = -\frac{\delta L_M}{\delta S_{\mu\nu}^\lambda},
\]
(4)

\[
\Gamma^\lambda_{(\mu\nu)} = \begin{pmatrix} \lambda \\ \mu \\ \nu \end{pmatrix} + 2 S_{(\mu\nu)}^\lambda,
\]
(5)

where \( M^\mu{}^\nu \) is obtained by the variation of the third term appearing in (1). From (5) and (2) we get

\[
M^\mu{}^\nu = \frac{1}{2} \left[ \nabla_\sigma (A^\mu{}^\sigma + A^\nu{}^\sigma - A^\sigma{}^\nu) \\
+ 2 \left[ 2 A^\sigma{}^\rho (S^\rho{}^\nu) + S^\sigma{}^\rho (A^\rho{}^\nu - 2 A^{(\rho\nu)\rho}) - 2 S^{(\nu_{(\rho\sigma)}} A^{\mu)}{}^{\rho\sigma} \right] \right],
\]
(6)

where \( \nabla_\sigma \) is the covariant derivative corresponding to \( \Gamma^\lambda_{(\mu\nu)} \). The constraint (5) is equivalent to the \( U_4 \) postulate, or to the assumption that the connection has the form

\[
\Gamma^\lambda_{(\mu\nu)} = \begin{pmatrix} \lambda \\ \mu \\ \nu \end{pmatrix} + K_{\mu\nu}^\lambda
\]
(7)

with

\[
K_{\mu\nu}^\lambda = S_{\mu\nu}^\lambda - S_{\nu\mu}^\lambda + S_{\mu\nu}^\lambda.
\]
(8)

For a given \( L \) and \( L_M \), Eqs. (2) and (4), after the elimination of the Lagrange multiplier, give explicitly two groups of field equations. The first (from (2)) will be called Einstein-type equations, the other (from (4)) Cartan-type equations. We notice that the introduction of the Lagrange multiplier considerably abbreviates the calculations of the variational derivatives.

We consider the gravitational Lagrangian density \( L = L_0 + L_1 \) with

\[
L_0 = \alpha \sqrt{-g} R,
\]
(9)

\[
L_1 = \sqrt{-g} \left[ R^2 + \beta R_{\mu\nu} R^{\mu\nu} - (\beta + 4) R_{\mu\nu} R^{\mu\nu} + \gamma R_{\alpha\beta\mu\nu} R^{\alpha\beta\mu\nu} \\
+ (1 - \gamma) R_{\alpha\beta\mu\nu} R^{\mu\nu\alpha\beta} + \lambda R_{\alpha\beta\mu\nu} R^{\alpha\beta\mu\nu} \right],
\]
(10)

where \( \alpha, \beta, \gamma \) and \( \lambda \) are free parameters. The square brackets in \( R_{[\alpha\beta\gamma]} \) denote the process of alternation. We follow the definitions of Ref. 16) for the curvature and Ricci tensors.

The Lagrangian \( L_1 \) is the most general quadratic Lagrangian in \( U_4 \) which depends only on the curvature invariants and which generates the Bach-Lanczos identity when the torsion vanishes, since in this case we have

\[
\delta \int L_1 d^4 x = \delta \int \sqrt{-g} \left( R^2 - 4 R_{\mu\nu} R^{\mu\nu} + R_{\alpha\beta\mu\nu} R^{\alpha\beta\mu\nu} \right) d^4 x = 0.
\]
(11)
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We observe that the symmetry properties of the Riemann-Christoffel and Ricci tensors
\[ R^\mu\nu\rho\sigma = R^{\sigma\rho\mu\nu} \quad \text{and} \quad R^\nu\rho = R^{\rho\nu} \]
hold in \( V \), but not in \( U \), although the properties
\[ R^{\mu\nu\rho\sigma} = - R^{\rho\mu\sigma\nu} = - R^{\mu \nu \rho \sigma} \]
remain valid in \( U \) as well as in \( V \). This fact has led to a misquotation in the literature, being (11) frequently referred to as an identity in \( U \). The true extended Bach-Lanczos identity in Riemann-Cartan space is in fact

\[ \delta \sqrt{-g} \left( R^2 - 4 R_{\mu\nu} R^{\mu\nu} + R_{\sigma\rho\mu\nu} R^{\mu\nu\rho\sigma} \right) d^4 x \equiv 0 , \]

as clarified by Hayashi and Shirafuji. The last term in (10) vanishes in the torsionless limit in view of the identity \( R_{\sigma\rho\mu\nu} R^{\rho\mu\sigma\nu} \), that holds in Riemann space (Bianchi identity). This term contains the invariant \( R_{\sigma\rho\mu\nu} R^{\alpha\beta\rho\sigma} \), as well as the invariant \( R_{\sigma\rho\mu\nu} R^{\alpha\beta\rho\sigma} \). Then all the quadratic invariants built with the curvature tensor alone are included in \( L \). We can also see that \( \delta L_1 d^4 x \), with \( L_1 \) expressed by (10), becomes an identity if we take \( \beta = 0 = \gamma = \lambda \). In \( V \) the Lagrangian \( L = L_0 + L_1 \), obtained from (9) and (10) by taking \( S_{\alpha\beta\gamma} = 0 \), is the most general one (with at most quadratic terms in \( R, R_{\mu\nu} \) and \( R_{\alpha\beta\gamma} \)) from which Einstein equations in vacuum are derived. (Neither cosmological nor parity violation terms are taken into account here). An adequate choice of the parameter \( \alpha \), namely, \( \alpha = 1/2 K \), where \( K \) is Einstein gravitational constant, is of course needed in order to yield the true GR limit when torsion vanishes.

The substitution of \( L \) in (2) and (4) leads to the Einstein-type and Cartan-type equations, which in vacuum read, respectively

\[ - \alpha G^{\mu\nu} + M^{\mu\nu} + N^{\mu\nu} = 0 , \]

where \( G^{\mu\nu} \) is the Einstein tensor, \( M^{\mu\nu} \) is given by (6), with

\[ A^{\mu\nu}_\alpha = -2 \alpha A_{\alpha}^{\mu\nu} - 2 B_{\alpha}^{\mu\nu} + C_{\alpha}^{\mu\nu} + D_{\alpha}^{\mu\nu} , \]

\[ A_{\alpha}^{\mu\nu} = g^{\mu\nu} S_{\alpha}^{\lambda} - \delta_{\alpha}^{(\mu} S_{\lambda}^{\nu)} + S_{\alpha}^{(\mu\nu)} , \]

\[ B_{\alpha}^{\mu\nu} = 2 \left( g^{\mu\nu} R - 4 R^{\mu\nu} \right) S_{\alpha}^{\lambda} + \overline{R} \delta_{\alpha}^{(\nu} S_{\mu)}^{\lambda} + R S^{\nu} g^{\mu\nu} R - \nabla_{\alpha} (g^{\mu\nu} R) \]

\[ + 2 \beta \left( R^{(\lambda\mu)} S_{\alpha}^{\nu} + R^{(\lambda\nu)} S_{\alpha}^{\lambda} \right) \],

\[ C_{\alpha}^{\mu\nu} = \beta \left[ 2 \left( R^{(\lambda\mu)} \delta_{\sigma}^{\nu} - \delta_{\sigma}^{(\lambda} R_{\mu)}^{\nu} \right) S_{\lambda}^{\nu} - \nabla_{\lambda} R^{(\mu\sigma)} \right] \]

\[ - 4 \left( 2 \delta_{\sigma}^{(\mu} R_{\lambda)\nu)} S_{\lambda}^{\nu} + \nabla_{\lambda} (\delta_{\sigma}^{(\nu} R_{\mu)}^{\lambda)} - \nabla_{\sigma} R^{(\mu\nu)} \right) \]

\[ + 2 \gamma \left[ 2 S_{\beta\gamma}^{\nu} \left( R_{\nu}^{(\lambda\sigma)} - R_{\sigma}^{(\lambda\gamma)} \right) + S_{\beta}^{\nu} \left( R^{(\mu\nu)}_{\gamma} - R^{(\nu\gamma)}_{\mu} \right) + \nabla_{\lambda} \left( R^{(\nu\gamma)}_{\sigma} \right) \]

\[ - R^{(\mu\nu)}_{\alpha} \right] + 2 S_{\lambda}^{\nu} \left( R_{\alpha}^{(\mu\nu)} - R^{(\mu\nu)}_{\alpha} \right) + 2 \nabla_{\lambda} \left( R^{(\nu\gamma)}_{\sigma} \right) \],

\[ D_{\alpha}^{\mu\nu} = - \frac{4}{3} \left[ S_{\beta\gamma}^{\nu} \left( - R_{\sigma}^{(\mu\nu)} + R^{(\mu\nu)}_{\sigma} \right) + S_{\lambda}^{\nu} \left( - R^{(\mu\nu)}_{\lambda} \right) \]

\[ + R^{(\lambda\mu\nu)}_{\sigma} \right] + \frac{1}{2} \nabla_{\lambda} \left( R_{\alpha}^{(\mu\nu)} - R_{\alpha}^{(\mu\nu)} \right) \],

for \( N^{\mu\nu} \) we have

\[ N^{\mu\nu} = -2 \left( \overline{R_{\alpha\beta\mu\nu}} + g^{\mu\nu} R_{\alpha\beta} \right) + \left[ \beta R_{\alpha\beta} - (\beta + 4) R_{\alpha\beta} \right] \]
\[ + \left( \gamma + \frac{\lambda}{3} \right) R^{(\nu\zeta\lambda\mu)}_{\lambda\mu\rho} + (1 - \gamma) R^{(\nu\zeta\lambda\mu)}_{\lambda\mu\rho} \]
\[ + \frac{\lambda}{3} \left[ R^{(\nu\zeta\lambda\mu)}_{\lambda\mu\rho} + R^{(\nu\zeta\lambda\mu)}_{\lambda\mu\rho} \right] \]
\[ + \frac{1}{2} g^{\mu\nu} \left[ R^2 + 2R_{\nu\rho}R^{\lambda\mu} - (\beta + 4) R_{\nu\rho}R^{\lambda\mu} \right] \]
\[ + \left( \gamma + \frac{\lambda}{3} \right) R_{\lambda\mu\rho\sigma}R^{\lambda\mu\rho\sigma} + (1 - \gamma) R_{\lambda\mu\rho\sigma}R^{\lambda\mu\rho\sigma} + \frac{2\lambda}{3} \left[ R_{\lambda\mu\rho\sigma}R^{\lambda\mu\rho\sigma} \right] \], \quad (18)

and
\[ \alpha(S^{\mu\nu\rho} - 2S^{\mu\nu\rho}_{\lambda\mu\nu}) - 4\delta^{\mu\nu\rho}_{\lambda\mu\nu} + 2E^\nu_{\lambda\mu\nu} + \frac{2}{3} R^{\mu\nu\rho} - 2F_{\mu\nu} = 0 \]

with
\[ E_{\sigma}^{\mu\nu} = \beta [ \mathcal{V} \delta R^{(\nu\zeta\lambda\mu)}_{\lambda\mu\rho} - \delta_{\sigma}^{\nu\zeta\lambda\mu} - \delta_{\sigma}^{\nu\zeta\lambda\mu} ] \]
\[ + R^{(\nu\zeta\lambda\mu)}_{\lambda\mu\rho} (S_{\sigma\rho\nu} + S_{\sigma\rho\mu}) - R^{(\nu\zeta\lambda\mu)}_{\lambda\mu\rho} (S_{\sigma\rho\nu} + S_{\sigma\rho\mu}) + 2R_{\sigma\rho\nu} S^{(\nu\zeta\lambda\mu)}_{\lambda\mu\rho} \], \quad (19)
\[ F_{\sigma}^{\mu\nu} = \gamma [ 2 \mathcal{V} \delta R^{(\nu\zeta\lambda\mu)}_{\lambda\mu\rho} - R^{(\nu\zeta\lambda\mu)}_{\lambda\mu\rho} ] \]
\[ + 2S^{(\nu\zeta\lambda\mu)}_{\lambda\mu\rho} (R^{(\nu\zeta\lambda\mu)}_{\lambda\mu\rho} - R^{(\nu\zeta\lambda\mu)}_{\lambda\mu\rho}) \]
\[ + 2S^{(\nu\zeta\lambda\mu)}_{\lambda\mu\rho} (R^{(\nu\zeta\lambda\mu)}_{\lambda\mu\rho} - R^{(\nu\zeta\lambda\mu)}_{\lambda\mu\rho}) \], \quad (20)
\[ H_{\sigma}^{\mu\nu} = \lambda [ \mathcal{V} \delta R^{(\nu\zeta\lambda\mu)}_{\lambda\mu\rho} - 2R^{(\nu\zeta\lambda\mu)}_{\lambda\mu\rho} ] + S_{\sigma\rho\nu} (2R^{(\nu\zeta\lambda\mu)}_{\lambda\mu\rho} - R^{(\nu\zeta\lambda\mu)}_{\lambda\mu\rho}) \]
\[ - 2R^{(\nu\zeta\lambda\mu)}_{\lambda\mu\rho} (R^{(\nu\zeta\lambda\mu)}_{\lambda\mu\rho} - R^{(\nu\zeta\lambda\mu)}_{\lambda\mu\rho}) + S_{\sigma\rho\nu} (R^{(\nu\zeta\lambda\mu)}_{\lambda\mu\rho} - R^{(\nu\zeta\lambda\mu)}_{\lambda\mu\rho}) \], \quad (21)

where, as usual
\[ \mathcal{V} \delta = \mathcal{V} \delta + 2S_{\sigma\rho\nu} \]

(22)

In the torsionless limit Eq. (II) vanishes identically and (I) corresponds to Einstein equations plus the Bach-Lanczos identity in $V_4$, so that we regain GR in this case. That (II) vanishes can be verified with the aid of identities such as (16)
\[ \mathcal{V} \delta R^{(\nu\zeta\lambda\mu)}_{\lambda\mu\rho} - 2\mathcal{V} \delta R^{(\nu\zeta\lambda\mu)}_{\lambda\mu\rho} = 4S_{\delta\rho\nu} R^{(\nu\zeta\lambda\mu)}_{\lambda\mu\rho}, \]
\[ \mathcal{V} \delta R^{(\nu\zeta\lambda\mu)}_{\lambda\mu\rho} + 4S_{\delta\rho\nu} R^{(\nu\zeta\lambda\mu)}_{\lambda\mu\rho} \]

(23)
\[ \mathcal{V} \delta R^{(\nu\zeta\lambda\mu)}_{\lambda\mu\rho} + 4S_{\delta\rho\nu} R^{(\nu\zeta\lambda\mu)}_{\lambda\mu\rho} \]
\[ \mathcal{V} \delta R^{(\nu\zeta\lambda\mu)}_{\lambda\mu\rho} + 2S_{\delta\rho\nu} R^{(\nu\zeta\lambda\mu)}_{\lambda\mu\rho} \]
\[ \mathcal{V} \delta R^{(\nu\zeta\lambda\mu)}_{\lambda\mu\rho} + 2S_{\delta\rho\nu} R^{(\nu\zeta\lambda\mu)}_{\lambda\mu\rho} \]

(24)

obtained from Bianchi identities in $U_4$. The existence of the GR limit implies that the theory based on (9) and (10) pass the classical gravitational tests when the effect of the torsion is negligible.

In Eqs. (I) and (II) the torsion propagates in the general case, as also occurs in many quadratic gravitational theories in $U_4$. In order to obtain explicitly propagating torsion solutions we shall analyse the particular case $g_{\sigma\rho} = \eta_{\sigma\rho},$ where $\eta_{\sigma\rho}$ is the Minkowski metric. Furthermore we shall neglect terms in which the torsion and/or its derivatives appear quadratically (weak torsion approximation). In this case the Einstein-type equations disappear, in fact (I) becomes an identity. And the Cartan-type equations reduce to
\[ \alpha(S^{\mu\nu\rho} - 2S^{\mu\nu\rho}_{\lambda\mu\nu} - 4\delta^{\mu\nu\rho}_{\lambda\mu\nu} + 2\beta(2\delta^{(\mu\nu\rho)}_{\lambda\mu\nu} - \delta^{(\mu\nu\rho)}_{\lambda\mu\nu}) \]
\[ + 2\delta^{(\mu\nu\rho)}_{\lambda\mu\nu} + 2\delta^{(\mu\nu\rho)}_{\lambda\mu\nu} - 2\Box S^{(\mu\nu\rho)}_{\lambda\mu\nu} - \delta^{(\mu\nu\rho)}_{\lambda\mu\nu} \]
\[ - \delta^{(\mu\nu\rho)}_{\lambda\mu\nu} \]
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\[-2\gamma(3\Box S_{\mu\nu} - 2\Box S_{\mu}^{[\nu]} + 6\partial^\lambda \partial^{[\nu} S_{\lambda\mu]} + \partial_\sigma \partial_{\mu} S_{\sigma\mu})
+ 2\partial_\lambda \partial_{\mu} S_{\mu}^{[\lambda]} - 2\partial_\lambda \partial^{\nu} S_{\mu}^{[\lambda]} + 2\partial_\sigma \partial_{\lambda} S_{\mu}^{[\nu\lambda]}\]
\[+ \frac{4}{3} A(2\partial_\lambda \partial^{\nu} S_{\mu}^{[\lambda\nu]} - \Box S_{\mu\nu}) = 0,\]  

(25)

where \(\Box \equiv \eta^{\mu\nu} \partial_\mu \partial_\nu\), as usual.

Now we decompose the torsion tensor into the irreducible parts\(^5\)

\[V_a = S_{a\beta}^\beta, \quad \text{(vector)}\]
\[A_a = \frac{1}{6} \varepsilon_{\lambda\mu\nu\sigma} S_{a}^{\lambda\mu\nu}, \quad \text{(axial vector)}\]
\[T_{ab} = S_{a(\beta\gamma)} + \frac{1}{3} \eta_{(ab} S_{c\gamma)} - \frac{1}{3} \eta_{ab} S_{c\gamma} \quad \text{tensor}\]

(26)  
(27)  
(28)

with

\[T_{ab} = T_{a(\beta\gamma)} = T_{a(\beta\gamma)} = 0,\]
\[T_{ab} + T_{b(a} + T_{a(\beta\gamma)} = 0.\]

(29)  
(30)

Then we have

\[S_{ab} = \frac{4}{3} T_{a(\beta\gamma)} + \frac{2}{3} \eta_{(ab} V_{c)} + \varepsilon_{ab\gamma} A^\gamma.\]

(31)

For the case \(S_{ab} = \frac{4}{3} \eta_{(ab} V_{c)}\) it follows from substitution into (25) and contraction of indices that

\[\Box V^\mu + \frac{15}{2} \alpha \frac{4\beta + 7\gamma + 2\lambda}{12\beta + 12\gamma + 2\lambda} V^\mu = 0.\]

(32)

The case \(S_{ab} = \varepsilon_{ab\gamma} A^\gamma\) gives

\[\Box A_{\sigma} + \frac{9}{2} \frac{3\beta + 3\gamma + 4\lambda}{3\beta + 12\gamma + 2\lambda} A_{\sigma} - \left( \frac{3\beta + 3\gamma + 4\lambda}{3\beta + 12\gamma + 2\lambda} \right) \partial_\sigma \partial_{\mu} A^\mu = 0;\]

(33)

the additional condition \(\partial_\mu A^\mu = 0\) leads to

\[\Box A_{\sigma} + \frac{9}{2} \frac{3\beta + 3\gamma + 4\lambda}{(3\beta + 12\gamma + 2\lambda)} A_{\sigma} = 0.\]

(34)

Note that (33) in the special case \(\gamma = 0 = \lambda\) implies

\[\partial_\mu A^\mu = 0.\]

Similarly, we can find the equations for \(T_{(\beta\gamma)}\). The resulting equations are not very illuminating and consequently we shall not list them. The Proca fields of (33) and (34) show the existence of mass torsion excitations in the gravitational theory under consideration. The positiveness of energy for these fields depends on conditions involving the parameters of the theory. We shall not treat here this question. Conditions of this type were derived in a similar context by Hayashi and Shirafuji.\(^{18}\) In this reference the particle spectrum associated with the general ten-parameter Lagrangian in \(U_4\) is studied.
We also observe that, according to (I) and (II), the theory predicts a coupling between gravitational (metric) and torsion waves in vacuum.

As a final remark we would like to point out that the study of torsion wave perturbations in Friedmann-Robertson-Walker cosmologies could throw some light on the problem of galaxy formation. This is also a question to be further investigated in the framework of the present theory.

References

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