Study of $^{16}\text{O}-^{16}\text{O}$ Potential by the Resonating Group Method. I

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(Received December 18, 1987)

The results of the analyses are reported of the $^{16}\text{O}-^{16}\text{O}$ potential calculated by the use of the resonating group method as a typical example of the potential between two light heavy-ion nuclei. The analyses are on such basic properties as the shape, its energy-dependence and its angular-momentum-dependence. Our main purpose is to study what features are similar and dissimilar between the heavy-ion potential of the $^{16}\text{O}-^{16}\text{O}$ system and the light-ion potentials.

§ 1. Introduction

The optical potential is now widely used in describing the interaction process between composite nuclei. However, as is well known the optical potentials have not been determined uniquely in many cases. Therefore, it is desirable that the knowledge at least about the basic properties of the inter-nucleus potential such as depth, range, energy-dependence, etc. are obtained by the microscopic theory of the inter-nucleus interaction.

In spite of this requirement, the field of the microscopic theory of the inter-nucleus potential is now also in a rather confused situation. This is because there exist contradictions among the results about the basic potential properties obtained by different theoretical methods even when we take into consideration that the inter-nucleus potential can be defined more than one way.\(^1\)-\(^2\)

The present authors and their collaborators have studied the microscopic inter-nucleus local potentials by deriving them in the framework of the resonating group method (RGM).\(^1\)-\(^3\) They have also investigated the reasons why one obtains different results for some basic potential properties between different theories.\(^1\)-\(^4\)

The present authors consider that at present a decisive point to resolve the problem of the inter-nucleus potential is the fact that for light ion projectiles like $^3\text{He}$ and $\alpha$ the ambiguity of the real part of the optical potential is now widely regarded to be removed by the use of the high energy scattering data.\(^5\) Namely any microscopic theory of the inter-nucleus potential is now required to reproduce the uniquely determined light-ion optical potentials.

In order to meet this requirement the present authors have investigated the system of $\alpha-^{16}\text{O}$ by the RGM.\(^6\),\(^7\) They have found that the RGM with the introduction of the phenomenological imaginary potential reproduces quite well the scattering cross sections over a wide energy range and furthermore the equivalent local potential constructed from this RGM non-local potential has proved to be very similar to the optical potential by Michel et al.\(^8\) which fits the $\alpha-^{16}\text{O}$ scattering data very well in a wide energy range up to about 150 MeV. The present authors have been
investigating also the $\alpha^{40}\text{Ca}$ system and they are getting the same conclusion as in $\alpha^{16}\text{O}$ system also in this system.

The above-mentioned results of the investigations of the $\alpha^{16}\text{O}$ and $\alpha^{40}\text{Ca}$ systems show clearly that the RGM and the equivalent local potential derived from it are both quite suitable and reliable for the study of the inter-nucleus potential. Thus the importance of the RGM study of the potential between two light heavy-ion nuclei should be stressed even more than before.

The purpose of this paper is to report the results of the analyses of the equivalent local potential (ELP) of the $^{16}\text{O}^{16}\text{O}$ system calculated by the use of the RGM as a typical example of the potential between two light heavy-ion nuclei. The analyses are on such basic properties as the shape, its energy-dependence and its angular-momentum-dependence. Our main concern is the study about what features are similar and dissimilar between the heavy-ion potential of the $^{16}\text{O}^{16}\text{O}$ system and the potentials for the light-ion projectiles which have been studied in detail.1,10,9,11 A preliminary report on the present subject was reported in Refs. 2 and 1.

The construction of this paper is as follows. In § 2 we briefly recapitulate the method how we derive the ELP from the RGM non-local interaction. Then in § 3 we display the calculated ELP for the $^{16}\text{O}^{16}\text{O}$ system. Here we also check the accuracy of our localization procedure by comparing the phase-shifts by ELP with those by the original non-local potential. The study of the basic properties of the ELP is done in § 4. Finally in § 5 we give summarizing discussion.

§ 2. Formulation

In this section we briefly explain our framework which is discussed in detail in Refs. 12, 10 and 13. The many-body wave function of the RGM for the single-channel system composed of two nuclei $C_1$ and $C_2$ has the form

$$\mathcal{A}[\chi(r)\phi_1\phi_2],$$  \hspace{1cm} (2.1)

where $\mathcal{A}$ is the antisymmetrizer and $\phi_i$ are the internal wave functions of $C_i$. The relative wave function $\chi(r)$ is obtained by solving

$$\langle\phi_1\phi_2|(H-E_1-E_2-E)|\mathcal{A}[\chi(r)\phi_1\phi_2]\rangle=0,$$  \hspace{1cm} (2.2)

where $H$ is the total Hamiltonian, $E$ the energy of the relative motion and $E_i$ the binding energies of $C_i$. Equation (2.2) can be written as

$$\int[H(\rho, \rho')-E\cdot N(\rho, \rho')]\chi(\rho')d\rho'=0,$$

$$\left\{\begin{array}{c} H(\rho, \rho') \\ N(\rho, \rho') \end{array} \right\} \left[ \begin{array}{c} H-E_1-E_2 \\ 1 \end{array} \right] \{\mathcal{A}[\delta(\rho-\rho')\phi_1\phi_2]\}.$$  \hspace{1cm} (2.3)

Both $H(\rho, \rho')$ and $N(\rho, \rho')$ are sums of many local and non-local kernels, and we divide these kernels into Wigner- and Majorana-types.12,10,21 $H(\rho, \rho')=(1+\delta_{12})^{-1} \times [H_A(\rho, \rho') + H_b(\rho, \rho')]$, $N(\rho, \rho')=(1+\delta_{12})^{-1} [N_A(\rho, \rho') + N_b(\rho, \rho')]$, where the kernels with suffix $A$ denote the sums of the Wigner-type kernels and those with suffix $B$ the
sums of the Majorana-type kernels. The Wigner-type kernel has shorter non-locality range in $|\rho - \rho'|$ than in $|\rho + \rho'|$ and the Majorana-type kernel has just the opposite property. It is to be noticed that the present $^{16}$O-$^{16}$O system which is composed of two identical nuclei has the special property, namely, $H_b(\rho, -\rho') = H(\rho, \rho')$, $N_b(\rho, -\rho') = N_b(\rho, \rho')$. Thus for even partial wave $H_A$ and $N_A$ have entirely the same effect as $H$ and $N$,

$$\int H_A(\rho, \rho') \chi(\rho') d\rho' = \int H(\rho, \rho') \chi(\rho') d\rho', \tag{2.4}$$

$$\int N_A(\rho, \rho') \chi(\rho') d\rho' = \int N(\rho, \rho') \chi(\rho') d\rho'.$$

In a concise operator notation, Eq. (2.3) is rewritten as $[H_A - E \cdot N_A] \chi = 0$. Needless to say, odd partial waves are not allowed.

In the RGM study $\sqrt{N} \chi$ is usually regarded as a wave function quite appropriate for discussing the relative motion. This is largely due to the standard form of the orthonormalization relation satisfied by $\sqrt{N} \chi_E$, with $\chi_E$ standing for $\chi$ at the energy $E$; namely, $\langle \sqrt{N} \chi_E | \sqrt{N} \chi_{E'} \rangle \propto \delta_{EE'}$. This orthonormalization relation is the same as that satisfied by the scattering wave functions generated by the local and energy-independent potential. From Eq. (2.4) we have $\sqrt{N_A} \chi = \sqrt{N} \chi$ for even partial waves.

The equation of motion for $\sqrt{N_A} \chi$ has the form

$$\left( \frac{1}{\sqrt{N_A}} H_A - \frac{1}{\sqrt{N_A}} - E \right) \sqrt{N_A} \chi = 0. \tag{2.5}$$

For convenience's sake we introduce the following notations:

$$\tilde{H}_A = \frac{1}{\sqrt{N_A}} H_A - \frac{1}{\sqrt{N_A}}, \quad \tilde{\chi} = \sqrt{N_A} \chi. \tag{2.6}$$

When we use instead of the integral operators their equivalent differential operators, we can express Eq.(2.5) as

$$(\tilde{H}_A(r, p_{op}) - E) \tilde{\chi}(r) = 0,$$

$$\tilde{H}_A(r, p_{op}) \equiv \frac{1}{\sqrt{N_A(r, p_{op})}} \tilde{H}_A(r, p_{op}) \frac{1}{\sqrt{N_A(r, p_{op})}}, \tag{2.7}$$

where

$$O(r, p_{op}) u(r) = \int O(r, r') u(r') dr',$$

$$O(r, p_{op}) = \int ds \ e^{(i/\hbar)s \cdot p_{op}} O(\frac{r-s}{2}, r + \frac{s}{2}) e^{(i/\hbar)s \cdot p_{op}},$$

$$p_{op} = \frac{\hbar}{i} \frac{\partial}{\partial r}. \tag{2.8}$$

By expressing $\tilde{\chi}(r)$ as $\exp \left((i/\hbar) \tilde{S}(r)\right)$, we get from Eq. (2.7)
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\[ \hat{H}(r, p_o) + \nabla \hat{S}(r) \cdot 1 = E. \quad (2.9) \]

Now we expand $\hat{S}(r)$ in power series of $\hbar$ as $\hat{S}(r) = \hat{S}_0(r) + \hbar \hat{S}_1(r) + \cdots$, and then we obtain the following relations from the zeroth power term of $\hbar$ in the expansion of Eq. (2.9),

\[ \hat{H}_A^w(r, \nabla \hat{S}_0(r)) = E, \quad (2.10) \]

where $\hat{H}_A^w(r, \mathbf{p})$ is the Wigner transform of $\hat{H}_A$,

\[ \hat{H}_A^w(r, \mathbf{p}) = \int ds \ e^{i(h/\hbar)\cdot \mathbf{p}} \hat{H}_A \left( r - \frac{s}{2}, r + \frac{s}{2} \right). \quad (2.11) \]

Equation (2.10) is the Hamilton-Jacobi equation from which $\hat{S}_0(r)$ is obtained. Once $\hat{S}_0(r)$ is obtained, we can define the equivalent local potential $V^{eq}(r)$ by

\[ \frac{1}{2\mu} \hat{p}_r^2(r) + \frac{\hbar^2 l(l+1)}{2\mu r^2} + V^{eq}(r) = E, \quad (2.12) \]

where $p_r(r)$ is obtained for each partial wave from

\[ \nabla \hat{S}_0(r) = p_r(r) \frac{r}{r} + \frac{\hbar \sqrt{l(l+1)}}{r} \mathbf{n}, \quad (\mathbf{n} \perp r). \quad (2.13) \]

In the actual calculation we utilize the following approximate relation,\(^{13)}\)

\[ \frac{\hat{H}_A^w(r, \mathbf{p})}{\hat{N}_A^w(r, \mathbf{p})}, \quad (2.14) \]

where $O^w(r, \mathbf{p})$ stands for Wigner transform of $O(r, r')$, $O^w(r, \mathbf{p}) = \int ds \ e^{i(h/\hbar)\cdot \mathbf{p}} O(r - s/2, r + s/2)$. Use of Eq. (2.14) in Eq. (2.10) gives

\[ \hat{H}_A^w(r, \nabla \hat{S}_0(r)) - E \cdot \hat{N}_A^w(r, \nabla \hat{S}_0(r)) = 0. \quad (2.15) \]

In the present calculation the Coulomb interaction is included by approximating the Coulomb kernel $V_C \approx \sqrt{N} V_{DC} \sqrt{N}$ where $V_{DC}(r)$ is the direct (or double folding) potential of the Coulomb interaction. Thus in $\hat{H}_A$ the Coulomb interaction is included as

\[ \frac{1}{\sqrt{N_A} V_C - \frac{1}{\sqrt{N_A} V_{DC} \sqrt{N}} - \frac{1}{\sqrt{N_A} V_{DC}}} = V_{DC}. \quad (2.16) \]

For the classically forbidden region (CFR) where $p_r^2 < 0$, we set $p_r = i \sqrt{-p_r^2}$. Another possible choice is $p_r = -i \sqrt{-p_r^2}$, but there is no difference between the two since the Wigner transforms $H_A^w(r, \mathbf{p})$ and $N_A^w(r, \mathbf{p})$ depend only on $r^2, p^2$ and $(\mathbf{r} \cdot \mathbf{p})^2$.

Finally we explain some notations. The equivalent local nuclear potential $V^{eq}_n(r)$ is defined by

\[ V^{eq}_n(r) \equiv V^{eq}(r) - V_{DC}(r). \quad (2.17) \]

By denoting the direct (or double folding) potential due to the nuclear force as $V_D(r)$, the exchange potential $\Delta V(r)$ is expressed as
\[ \Delta V(r) = V_{\text{eq}}(r) - (V_D(r) + V_{\text{nc}}(r)) = V_{\text{eq}}(r) - V_D(r). \]

(2.18)

§ 3. The equivalent local potentials and their accuracy

In this section we present the calculated equivalent local potential \( V_{\text{eq}}(r) \) for \(^{16}\text{O}\) \(^{16}\text{O}\) system. Furthermore, by comparing the phase shifts by \( V_{\text{eq}}(r) \) with those by the original RGM non-local potential we check the accuracy of our localization procedure. The adopted effective two-nucleon force is Volkov No. 2\(^{14}\) with Majorana mixture \( m=0.65 \). The oscillator parameter \( \nu(=\alpha \omega/2\hbar) \) of the internal wave function of \(^{16}\text{O}\) is set to \( 0.195 \text{ fm}^{-2} \). These values of the parameters are the same as those adopted by Ando et al.\(^{15}\)

We show in Fig. 1 the equivalent local nuclear potential \( V_{\text{eq}}^0(\mathbf{r}) = V_{\text{eq}}(\mathbf{r}) - V_{\text{nc}}(\mathbf{r}) \) at the incident energy per nucleon \( E=5, 10, 20 \) and \( 100 \text{ MeV/}\mu \) for the orbital angular momentum \( l=0 \). In Fig. 1 the direct nuclear potential \( V_D(r) \) is also shown by the dotted line for comparison. In Fig. 2 we show by crosses the phase shifts by \( V_{\text{eq}}(r) \) and compare them with those by the original RGM non-local potential drawn by solid lines. We also show in Fig. 2 by using dotts the phase shifts by the potentials \( 0.9 \times V_{\text{eq}}^0(\mathbf{r}) + V_{\text{nc}}(\mathbf{r}) \). Since two kinds of relative wave function \( \chi(\mathbf{r}) \) and \( \tilde{\chi}(\mathbf{r}) \) have the same phase shifts, the exact RGM phase shifts are calculated for \( \chi(\mathbf{r}) \) by solving
Eq. (2·3) and by using the variational method of Ref. 16). The present results of the exact RGM phase shifts are in good agreement with those obtained by Ando et al.\textsuperscript{15} However, since the Coulomb interaction kernel is approximated by \( V_c \approx \sqrt{N} V_{dc} \sqrt{N} \) in our treatment, there exist some slight differences between the two due to the different treatment of the Coulomb interaction. In the case of the light-ion-projectile systems the original RGM phase shifts can be reproduced by the small (within 5\%) modification of \( V^{eq}(r) \). For the present system \( V^{eq}(r) \) are a little too deep; \( V^{eq}(r) \) have an extra bound state than the original non-local potentials have. However, as is seen in Fig.2 the original RGM phase shifts can be reproduced by about 10\% modification of \( V^{eq}(r) \). Therefore, various kinds of quantities which are calculated by \( V^{eq}(r) \) are to be regarded to have the accuracy more than 90\%.

§ 4. Properties of the equivalent local potentials

In the following, we discuss the characteristic properties of the equivalent local potential of \(^{16}\text{O}-^{16}\text{O}\) system by comparing them with those of the light-ion-projectile systems.

4.1. Depth of the potential

First of all we see in Fig. 1 that \( V^{eq}(r) \) is deep. This feature is common to all systems we have investigated and is irrespective of the adopted effective two-nucleon force.\textsuperscript{13-30,99} This is due to the existence of the Pauli forbidden region (FR) in the phase space of the relative motion.\textsuperscript{17}

In the RGM theory, it is well known that there exist the so-called Pauli forbidden states (FS) \( \chi_F(r) \): FS are defined as the functions which satisfy \( \mathcal{H}(\chi_{F}(r)\phi_{1}\phi_{2})=0 \), and therefore they are redundant solutions of the RGM equation of motion given in Eq. (2·1). FS are expressed by the harmonic oscillator wave functions \( R_{n\ell}(r,\gamma) \times Y_{m}(\phi) \) with \( N=2n+l \leq N_F \) where \( \gamma=\nu N_1 N_2/(N_1+N_2) \) with \( N_i \) standing for the mass number of the nucleus \( C_i \). \( N_F \) is the largest number of the harmonic oscillator quanta of the FS: \( N_F=22 \) for \(^{16}\text{O}-^{16}\text{O}\) system. When we apply the WKB treatment to the RGM equation of motion, the harmonic oscillator functions are transformed into the harmonic oscillator trajectories \( p^2/2\mu+\mu\omega^2r^2/2=\hbar\omega(2n+l+3/2) \). Therefore the region of the phase space defined by \( p^2/2\mu+\mu\omega^2r^2/2\leq\hbar\omega(N_F+3/2) \) is the FR which the physical trajectory cannot enter. The FR is the semi-classical image of the functional space spanned by the FS.

The physical solution of the Hamilton-Jacobi equation, Eqs. (2·10) and (2·15), should be found outside of the FR. Therefore the local momentum \( p(r) \) should satisfy

\[
\frac{1}{2\mu}p^2(r)+\frac{1}{2}\mu\omega^2r^2>\hbar\omega\left(N_F+\frac{3}{2}\right),
\]

which, in terms of \( V^{eq}(r) \), takes the form

\[
V^{eq}(r)<E-\hbar\omega\left(N_F+\frac{3}{2}\right)+\frac{1}{2}\mu\omega^2r^2.
\]
It is clear from Eq. (4.1) that in the inner spatial region with $r \approx 0$ the local momentum $p(r)$ cannot be small because $p^2/2\mu \geq \hbar \omega (N_F+3/2)$. From Eq. (4.2) we see that $V^{eq}(r)$ is necessarily deep except at very high energy where $E$ is comparable with $\hbar \omega (N_F+3/2)$. This is the kinematical requirement and is independent of the choice of the effective two-nucleon force.

In Fig. 3 the FR is displayed by the shaded region and the trajectory by $V^{eq}(r)$ at $E=5$ MeV/μ is drawn by the solid line. Furthermore, we show the trajectory by the direct potential $V_D(r) + V_{DC}(r)$ at the same incident energy by the dashed line. We see that without the attractive exchange potential the trajectory penetrates slightly into the Pauli-forbidden region.

Although the feature that the ELP $V^{eq}(r)$ is deep is common to that in the light-ion projectile systems, the magnitude of the exchange potential $\Delta V(r)$ is fairly larger for the $^{16}\text{O} - ^{16}\text{O}$ potential than for the light-ion potentials. This can be seen, for example, by comparing the ratios $r_m = (\Delta V)_m / (V^{eq})_m$ where $(\Delta V)_m$ and $(V^{eq})_m$ represent the maximum depth of $\Delta V(r)$ and $V^{eq}(r)$, respectively. For $l=0$ and at $E=5$ MeV/μ, the ratio $r_m$ is about 0.23 for the $^{16}\text{O} - ^{16}\text{O}$ system while it is about 0.21 for the $\alpha - ^{40}\text{Ca}$ system studied in Ref. 18). Since in general the magnitude of the exchange potential $\Delta V(r)$ is larger for larger Majorana component of the effective two-nucleon force, the comparison of $\Delta V(r)$ should be done by taking care of this dependence on the Majorana component. An important point of the present comparison is that $\Delta V(r)$ is larger in $^{16}\text{O} - ^{16}\text{O}$ than in $\alpha - ^{40}\text{Ca}$ although the Majorana component $m$ is smaller in $^{16}\text{O} - ^{16}\text{O} (m=0.65)$ than in $\alpha - ^{40}\text{Ca} (m=0.658)$.

In Ref. 19) one of the present authors (T. W.) has reported that $\Delta V(r)$ of the $^{16}\text{O} - ^{40}\text{Ca}$ potential is also fairly larger than those of light-ion potentials. Therefore, it may be concluded that the exchange potential of the light-heavy-ion potential is larger in general than that of the light-ion potential. It is to be noted that the larger exchange potential means the larger energy-dependence of the total potential. The energy-dependence of the ELP is discussed in § 4.3.

4.2. Angular-momentum-dependence

In the following discussion of the angular-momentum-dependence ($l$-dependence)
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Fig. 4. Angular-momentum dependence of the exchange potential at $E=5$ MeV/u (a), at $E=10$ MeV/u (b) and at $E=20$ MeV/u (c). The ordinate shows the quantity $\delta V_i(r) = V_{\text{eq}}(r) - V_{\text{eq}}(r_i)$.

For the sake of comparison of the $l$-dependence between the $^{16}\text{O}-^{16}\text{O}$ potential and the light-ion potentials, we give here, as an example, the maximum value of $|\delta V_i(r)|$ in the case of $l=8$ and $E=10$ MeV/u for three systems, $\alpha-^{16}\text{O}$, $\alpha-^{40}\text{Ca}$ and $^{16}\text{O}-^{16}\text{O}$. The potentials for $\alpha-^{16}\text{O}$ and $\alpha-^{40}\text{Ca}$ are those given in Ref. 9. The maximum values are 3.5 MeV, 2.0 MeV and 1.5 MeV for $\alpha-^{16}\text{O}$, $\alpha-^{40}\text{Ca}$ and $^{16}\text{O}-^{16}\text{O}$, respectively.

In analyzing the data, the optical potentials are usually assumed to be independent of the angular momentum. Therefore in order to compare our $V_{\text{eq}}(r)$ with the phenomenological optical potential, it is necessary to make some kind of average over
the $l$-dependence of $V^{eq}(r)$. In Ref. 6) we made such average by using the weight factor $(2l+1)(1-\exp(2i\delta_i))^2$ for each partial wave with $\delta_i$ standing for the nuclear phase shift. Here in this paper, for the sake of simplicity we assume that the ELP for a quasi-grazing angular momentum $l_0$ is close to such $l$-averaged ELP. For $l_0$, we adopt the value determined by

$$l_0 = \frac{D}{\hbar} \sqrt{2\mu E}, \quad D = 5.2 \text{ fm}.$$  \hfill (4.4)

The distance parameter $D$ is calculated by

$$D = 2R_0, \quad R_0 = \sqrt{\langle r^2 \rangle_{16O}},$$

$$\langle r^2 \rangle_{16O} = \langle \phi^{(16O)}(r) \sum_{i=1}^{16} (r_i - X_E)^2 \phi^{(16O)}(r) \rangle,$$  \hfill (4.5)

where $\phi^{(16O)}$ is the harmonic oscillator shell model internal wave function of $^{16}$O with the oscillator parameter $\nu = 0.16 \text{ fm}^{-2}$. The value of $l_0$ calculated by Eq. (4.4) is not an integer but we adopt this non-integer value since our ELP can be calculated for any real value of $l$. It is to be noticed that the construction of the $l$-averaged ELP means the conversion of the $l$-dependence into the additional energy-dependence piled over the original energy-dependence of the ELP. We discuss the energy-dependence of such ELP for $l = l_0$ in the next subsection. There we will see that the additional energy-dependence due to the conversion of the $l$-dependence is larger for the $^{16}$O-$^{16}$O potential than for the light-ion potentials.

4.3. Energy-dependence

We show in Figs. 5(a) and (b) the energy-dependence of the exchange potentials of $V^{eq}$ for $l = 0$ and for $l = l_0$ of Eq. (4.4), respectively. The characteristic feature of the energy-dependence is as follows: As the incident energy gets higher up to about 50 MeV/u, the exchange potential becomes less attractive in the inner spatial region while it becomes more attractive in the outer spatial region. However, when the
incident energy gets much higher than the above-mentioned characteristic energy, about 50 MeV/\mu, the exchange potential in the whole spatial region becomes less attractive. This characteristic feature of the energy-dependence of the exchange potential for the $^{16}\text{O}^{16}\text{O}$ system is the same qualitatively as that for the light-ion projectile systems. When we compare them quantitatively, however, the energy-dependence of the exchange potential is larger in $^{16}\text{O}^{16}\text{O}$ system than in the light-ion projectile systems as noticed in § 4.1, and the characteristic energy in the $^{16}\text{O}^{16}\text{O}$ system which is about 50 MeV/\mu is higher than that in the light-ion projectile systems which is $30 \sim 40$ MeV/\mu.

In order to study the energy-dependence more quantitatively, we show in Fig. 6 the energy-dependence of the volume integral per nucleon pair

$$j_\nu = \frac{4\pi}{16 \times 16} \int_0^\infty dr \cdot r^2 V_{t_{\text{eq}}}(r),$$

and in Fig. 7 that of the root-mean-square radius

$$\sqrt{\langle r^2 \rangle_{\nu}} = \sqrt{\int_0^\infty dr \cdot r^4 V_{t_{\text{eq}}}(r)/\int_0^\infty dr \cdot r^2 V_{t_{\text{eq}}}(r)}.$$

For both $j_\nu$ and $\sqrt{\langle r^2 \rangle_{\nu}}$, we show the results for two angular momenta, $l=0$ and $l=l_\nu$ of Eq. (4.4). The qualitative feature of the energy-dependence of $j_\nu$ and $\sqrt{\langle r^2 \rangle_{\nu}}$ of the $^{16}\text{O}^{16}\text{O}$ potential is not so much different from that of the light-ion potentials. To see this point explicitly, we give in Figs. 8 and 9 the energy-dependence of $j_\nu$ and $\sqrt{\langle r^2 \rangle_{\nu}}$ of the $\alpha^{16}\text{O}$ potential, respectively, for two angular momenta $l=0$ and $l=l_\nu$. The $\alpha^{16}\text{O}$ potential is obtained by using the same parameter values as Ref. 6); namely, the oscillator parameter $\nu$ is $\nu=0.16$ fm$^{-2}$ and the effective two nucleon force is chosen to be the HNY force$^{20}$ with the $\Delta$ parameter being $\Delta=21.3$ MeV. The value
of the parameter $D$ of $l_0$ of Eq. (4.4) for the $\alpha\cdot^{16}\text{O}$ potential is chosen to be $D=4.0$ fm, which is because $j_r$ for $l_0$ evaluated by this $D$ value reproduces well the values of $j_r$ of the $l$-averaged $\alpha\cdot^{16}\text{O}$ potential of Ref. 6) with the weight factor $(2l+1)|1-\exp (2i\delta l)|^2$. The dashed line stands for $j_r$ calculated by the direct potential $V_d(r)$.

We see in Figs. 6 and 8 that $j_r$ for $l=0$ has a peak at $E\approx 20$ MeV/$\hbar$ for $^{16}\text{O}-^{16}\text{O}$ and at $E\approx 10$ MeV/$\hbar$ for $\alpha\cdot^{16}\text{O}$. The existence of this kind of peak is common to other angular momenta for both systems and its origin is commonly due to the rapid energy-dependence of the tail part of the exchange kinetic potential which was discussed in detail in Ref. 11) for the cases of $z\cdot^{16}\text{O}$ systems with $z$ standing for light-ions. When we adopt $l_0$ for $l$, the peak of $j_r$ is seen to be absent for both $^{16}\text{O}-^{16}\text{O}$ and $\alpha\cdot^{16}\text{O}$ systems. However, in the low energy region for $\alpha\cdot^{16}\text{O}$ and in the whole energy region for $^{16}\text{O}-^{16}\text{O}$ it has not been investigated whether the ELP with $l=l_0$ can be really a good approximation to the $l$-averaged ELP. Therefore, this disappearance of the peak of $j_r$ for $l=l_0$ should not be taken so much seriously at present.

When we compare Fig. 6 with Fig. 8, we see that the induced $E$-dependence due to the $l$-dependence of the ELP is not so much smaller in $^{16}\text{O}-^{16}\text{O}$ than in $\alpha\cdot^{16}\text{O}$ although $\delta V_l(r)$ of Eq. (4.3) is much smaller in $^{16}\text{O}-^{16}\text{O}$ than in $\alpha\cdot^{16}\text{O}$. This is of course due to the fact that the change rate of the quasi-grazing angular momentum ($l_0$) with the incident energy per nucleon is much larger in $^{16}\text{O}-^{16}\text{O}$ than in $\alpha\cdot^{16}\text{O}$.

§ 5. Discussion and summary

We have reported the results of the analyses of the equivalent local potential (ELP) of the $^{16}\text{O}-^{16}\text{O}$ system calculated by the use of the RGM as a typical example of
the potential between two light heavy-ion nuclei.

After the brief recapitulation in § 2 of the formulation to derive the ELP, we have compared in § 3 the phase shifts by the ELP with those by the original RGM non-local potentials in order to check the accuracy of our construction method of the ELP. Since the original RGM phase shifts can be reproduced by the ELP, within about 10% modification of the calculated ELP, various kinds of the quantities derived from the ELP are concluded to have the accuracy more than 90%.

The calculated ELP of $^{16}\text{O}-^{16}\text{O}$ is found to be quite deep and the reason of this result is discussed in § 4.1 to be due to the existence of the Pauli-forbidden region in the phase space of the inter-nucleus relative motion. This situation is common to light-ion projectile systems. Although the feature that the ELP is deep is commonly true both in $^{16}\text{O}-^{16}\text{O}$ and in light-ion potentials, the magnitude of the exchange potential has been reported in § 4.1 to be fairly larger for the $^{16}\text{O}-^{16}\text{O}$ potential than for the light-ion potentials. This result has been checked to hold under the proper consideration of the fact that the magnitude of the exchange potential depends largely on the magnitude of the Majorana exchange mixture of the effective two-nucleon potential.

It is here to be noted that though the magnitude of the exchange potential changes for the change of the Majorana exchange mixture, the net depth of the ELP does not change so much. This is because when the Majorana exchange mixture becomes larger (smaller) the direct potential becomes in general less (more) attractive oppositely to the exchange potential which becomes more (less) attractive. This situation is displayed in Fig. 10 for the $^{16}\text{O}-^{16}\text{O}$ potential. As is seen in this figure we should note, however, that the tail part of the potential changes rather largely in the same way as the direct potential does. This is because the spatial range of the direct potential is larger than that of the exchange potential.\textsuperscript{21,10)}

The angular momentum-dependence ($l$-dependence) of the $^{16}\text{O}-^{16}\text{O}$ potential is reported in § 4.2 to be quite small compared to that of the light-ion potentials. This conclusion is obtained from the fact that the quantity $\delta V(r)$ of Eq. (4.3) is much smaller for the $^{16}\text{O}-^{16}\text{O}$ potential than for the light-ion potentials when compared for the same $l$ at the same incident energy per nucleon.

Finally in § 4.3 we have studied the energy-dependence of the ELP. It has been shown that the energy-dependence
of the $^{16}$O-$^{16}$O ELP is similar to that of the light-ion ELP in its qualitative feature although the magnitude of the former is larger than that of the latter.

In order to compare the energy-dependence of our ELP with that of the phenomenological optical potential which is $l$-independent, we need to construct the $l$-averaged ELP. The weight factor which is adopted in Ref. 6) for this $l$-average is $(2l+1)|(1-\exp(2i\delta_l)|^2$ with $\delta_l$ being the nuclear phase shift. In this paper we have presumed that such $l$-averaged ELP is well approximated by the ELP for $l=l_0$ with $l_0$ denoting the quasi-grazing angular momentum of Eq. (4.4). The use of the ELP for $l=l_0$ means the conversion of the $l$-dependence of the ELP into the effective energy-dependence. We have seen that the induced energy-dependence due to the $l$-dependence of the ELP is not so much smaller in the $^{16}$O-$^{16}$O system than in the light-ion projectile systems in spite of the fact that the quantity $\delta V_l(r)$ of Eq. (4.3) is very small for the $^{16}$O-$^{16}$O ELP. This is of course due to the larger reduced mass and larger contact distance of the $^{16}$O-$^{16}$O system compared with the light-ion projectile systems.

Like the light-ion ELP, $j_\nu$ of the $^{16}$O-$^{16}$O ELP with a definite $l$ shows a peak in its energy dependence and the position of the peak is at about 20 MeV/\(u\). On the other hand, $j_\nu$ for the $^{16}$O-$^{16}$O ELP with $l=l_0$ has been found not to have such peak around and below about 20 MeV/\(u\). This has been shown to be also the case for $\alpha$-$^{16}$O. We have remarked, however, that in the low energy region of the $\alpha$-$^{16}$O system and in the whole energy region of the $^{16}$O-$^{16}$O system it has not been justified whether the ELP with $l=l_0$ gives really a good approximation to the properly $l$-averaged ELP.

Acknowledgements

The authors thank Dr. K. Aoki and Dr. K. Yabana for their discussions in the early stage of this work.

The computer calculations for this work were financially supported in part by the Research Center for Nuclear Physics, Osaka University.

References