Susceptibility Discontinuity at the He$^3$-A-Normal Transition

Shin Takagi

Department of Physics, University of Tokyo
Tokyo

February 4, 1974

When a Fermi liquid undergoes a transition from the normal phase to an anisotropic superfluid of ESP (equal spin pairing) type under an external magnetic field, there appear two transition temperatures $T_1$ and $T_2$; the higher one $T_1$ is associated with pairing of the spins parallel to the field, the lower one $T_2$ with pairing of the spins antiparallel to the field. This is probably due to the increase in the magnetic field of the density of states of spin-up particles relative to that of spin-down particles, as has been suggested by Ambegaokar and Mermin. Whether this is so or not, the occurrence of these two transitions implies an anomalous behavior of the magnetization and the susceptibility. In phase I ($T_1<T<T_1$), only spin-up particles can form a pair. It is expected that some of the spin-down particles emigrate to the spin-up state in order to gain condensation energy at the cost of kinetic energy, which leads to an anomalous magnetization added to the normal one. In phase II ($T<T_2$), spin-down particles cease to emigrate, now that they can form a pair, but a temperature-independent anomalous magnetization remains. Thus
the field derivative of the magnetization exhibits a two-step jump, and its zero-field limit (i.e., the susceptibility) shows a discontinuity at the zero-field transition temperature $T_e$.

A quantitative estimate of this effect may be given using the following free energy near $T_e$:

$$F - F_n = -(N(0)/2) \left\{ (t+\eta h) \Delta t^2 + (t-\eta h) \Delta t^3 + \beta \Delta t^2 \Delta t^4 / T_e^2 \right\} - \beta (\Delta t^4 + \Delta t^5) / 2T_e^3,$$

(1)

where $F_n$ is the normal-state free energy in the magnetic field, $N(0)$ is the density of states of one spin at the Fermi surface in the absence of a field, $t = 1 - T/T_e$, $h = \mu H / T_e$, $\mu$ is the magnetic moment of the particle, and $\eta$ is the Ambegaokar-Mermin parameter which is related to the variation in the density of states. This form has been proposed by Brinkman and Anderson on the basis of the paramagnon model. However symmetry considerations shows that this is a general form for ESP states independent of microscopic theories. Thus we regard $\eta$, $\beta$ and $\delta$ as phenomenological parameters; $\beta$ and $\delta$ are supposed to take care of the angular dependence of the energy gap and various fourth order terms. We assume that $\beta > 0$ and $|\delta| < 1$, so that the transitions are of second order. We quote the results of Brinkman and Anderson for the field-splitting of $T_e$ and the specific heat discontinuity (suffixes I and II refer to phases I and II, respectively),

$$\left( T_I - T_e \right) / T_e = \eta h,$$

(2)

$$\left( T_e - T_\alpha \right) / T_e = \eta h (1 - \delta) / (1 + \delta),$$

$$\Delta C_1 / C_n = \left[ (C_1 - C_n) / C_n \right] T_1 = 1 / 2T_\beta,$$

$$\Delta C_2 / C_n = \left[ (C_2 - C_1) / C_n \right] T_2$$

$$= (1 + \delta) / 2T_\beta (1 - \delta),$$

(3)

where we have put $C_n = \gamma N(0) T_e$, $\gamma \equiv 2\pi / 3$.

The anomalous magnetic behavior is as follows:

$$M_1 - M_n = N(0) \mu \eta \Delta t^2 / 2T_e$$

$$= N(0) T_e \mu \eta (t + \eta h) / 2\beta,$$

(4)

$$M_\alpha - M_n = N(0) \mu \eta (\Delta t^3 - \Delta t^4) / 2T_e$$

$$= N(0) T_e \mu \eta h / \beta (1 + \delta),$$

(5)

$$\Delta \chi / \chi_n = \left[ (\chi - \chi_n) / \chi_n \right] T_1$$

$$= (1 + Z_0 / 4) \eta^2 / 4\beta,$$

(6)

$$\Delta \chi / \chi_n = \left[ (\chi - \chi_n) / \chi_n \right] T_2$$

$$= (1 + Z_0 / 4) \eta^2 (1 - \delta) / 4\beta (1 + \delta),$$

(7)

where $\chi = (\partial M / \partial H)|_T$ and we have put $\chi_n = 2\mu^2 N(0) / (1 + Z_0 / 4)$; $Z_0$ is the Landau parameter.

Let us apply these formulæ to liquid He$^3$, where phases I and II correspond to A phase between "A$_1$" and "A$_\alpha$" and below "A$_\alpha$", respectively, and estimate the order of magnitude of the susceptibility discontinuity. Since detailed information on the quantities (2) ~ (7) are not available as yet, we shall use coarser relations

$$\Delta T_e / \mu H = (T_1 - T_\alpha) / \mu H$$

$$= 2\eta / (1 + \delta),$$

(8)

$$\Delta C / C_n = \left[ (\Delta C_1 + \Delta C_2) / C_n \right] T_1$$

$$= 1 / \gamma \beta (1 - \delta),$$

(9)

$$\Delta \chi / \chi_n = \left[ (\Delta \chi_1 + \Delta \chi_2) / \chi_n \right] T_2$$

$$= (1 + Z_0 / 4) \eta^2 / 2\beta (1 + \delta).$$

(10)

Equations (9) and (10) are equal to the zero-field discontinuities of the specific heat and the susceptibility, respectively. Eliminating $\eta$ and $\beta$ from Eqs. (8) ~ (10), we obtain

$$\Delta \chi / \chi_n = (\pi^3 / 12) (1 + Z_0 / 4) (1 - \delta^2)$$

$$\times (\Delta T_e / \mu H)^2 \Delta C / C_n.$$  

(11)

By the use of the empirical estimates

$$Z_0 \sim 2.9^{6}$$

$$\Delta T_e / \mu H \sim 0.08^{1,3}$$

$$\Delta C / C_n \sim 1.7^{7}$$

(12)
we find $\Delta x/x_n \sim 2.5\times10^{-3}(1-\delta^2)$. It is seen that so long as $\delta$ is not too close to 1, the order of magnitude of $\Delta x/x_n$ is not seriously affected by $\delta$.

A comparison of the theoretical expression (11) with experiment would fix the last unknown parameter $\delta$ and hence provide a test of the paramagnon model.

I would like to thank Professors A. J. Leggett, Y. Wada, R. Kubo, S. Nakajima, K. Maki, T. Tsuzuki, K. Yoshida, and Dr. Y. Kuroda for valuable advice and instructions, and to Professor J. C. Wheatley for some very helpful correspondence.


Note added: The assertion that Eq. (1) is general for ESP states is correct if only the form of the angular dependence of the order parameters does not vary with temperature. I am grateful to Professor N. D. Mermin for pointing out this omission.