Proton Compton scattering and photoproduction of vector mesons

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Proton Compton scattering and photoproduction of vector mesons at high energies are studied in terms of the quark model. It is shown that under the vector meson dominance hypothesis the quark model gives the $\gamma P$ total hadronic cross section in good agreement with the experimental data. The real-to-imaginary ratio of forward proton Compton scattering is related to the total cross sections and the real-to-imaginary ratios of amplitudes for meson-proton scattering. The differential cross section for photoproduction of vector mesons at zero momentum transfer is calculated without neglecting the contribution of the real part of the amplitude. A comment on Harari's conjecture is also given in connection with photoproduction of $\phi$ meson.

§ 1. Introduction

It is known that under the vector meson dominance hypothesis, the $\gamma P$ total hadronic cross section can be related to the differential cross sections for photoproduction of vector mesons at zero momentum transfer by assuming that the amplitudes for photoproduction of vector mesons are purely imaginary. However, we have a small number of experimental data which involve large errors not only for the differential cross sections for $\omega$ and $\phi$ productions, but also for the one for $\rho^0$ production at higher energies ($\geq 10$ GeV). Furthermore it is questionable to neglect the real part of amplitudes for photoproduction of vector mesons. So the above method does not provide a good test for the vector-meson-dominance hypothesis, though the above $\gamma P$ total hadronic cross section is consistent with the experimental data.

On the other hand, it was shown in a previous note\(^5\) that under the vector-meson-dominance hypothesis the quark model gives $\gamma P$ total hadronic cross section in agreement with the experimental data at the energy 7.5 GeV\(^5\) and up to 5.4 GeV.\(^4\) Our prediction for the $\gamma P$ total hadronic cross section has smaller errors than the one obtained by assuming that the amplitudes for photoproduction of vector mesons are purely imaginary, and is also in agreement with the more accurate data which were recently obtained.\(^5\)

The quark model enables us to estimate easily the contribution of the real part of the amplitudes for photoproduction of vector mesons. The contribution of the real part of the proton Compton scattering amplitude can also be estimated.
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in the same way. Our estimation shows that the ϕ production amplitude has a large real part. Thus, the contribution of the real part of the ϕ production amplitude cannot be neglected, though the contribution of the real part for proton Compton scattering and photoproduction of ρ₀ and ω mesons is small.

Under the vector-meson-dominance hypothesis the amplitudes for photoproduction of vector mesons can be described by the vector-meson-proton scattering amplitudes. So photoproduction of vector mesons enables us to give a test for Harari’s conjecture on duality. The ϕ photoproduction is interesting since the process contains no resonance in either of s- and u-channels.

We shall review the vector-meson-dominance hypothesis for photoproduction of vector mesons and proton Compton scattering in § 2, and the quark model for high energy scattering in § 3. We shall deal with proton Compton scattering in § 4 and photoproduction of vector mesons in § 5. A comment on Harari’s conjecture will be provided in § 6. Some remarks will be given in § 7.

§ 2. Vector meson dominance

At first, we assume that proton Compton scattering is given by the vector-meson-dominance model by analogy with photoproduction of vector mesons. Figure 1 shows photoproduction of vector mesons and proton Compton scattering under the vector-meson-dominance model.

We have then

\[ (γP|VP) = \sum_{V'} X_{V'} (VP|VP) \]  

(1)

for photoproduction of vector mesons and

\[ (γP|γP) = \sum_{V} X_{V} (γP|VP) \]  

(2a)

and

\[ = \sum_{V'} X_{V'} X_{V'} (VP'|VP) \]  

(2b)

for proton Compton scattering, where V and V′ denote transverse ρ₀, ω and ϕ mesons, and the X_{V'}s are related to the V → γ transition moments γ_{V},

\[ X_{V}^{2} = \frac{\alpha}{4} \left( \frac{4\pi}{γ_{V}} \right)^{2} \]  

(3)

If the photon is written as a U-spin scalar with ω-ϕ mixing, it is well known that

\[ X_{ϕ₀} : X_{ω} : X_{ϕ} = 3 : 1 : -\sqrt{2} \]  

(4)

The experimental values of the γ_{V}’s are \((4\pi/γ_{ϕ₀})^{-1} = 0.50 ± 0.07\), \((4\pi/γ_{ω})^{-1} = 4.0 ± 0.9\) and \((4\pi/γ_{ϕ})^{-1} = 3.1 ± 0.7\), which are determined from the leptonic decays...
of vector mesons and consistent with Eq. (4).

Assuming that \((\gamma P|VP)\) is purely imaginary at zero momentum transfer, from Eq. (2a) we obtain the following equation\(^5\) by using the optical theorem,

\[
\sigma_\pi(\gamma P) = \sqrt{16\pi} \left\{ X_\pi \left[ \frac{d\sigma}{dt} (\gamma P \rightarrow \rho^0 P)_{t=0} \right]^{1/2} + X_\sigma \left[ \frac{d\sigma}{dt} (\gamma P \rightarrow \omega P)_{t=0} \right]^{1/2} \right\},
\]

Substitution of the experimental values into the right-hand side of the above equation enables us to compare the experimental values of the \(\gamma P\) total hadronic cross section \(\sigma_\pi(\gamma P)\) with the prediction of the vector-meson-dominance model. Although the experimental data below about 8 GeV seem to be consistent with the above prediction, there are large errors in a small number of data on the differential cross section for vector meson photoproduction except \(\rho^0\) production, as well as large errors for \(\rho^0\) production at higher energies \((\geq 10\ \text{GeV})\).\(^6\) Furthermore it is questionable whether or not we can neglect the real part of amplitudes for photoproduction of vector mesons.

A more accurate prediction for \(\gamma P\) total hadronic cross section will be given using the quark model in § 4 and the contribution of the real part of amplitudes for photoproduction of vector mesons will be discussed in § 5.

§ 3. Additivity assumption

It is well known that the quark model is successful in the high energy scattering of hadrons. In this model it is assumed that the amplitude for the elastic scattering of hadrons is approximated by a sum of terms which describe the scattering of individual quarks and antiquarks treated as "quasi free" particles and that the amplitude for quark-quark (or antiquark) scattering does not depend on whether the quarks (or antiquarks) belong to a meson or a baryon.\(^10\) It should be noted that the assumptions lead to \(\sigma_\pi(PP)/\sigma_\pi(\pi P) = 3/2\) in the Pomeranchuk limit (experiment\(^11\) gives \(2\sigma_\pi(PP)/[\sigma_\pi(\pi^- P) + \sigma_\pi(\pi^+ P)] = 1.58 \pm 0.05\), which is not understandable apart from the quark model. Considering these successes, it would be interesting to apply the model to our case.

First, we assume, in analogy to the case of elastic scattering of hadrons, that the amplitudes for the vector meson-proton scattering in Eqs. (1) and (2b) are described by the sum of amplitudes for the scattering of individual quarks which construct vector mesons and a proton.

Next, the charge independence of quark-quark scattering amplitudes leads to the following equalities,

\[
(p\bar{p}) = (pn) = (np) = (nn), \tag{6}
\]

\[
(\bar{p}n) = (\bar{n}p), \tag{7}
\]
where $p(p), n(n)$ and $\lambda(\bar{\lambda})$ denote three quarks (antiquarks), and the notation $(AB)$ represents the elastic quark $(A)$-quark $(B)$ scattering amplitude $(AB|AB)$. Since the real part of quark-quark scattering amplitude is not generally equal to that of corresponding quark-antiquark scattering amplitude, the equality between quark-quark and corresponding quark-antiquark scattering amplitudes is not assumed. The amplitudes $(p\bar{p})$ and $(n\bar{n})$ are not equal to the amplitudes $(p\bar{n})$ and $(n\bar{p})$ due to the contribution of an isosinglet annihilation channel.

Under these assumptions, neglect of the spin dependence of the amplitudes leads to the following relations,

$$\langle p\bar{p} \rangle = \langle n\bar{n} \rangle,$$  \hspace{1cm} (8)  

$$\langle p\bar{n} \rangle = \langle n\bar{p} \rangle,$$  \hspace{1cm} (9)  

$$\langle \bar{p}p \rangle = \langle \bar{n}n \rangle,$$  \hspace{1cm} (10)  

These relations will be used in the discussion of proton Compton scattering in § 4 and photoproduction of vector mesons in § 5.

### § 4. Proton Compton scattering

Substituting Eqs. (11)~(14) into the right-hand side of Eq. (2b), we get

$$\langle \gamma P | \gamma P \rangle = \frac{1}{2} \left( X_{\rho^2} + X_{\omega^2} \right) \left\{ \left( \pi^- P | \pi^- P \right) + \left( \pi^+ P | \pi^+ P \right) \right\}$$

$$+ X_{\rho} X_{\omega} \left\{ \left( \pi^- P | \pi^- P \right) - \left( \pi^+ P | \pi^+ P \right) \right\}$$

$$+ X_{\omega^2} \left\{ \left( K^+ P | K^+ P \right) + \left( K^- P | K^- P \right) - \left( \pi^- P | \pi^- P \right) \right\}. \hspace{1cm} (15)$$

Using the optical theorem, we obtain the following $\gamma P$ total hadronic cross section,

$$\sigma_T(\gamma P) = \frac{1}{2} \left( X_{\rho^2} + X_{\omega^2} \right) \left\{ \sigma_T(\pi^- P) + \sigma_T(\pi^+ P) \right\}$$

$$+ X_{\rho} X_{\omega} \left\{ \sigma_T(\pi^- P) - \sigma_T(\pi^+ P) \right\}$$

$$+ X_{\omega^2} \left\{ \sigma_T(K^+ P) + \sigma_T(K^- P) - \sigma_T(\pi^- P) \right\}. \hspace{1cm} (16)$$

The experimental data can be compared with the prediction of the vector meson dominance model obtained after substituting the experimental data on the $X_{\rho^2}$s and total cross sections for $\pi^\pm P$ and $K^\pm P$ into the right-hand side of Eq. (16). Figure 2 shows that the predicted values, which have about 15 per cent errors due to the errors of the $X_{\rho^2}$s, are in good agreement with all of the known data.
The real-to-imaginary ratio of forward proton Compton scattering amplitude is also obtained by using Eq. (15) and the optical theorem. It is given by

\[
\alpha(\gamma P) = \frac{1}{\sigma_T(\gamma P)} \left\{ \frac{1}{2} (X^x_\rho + X^s_\rho) \left[ \alpha(\pi^- P) \sigma_T(\pi^- P) + \alpha(\pi^+ P) \sigma_T(\pi^+ P) \right] \\
+ X^x_\sigma [\alpha(\pi^- P) \sigma_T(\pi^- P) - \alpha(\pi^+ P) \sigma_T(\pi^+ P)] \\
+ X^x_\rho [\alpha(K^+ P) \sigma_T(K^+ P) + \alpha(K^- P) \sigma_T(K^- P) - \alpha(\pi^- P) \sigma_T(\pi^- P)] \right\},
\]

(17)

where \( \alpha(AB) \) denotes the real-to-imaginary ratio of the amplitude for \( AB \) elastic scattering in the forward direction.

The values of \( \alpha(\pi^\pm P) \), which are determined in the region of about 8~26 GeV/c \( \pi^- \) momenta and about 8~20 GeV/c \( \pi^+ \) momenta,\(^{16} \) are 
\(-0.15 \leq \alpha(\pi^- P) \leq -0.13 \) and 
\(-0.23 \leq \alpha(\pi^+ P) \leq -0.14 \), respectively, and their absolute values are gradually decreasing with increasing energies, but they are not known below 8 GeV/c. It is seen that above 3 GeV/c there is no evidence for the existence of a real part of the \( K^- P \) scattering amplitude,\(^{25} \) so we put \( \alpha(K^- P) = 0 \) above 3 GeV/c. It is well known that the \( K^+ P \) total cross section is constant above 3 GeV/c, and the differential cross section for \( K^+ P \) elastic scattering in the forward direction seems to be nearly constant in the region of 3~15 GeV/c \( K^+ \) momenta.\(^{18} \) A recent experiment gives \( |\alpha(K^+ P)| = 0.60 \pm 0.14 \) at 7.3 GeV/c,\(^{19} \) and by using the dispersion relation technique Bialkowski and Pokorski\(^{20} \) predict the existence of a large negative real part for the \( K^+ P \) elastic scattering.

As seen above, the values of the real-to-imaginary ratios \( \alpha(K^\pm P) \) except \( |\alpha(K^\pm P)| \) at 7.3 GeV/c are not clear at the present stage, though the values of \( \alpha(\pi^\pm P) \) are determined in the region of 8~20 GeV/c \( \pi^- \) and 8~26 GeV/c \( \pi^+ \) momenta. Moreover, the values of \( \alpha(\pi^\pm P) \) are also not known below about 8 GeV/c. Therefore we cannot estimate accurately the value of \( \alpha(\gamma P) \). For the rough estimation of the value of \( \alpha(\gamma P) \), we substitute the experimental data on total cross sections\(^{16} \) and the approximate values \( \alpha(\pi^- P) \approx -0.15, \alpha(\pi^+ P) \approx -0.25, \alpha(K^- P) \approx 0 \) and \( \alpha(K^+ P) \approx -0.60 \) above 3 GeV/c into the right-hand side of Eq. (17), though the values may give a little over-estimation for \( \alpha(\gamma P) \) above 8 GeV/c. The result is \( \alpha(\gamma P) \approx -0.2 \).
The contribution of the real part to the differential cross section for proton Compton scattering at zero momentum transfer is smaller than several per cent, since \( \alpha (\gamma P) \simeq -0.2 \). If we neglect the contribution of the real part, we obtain

\[
\frac{d\sigma}{dt}(\gamma P \to \gamma P)_{t=0} = \frac{1}{16\pi} \sigma_{\gamma}(\gamma P). \tag{18}
\]

However, we have not enough experimental data to compare Eq. (18) with experiment at the present stage.

§ 5. Photoproduction of vector mesons

For the forward scattering, the produced vector mesons carry the same polarization as the initial photons. If we neglect the spin dependence of the forward amplitude for \( \gamma P \to VP \), we obtain the following real-to-imaginary ratios \( \alpha_{\gamma} \) (\( V = \rho^{0}, \omega \) and \( \phi \)) and differential cross sections for photoproduction of vector mesons at zero momentum transfer by using Eqs. (1) and (11) ∼ (14):

\[
\alpha_{\rho} = \frac{(X_{\rho} + X_{\omega}) \alpha(\pi^{-}P) \sigma_{\pi}(\pi^{-}P) + (X_{\rho} - X_{\omega}) \alpha(\pi^{+}P) \sigma_{\pi}(\pi^{+}P)}{(X_{\rho} + X_{\omega}) \sigma_{\pi}(\pi^{-}P) + (X_{\rho} - X_{\omega}) \sigma_{\pi}(\pi^{+}P)}, \tag{19}
\]

\[
\alpha_{\omega} = \frac{(X_{\rho} + X_{\omega}) \alpha(\pi^{-}P) \sigma_{\pi}(\pi^{-}P) - (X_{\rho} - X_{\omega}) \alpha(\pi^{+}P) \sigma_{\pi}(\pi^{+}P)}{(X_{\rho} + X_{\omega}) \sigma_{\pi}(\pi^{-}P) - (X_{\rho} - X_{\omega}) \sigma_{\pi}(\pi^{+}P)}, \tag{20}
\]

\[
\alpha_{\phi} = \frac{\alpha(\pi^{+}P) \sigma_{\pi}(\pi^{+}P) + \alpha(\pi^{-}P) \sigma_{\pi}(\pi^{-}P)}{\sigma_{\pi}(\pi^{+}P) + \sigma_{\pi}(\pi^{-}P)} \tag{21}
\]

and

\[
\frac{d\sigma}{dt}(\gamma P \to \rho^{0}P)_{t=0} = \frac{1}{64\pi} \left[(X_{\rho} + X_{\omega}) \sigma_{\pi}(\pi^{-}P) + (X_{\rho} - X_{\omega}) \sigma_{\pi}(\pi^{+}P)\right]\left(1 + \alpha_{\rho}^{2}\right), \tag{22}
\]

\[
\frac{d\sigma}{dt}(\gamma P \to \omega P)_{t=0} = \frac{1}{64\pi} \left[(X_{\rho} + X_{\omega}) \sigma_{\pi}(\pi^{-}P) - (X_{\rho} - X_{\omega}) \sigma_{\pi}(\pi^{+}P)\right]\left(1 + \alpha_{\omega}^{2}\right), \tag{23}
\]

\[
\frac{d\sigma}{dt}(\gamma P \to \phi P)_{t=0} = \frac{X_{\phi}^{2}}{16\pi} \left[\sigma_{\pi}(\pi^{+}P) + \sigma_{\pi}(\pi^{-}P) - \sigma_{\pi}(\pi^{-}P)\right]\left(1 + \alpha_{\phi}^{2}\right). \tag{24}
\]

For rough estimation of the values of the \( \alpha_{\gamma} \)'s we substitute the same values of meson-baryon total cross sections, \( \alpha(\pi^{\pm}P) \) and \( \alpha(K^{\pm}P) \) as those used in § 4 into the right-hand side of Eqs. (19) ∼ (21). The calculated values are \( \alpha_{\rho} \simeq -0.2 \), \( \alpha_{\omega} \simeq -0.07 \) and \( \alpha_{\phi} \simeq -0.5 \).

The roughly estimated values of real-to-imaginary ratios show that the contribution of the real part of amplitudes to the differential cross sections for photoproduction of \( \rho^{0} \) and \( \omega \) mesons may be neglected while the real part of the \( \phi \) production amplitude contributes significantly to the differential cross section.
However, our values of the $\alpha_\gamma$'s seem to be inconsistent with those recently given by Swartz and Talman, which are $\alpha_\rho = -0.45$ and $\alpha_\omega = -0.35$.

The differential cross sections obtained by substituting the above values of the $\alpha_\gamma$'s into the right-hand side of Eqs. (22)~(24) are compared with the experimental data in Fig. 3. The $\rho^0$ and $\omega$ productions are consistent with experiments while the $\phi$ production seems to have a discrepancy at near 4 GeV.

§ 6. Harari's conjecture and photoproduction of vector mesons

Harari made a conjecture that the Pomeranchuk trajectory is mostly built by nonresonating background in the low energy amplitude, while the "ordinary" trajectories can usually be described in terms of the resonance approximation in the low energy region and be explained the following "strange puzzle"; (a) all total cross sections for reactions including no $s$-channel resonances are approximately constant in energy over a wide energy range, but (b) total cross sections for reactions including strong resonances in $s$-channel decrease gradually to their asymptotic value.

Using Eqs. (1) and (14), we obtain the following $\phi$ production amplitude,
The quark model tells us that baryons and their resonances contain only three quarks and mesons are quark-antiquark pair systems. The \( \varphi P \) elastic scattering amplitude in Eq. (25) has no contribution from either of \( s \)- and \( u \)-channel resonances, since the process contains no isosinglet annihilation channel for quark-antiquark scattering in either of \( s \)- and \( u \)-channels. Therefore, by Harari's conjecture, only the Pomeranchuk trajectory contributes to the \( \varphi P \) elastic scattering in \( t \)-channel. Thus we obtain

\[
\frac{d\sigma}{dt}(\gamma P \rightarrow \varphi P) = X'_s (\varphi P \rightarrow \varphi P) \sim \text{const } s^{2\alpha(0)-1}.
\]  

Most of the total cross sections comes from a small region of \( t \) near \( t=0 \), so the total cross section for photoproduction of \( \varphi \) meson is given by

\[
\sigma(\gamma P \rightarrow \varphi P) \sim \text{const } s^{2\alpha(0)-1}.
\]  

Since \( \alpha(0) = 1 \), the above equation (27) shows that the total cross section for photoproduction of \( \varphi \) meson is approximately constant over a wide range of energy. Experimental data on the total cross section for \( \varphi \) production,

\[
\sigma(\gamma P \rightarrow \varphi P) = (0.41 \pm 0.14) \mu b \quad \text{in } 2.5 \sim 3.5 \text{ GeV},
\]

\[
\sigma(\gamma P \rightarrow \varphi P) = (0.45 \pm 0.13) \mu b \quad \text{in } 3.5 \sim 5.8 \text{ GeV}
\]

and

\[
\sigma(\gamma P \rightarrow \varphi P) = (0.38 \pm 0.18) \mu b \text{ or } (0.52 \pm 0.25) \mu b \text{ at } 15 \text{ GeV},
\]

seem to suggest the constancy in the energy region \( \geq 3 \text{ GeV} \), though it cannot be concluded due to large errors. So Harari's conjecture seems to be supported by photoproduction of \( \varphi \) meson.

Photoproduction of \( \rho^0 \) and \( \omega \) mesons can be treated in the same way as the case of \( \varphi \) production. Since \( \rho^0 \) and \( \omega \) production processes can contain strong resonances in \( s \)-channel, the second half of Harari's conjecture predicts that \( \sigma(\gamma P \rightarrow \rho^0 P) \) and \( \sigma(\gamma P \rightarrow \omega P) \) decrease gradually at high energies. The experimental data on \( \rho^0 \) and \( \omega \) productions are consistent with the above prediction.

\section*{§ 7. Summary and remarks}

It was shown in § 4 that the unified version of the vector meson dominance hypothesis and the quark model with additivity assumption provides the \( \gamma P \) total hadronic cross section in good agreement with all of the known data above about 3 GeV. This success seems to support the vector-meson-dominance hypothesis. From Eq. (16) we can also predict the values of \( \gamma P \) total hadronic cross section at very high energies. However, the errors of our predicted values are not sufficiently small owing to the errors of the \( X'_s \)'s. Thus the accurate determi-
nation of the values of the $X_v$'s is highly desirable.

The differential cross section for photoproduction of $\omega$ meson at zero momentum transfer is in good agreement with the experimental data though they have large errors, and that of $\rho^0$ meson is also consistent within about 15 per cent errors. The differential cross section for $\varphi$ production is consistent with the experiment at 6.5 GeV within about 25 per cent errors while it is inconsistent at near 4 GeV. But it seems too early to say how serious this discrepancy is.

The constancy of the total cross section for photoproduction of $\varphi$ meson above about 3 GeV is consistent with the result from Harari's conjecture that only the Pomeranchuk trajectory in $t$-channel contributes to the process. It also seems to be consistent with the constancy of the differential cross section for $\varphi$ production at zero momentum transfer in the same energy region, as was shown in § 5. But it should be noted that the amplitude for $\varphi$ production has a large negative real part as was also shown in § 5, which contradicts the ordinary picture for the Pomeranchuk trajectory.

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