Determination of $|g_A|$ from Veneziano Amplitude and Current Algebra Sum Rule for $\pi$-$N$ Scattering

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In the present note, an attempt has been made to investigate the compatibility of current-algebraic sum rule (AWSR) with the Veneziano-type amplitude for $\pi$-$N$ scattering, constructed mainly on the hypotheses of high-energy behavior, crossing symmetry and duality. Our investigation yields $|g_A|$ equal to 1.33, which compares favourably with the corresponding experimental value. Yellin and Corrigan have similarly considered the case of $\pi\pi$ and $\pi K$ scattering.

Several authors have constructed the V-A (Veneziano amplitude) for $\pi$-$N$ scattering by incorporating different experimental findings in their formulations. We have used the simplest amplitude as suggested by Gupta and Bose, who have neglected the magneton-type $pNN$ coupling in their determination of parameters.

The V-A for our case runs as

$$A^-(s, t) = \beta_1 [B(1-\alpha(t); 3/2-\alpha(s)) - B(1-\alpha(t); 1/2-\alpha(u))] + \beta_2[C(1-\alpha(t); 1/2-\alpha(s)) + \beta_3 C(1/2-\alpha(s); 1/2-\alpha(u))],$$

$$B^-(s, t) = \beta_4 [C(1-\alpha(t); 1/2-\alpha(s)) + \beta_5 C(1/2-\alpha(s); 1/2-\alpha(u))],$$

(1)

where $B(x, y) = (\Gamma(x) \Gamma(y)) / (\Gamma(x+y))$ and $C(x, y) = (\Gamma(x) \Gamma(y)) / (\Gamma(x+y-1))$ and $\alpha$ is the $p\cdot f^0$ trajectory given by $\alpha(t) = .48 + .89t$. $\alpha(s)$ is the degenerate $N_m$, $d_s$ and $N_s$ trajectory taken to be $\alpha(s) = -.26 + .86s$. The amplitudes $A^+$, $B^+$ do not occur in the AWSR and as such are unnecessary in the present context.

In order to determine the parameters, we extrapolate the amplitude (1) to the vector mesons ($\rho$) pole in $t$-channel and nucleon pole in the direct channel. If tensorial coupling of $\rho$ is considered, then it can be easily seen

$$\beta_1 = \frac{g_{\rho NN}^2}{4\pi} \frac{K}{2M},$$

$$\beta_2 = -\frac{g_{\rho NN}^2}{16\pi} b(1+K),$$

$$\beta_3 + \beta_5 = \frac{g_b^2}{4\pi}$$

(2)

In Eq. (2) we have assumed that $\rho$ is universally coupled to the isospin current. Equations (1) and (2) are enough for our purpose and determine the amplitude completely.

The AWSR for $\pi$-$N$ scattering is written as

$$g_A^2 = 1 - \frac{2f_\pi^2}{\pi} \int_{max}^{\infty} \frac{\text{Im}(A^+ + \nu B^-)}{\nu^2} d\nu,$$

(3)

which can be looked upon as an equation for the determination of the renormalized value of $g_A$.

Since the amplitudes are written in narrow-resonance approximation, their imaginary part can be written as

$$\text{Im} A^- = 2\pi \beta_1 \sum_{N=0}^{\infty} \frac{(-1)^N}{N!} \delta(3/2-\alpha(s)+N) \Gamma(3/2-\alpha(s)-\alpha(t)) \times \Gamma(1-\alpha(t)), $$

$$\text{Im} B^- = 2\pi \beta_3 \sum_{N=0}^{\infty} \frac{(-1)^N}{N!} \delta(1/2-\alpha(s)+N) \Gamma(1/2-\alpha(s)-\alpha(t)) \times \Gamma(1/2-\alpha(s)-\alpha(t)),$$

(4)

$$+ 2\pi \beta_3 \sum_{N=0}^{\infty} \frac{(-1)^N}{N!} \delta(1/2-\alpha(s)+N) \Gamma(1/2-\alpha(s)-\alpha(t)) \times \Gamma(1/2-\alpha(s)-\alpha(t)),$$

$$+ 2\pi \beta_3 \sum_{N=0}^{\infty} \frac{(-1)^N}{N!} \delta(1/2-\alpha(s)+N) \Gamma(1/2-\alpha(s)-\alpha(t)) \times \Gamma(1/2-\alpha(s)-\alpha(t)),$$

$$+ 2\pi \beta_3 \sum_{N=0}^{\infty} \frac{(-1)^N}{N!} \delta(1/2-\alpha(s)+N) \Gamma(1/2-\alpha(s)-\alpha(t)) \times \Gamma(1/2-\alpha(s)-\alpha(t)).$$
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Substituting from Eqs. (4) and (3), the sum rule reads

\[ g_A^2 = 1 - S_1 - S_2, \]

\[ S_1 = \frac{2f_2}{\pi} \sum_{N=0}^{\infty} 2\pi \beta_1 \frac{(-N)^N}{N!} \]
\[ \times \frac{\Gamma(N+1/2)}{\Gamma(1/2 - \alpha(u))} \quad \Gamma(1 - \alpha(s) - \alpha(u)) \]
\[ \times \frac{\Gamma(N+5/2)}{\Gamma(-N - 48) \Gamma(58N + 577)^2}, \]
\[ (5) \]

\[ S_2 = \frac{2f_1}{\pi} \sum_{N=0}^{\infty} 2\pi \beta_1 \frac{(-1)^N}{N!} \]
\[ \times \frac{\Gamma(N+5/2)}{\Gamma(52 - N) \Gamma(623N - 0.04)}, \]
\[ (6) \]

The summation involved in Eq. (6) is over all the nucleon resonances in the direct channel. The \( N=0 \) term in \( S_2 \) of Eq. (6) is dropped, as it gives a rise to a negative value of \( \nu \) outside the range of integration. Using the following formula

\[ \sum_{N=1}^{\infty} \frac{\Gamma(N+3/2)}{\Gamma(N+1) (N+1-d)^3} \]
\[ = \frac{\Gamma(1-d)}{\Gamma(1/2+d)} \{ \psi(1-d) - \psi(1/2-d) \}, \]

where \( \psi(Z) = (d/dZ) \ln \Gamma(Z) \), the r.h.s. of Eq. (5) is seen to be equal to 1.7864, yields \( |g_A| = 1.33 \).

A recent experimental determination of \( g_A \) from neutron life-time sets an upper limit equal to 1.26, very close to the value obtained above, whereas the theoretical value predicted by Adler is 1.25. On the other hand, predictions from \( SU_3 \times SU_3 \) model representative with degenerate baryons gives \( |g_A| = 1.67 \), which goes down to 1.44 taking into account the mass splitting and finite \( N \) width. In view of the approximations made in our calculations, viz. assumption of degenerate baryon trajectories, presence of parity doublets and neglect of non-leading terms in the amplitude, our value for \( |g_A| \) is in sufficiently close agreement with experiment. It is expected that a more realistic model, free from above-mentioned unwanted features, will reduce the value of \( |g_A| \) so as to bring about better agreement.

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