Commutation Relations of Current and Meson Fields

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In general it is difficult to give the axial-vector current, which satisfies the algebra of current, in terms of the meson fields with the same spin and parity.\(^1\) The only one go-well example is the pseudo-scalar meson model presented by Kuo and Sugawara.\(^2\) In their model, the axial-vector current consists of linear and trilinear terms with respect to the fields, and the terms contain the space-time derivative operators linearly. For the scalar and vector fields, such an axial-vector current does not exist. On the other hand, it is possible to define the axial-vector current when one introduces pseudo-vector fields instead of the derivatives of pseudo-scalar fields. Explicitly the time-component of the axial-vector current density is given in terms of the pseudo-vector field, \(B^\mu(x)\), as\(^*\)

\[
\alpha^\mu(x) = MB^\mu(x) + \frac{1}{M} M^{ij} C_{ij} B^i(x) \cdot B^j(x), \quad (1)
\]

where \(M\) is a constant with the dimension of mass and \(C_{ij}\) is a function of the appropriate structure constant.

Now, let us consider a massive gauge field \(b^\mu(x)\) whose Lagrangian density is

\[
\begin{align*}
L &= -\frac{1}{4} b_{\mu\nu} \cdot b_{\mu\nu} - 2 i \frac{\mu}{m^2} b_{\mu} \cdot b_{\mu}, \\
b^\mu(x) &= \Theta^\mu b^\mu(x) - \Theta^\nu b^\nu(x) \\
&\quad + 2 i \alpha_f b^\mu(x) \cdot \phi^\mu(x).
\end{align*}
\]

The canonical conjugate variable to the field \(b^\mu(x)\) is

\(^*\) The Greek indices are used for Lorentz space and the Latin indices are for unitary space.


From the equation of motion, $b_i^r(x)$ is represented as

\begin{equation}
   b_i^r(x) = -m^{-2} \{ \partial_\mu \pi_i^r(x) - 2 i \alpha f_{ij} b_j^r(x) \cdot \pi_i^s(x) \},
\end{equation}

where the symbol $\cdot$ denotes the space-component. Then, the following commutation relations hold

\begin{equation}
   [b_i^r(x), \pi_i^s(y)] = \frac{2g}{m^2} f_{ij} \partial_j \delta(x-y),
\end{equation}

These commutation relations, however, do not stand for pseudo-vector gauge fields solely, and even if we identify $b_i^r(x)$ to $B_i^r(x)$ taking the limit of vanishing $g$ with finite $m$, the vanishing commutator between $b_i^r(x)$ and $\pi_i^s(x)$ prevents us from forming the algebraic commutation relations of current for the currents involving (1).

As the next step we loose the limitation that the axial-vector current consisting of mesons with the same spin but different parity. In the following, we will derive commutation relations of currents in the case of spin one meson nonet.

The octet current densities, we will discuss, are defined in terms of the vector meson nonet, $W^a_v$, and the pseudo-vector meson nonet, $B^a_v$, as follows:

\begin{equation}
   v_i^a = (F_{v} f_{iab} + D_{v} d_{iab}) V_{v}^a,
\end{equation}

\begin{equation}
   a_i^a = (F_{a} f_{iab} + D_{a} d_{iab}) A_{a}^a,
\end{equation}

where \( F_{v}, D_{v} \) and \( F_{a}, D_{a} \) are constant c-numbers.

Using the commutation relations obtained from the identity of Jacobi.

When we assume the vector current to be purely F-type, i.e. $D_{v}=0$, the commutation relations of currents (9) form algebra in the case: (a) \( F_{a}=0, D_{a}=0 \) or (b) \( F_{a}=0, D_{a}=0 \).

Taking both of the vector current and axial-vector current as purely F-type, more explicitly: \( F_{a}=F_{v}=i, D_{a}=D_{v}=0 \), we have the same algebra of current as quark model and also as the chiral gauge formalism in which there appears F-type current only.

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4) See also P. Narayanaswami and T. Pradhan, TRIESTE preprint, IIC/66/42.