Charged Higgs Effects in Inclusive and Exclusive Weak Radiative $B$-Decays

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The decay $B 	o X_s\gamma$ is analysed with the non-standard physical effects due to the charged Higgs contribution in the two-Higgs-doublet model. The hard $\gamma$ emission of $B$-meson, being induced at one-loop level, is an important process to search for the non-standard contributions in the low energy region. It is shown that the charged Higgs contribution can provide sizable enhancements for the weak radiative $B$-decay compared to the standard model. It is also found that the process such as $B \to K^*(1410) + \gamma$ has the largest fraction in the exclusive radiative decays, which amounts to almost 20%.

§ 1. Introduction

Recent LEP experiments show no indication beyond the standard model (SM) of electroweak interactions. Below the Fermi scale the existing data are all consistent with the predictions of SM. The remaining main problem is to reveal the structure of the Higgs sector. It seems that the gauge coupling unification is realized by the minimal supersymmetric model (MSSM) at the GUT scale $10^{16}$ GeV. The MSSM has two Higgs-doublets, which appear also in the other various models, e.g., in the axion resolution of the strong CP problem or in the superstring inspired E6-model. Though in SM the only physical Higgs particle is a neutral Higgs, the two doublets model has a physical spectrum consisting of three neutral scalars and a charged scalar. At the energies probed by TRISTAN, LEP, SLC and Fermilab TEVATRON, there is no indication of the existence of any scalar particle. The direct search at LEP for charged Higgs scalar puts the upper limit on its mass with $m_H \leq m_Z/2$ and SSC is now setting up to study physics of the Higgs sector. In the low energy region, we consider it is of much importance to study the effects of the virtual charged Higgs scalar based on the two-Higgs-doublet model (THDM) to open, if any, the window of new physics beyond SM. From this standpoint we look for the signature of the virtual charged Higgs scalar through the analysis of the weak radiative decays of $B$-mesons.

The experimental search for the virtual effects will be able to be carried out at the $B$-factory which is under proposal and is expected to give a sensitive test of SM or to provide important clues beyond SM. Already some analyses have been performed on the weak radiative process of $B$-meson from the viewpoint of SM or of the models beyond SM, which include charged Higgs scalars, or super particles or the fourth-generation of quarks. The charged Higgs scalar is expected to give the most sizable effects among these extra candidates beyond SM, because of the large Higgs-coupling with the top quark, as shown in the following. So in this paper we...
concentrate on the effects of the charged Higgs scalar within THDM (so-called as Model II in the literatures\(^3\)) on the \(\gamma\) emission accompanied by the \(s\)-quark from the \(b\)-quark.

Now the relevant Lagrangian in Model II is given by

\[
L_{\text{eff}} = -\frac{g_w}{2\sqrt{2}} \frac{1}{m_w} \text{cot} \beta \bar{u}_i V_{ij} (a_{ij} - b_{ij} \gamma_5) d_j H^+ + h.c.,
\]  

(1.1)

where \(g_w\) is the \(SU(2)_L\) coupling constant, \(V_{ij}\) is the KM matrix element and \(\text{tan} \beta = \frac{v_u}{v_d}\) is the ratio of the vacuum expectation values of the neutral sectors of the two Higgs-doublets \(\Phi_u\) and \(\Phi_d\) which couple to the up-quarks and down-quarks, respectively. The coefficients \(a_{ij}\) and \(b_{ij}\) (with \(i\) and \(j\) being generation indices) are given by

\[
a_{ij} = m_{ui} + m_{dj} \text{tan}^2 \beta, \\
b_{ij} = m_{ui} - m_{dj} \text{tan}^2 \beta.
\]  

(1.2)

This interaction induces the effective \(\gamma\) emission at one loop level corresponding to the SM process with the \(W\)-boson replaced by the charged Higgs \(H^+\) as will be discussed in the free quark picture in §2. Section 3 is devoted to the comparison of predictions with experiments.

The theoretical study on the exclusive processes is also significant, because it is expectable that the weak radiative processes of \(B\) decay might be saturated by the two body decays such as \(B \rightarrow K\) (higher resonance) + \(\gamma\). Then the analysis of the exclusive decays is given in §4. We present some discussion in §5.

§2. Weak radiative decay of \(B\)-meson in the two-Higgs-doublet model

The one-loop flavour changing processes \(s \rightarrow d\gamma\) and \(b \rightarrow s\gamma\) have received a good deal of attention in the literature. In SM these processes occur via the exchange of a virtual \(W\)-boson.\(^11\) In models with a charged Higgs scalar which couples to fermions, there is generally a corresponding process with a charged scalar appearing inside the loop in place of the \(W\)-boson. By using Eqs. (1.1) and (1.2) we obtain the following matrix element for the \(b \rightarrow s\gamma\) interaction:

\[
M^{2H}(b \rightarrow s + \gamma) = -\frac{e G_F}{2 \sqrt{2} \pi^2} C^{2H}_7 (y) V_{ts}^* V_{tb} \\
\times q^\nu \bar{\nu} (p) \sigma_{\mu\nu} (m_b R + m_s L) b(p + q),
\]  

(2.1)

where \(R = (1 + \gamma_5)/2\) and \(L = (1 - \gamma_5)/2\). The function \(C^{2H}_7(y)\) of \(y = m_t^2/m_H^2\) is

\[
C^{2H}_7(y) = \frac{y}{4(1 - y)^2} \left[ \frac{5y^2 - 8y + 3}{3} - \frac{2}{3} (3y - 2) \ln y \right] \\
- \text{cot}^2 \beta \frac{y}{4(1 - y)^2} \left[ \frac{8y^3 - 3y^2 - 12y + 7}{18} + \frac{2}{3} y \left( \frac{1}{2} - \frac{3}{2} y \right) \ln y \right].
\]  

(2.2)

Now SM matrix element of the radiative interaction is obtained\(^11\) by the replacement of \(C^{2H}_7(y) V_{ts}^* V_{tb}\) with the function \(\sum_{i=u,c,t} C^{SM}_i(x_i) V_{ts}^* V_{tb}\) of \(x_i = m_t^2/m_w^2\) yielding
As this function is proportional to $x_i$ at small $x_i$ and converges to $-1/3$ at large $x_i$, the contribution from the top quark dominates SM matrix element.

Since the $b$-quark is heavy compared to the QCD scale, the short distance contribution to the inclusive $B \to X_s + \gamma$ decay rate, where $X_s$ is the hadronic state involving one $s$-quark, can be approximated by the rate for the free quark decay $b \to s + \gamma$. This free quark approximation successfully interprets the semi-leptonic $B$-meson decay.\textsuperscript{12) In the following we take this approximation to estimate the rate of the inclusive $\gamma$ emission of $B$-meson decays.

Then the branching fraction $B(B \to X_s + \gamma)$ is given by

$$B(B \to X_s + \gamma) \approx B(b \to s + \gamma) = \frac{6a}{\pi \rho(m_c/m_b)} B(b \to c e \nu_e)$$

with $C_7(m_W) = C_7^{\text{SM}}(x_i) + C_7^{\text{ch}}(y)$. Note that the argument in the parenthesis of $C_7$ denotes the energy scale throughout this paper. In the above equation, $\rho(x) = 1 - 8x^2 + 8x^6 - 24x^8 \ln x$ and $\rho(m_c/m_b) = 0.5644$ for $m_c = 1.4$ GeV and $m_b = 5.0$ GeV. However, it is noted that the QCD correction is important in such radiative $b$-decay processes and $C_7(m_W)$ should be replaced with the QCD corrected factor $C_7(m_b)$. First this was pointed out by Grinstein and Wise\textsuperscript{13) and they improved the calculation.\textsuperscript{14) Recently all the elements of the anomalous dimension matrix have been given up to the two-loop order by Misiak.\textsuperscript{15) Following these references and Appendix A, we obtain the QCD corrected factor $C_7(m_b)$ in THDM as

$$C_7(m_b) = 0.686C_7(m_W) + 0.088C_8(m_W) - 0.191$$

where $C_8(m_W)$ is the coefficient of the so-called gluonic penguin interaction $O_8$ given in Appendix A. Including charged Higgs contribution, $C_8(m_W)$ is

$$C_8(m_W) = \frac{x_i}{4} \left[ \frac{1}{2} x_i \frac{5}{2} x_i - \frac{1}{2} \right] - 3x_i \ln x_i$$

$$= \frac{y}{2} \left[ 1 - \frac{3}{2} \frac{\ln y}{(1-y)^2} \right] + \cot^2 \beta \frac{y}{12} \left[ \frac{1}{2} y^2 - \frac{5}{2} y - 1 \right] - \frac{3y \ln y}{(1-y)^4}$$

In Eq. (2.5), the factor $-0.191$ is induced from mixing with mainly the interaction $O_2$ whose definition is also in Appendix A. In the approximation that the $O_7$ operator mixes with only $O_8$ and $O_9$, the coefficient $C_7(m_b)$ is

$$C_7(m_b) = 0.686C_7(m_W) + 0.050C_8(m_W) - 0.254$$

as discussed in Ref. 14). As shown in the next section our predicted decay rate given by Eqs. (2.4) and (2.5) is somewhat different from the value obtained in Ref. 14).

Numerically we find that, for the values of $\cot \beta$ much less than 1, the effect of
charged Higgs exchange on the weak radiative interaction is still visible in contrast with the case of gluonic penguin interaction.\textsuperscript{16) In the gluonic penguin interaction, the charged Higgs effect is noticeable only for the parameter $\cot\beta$ to be larger than 1. This circumstance is attributed to the fact that the soft GIM cancellation in SM works in the virtual gluon interaction and the contribution of the term derived from THDM is proportional to $\cot^2\beta$ in the process of $b \rightarrow s + p$.

\[
H_{\text{penguin}} = -\frac{\alpha_G e_F}{24\sqrt{2}\pi} V_{ts}^* V_{tb} \frac{\lambda^2}{2} (1 + \gamma_s) \left\{ g^{\text{SM}}(x_t) + \frac{1}{3} y \cot^2 \beta g^{2H}(y) \right\} \gamma_b \bar{q} \gamma^\mu \frac{\lambda^a}{2} q,
\]

where the functions of $x_t = m_t^2/m_W^2$ and $y = m_t^2/m_H^2$ are

\[
g^{\text{SM}}(x_t) = \frac{x_t(1-x_t)(18-11x_t-x_t^2)-2(4-16x_t+9x_t^2)\ln x_t + 8\ln \frac{m_e^2}{m_W^2}}{(1-x_t)^4}
\]

\[
-8 \frac{V_{tb}^* V_{ub}}{V_{ts}^* V_{ts}} \ln \frac{m_u^2}{m_c^2}
\]

and

\[
g^{2H}(y) = \frac{16-45y + 36y^2 - 7y^3 + 6(2-3y)\ln y}{(1-y)^2},
\]

respectively. This gives the remarkable contrast between the radiative $b$-decay and the gluonic penguin case.

For larger values of $\cot\beta$, the second term of the r.h.s. in Eq. (2.2) is enhanced for the appropriate values of $m_t$ and $m_H$. So the effect of the charged Higgs exchange on the weak radiative interaction is rather distinguishable compared to the case of small values of $\cot\beta$ which is close to zero. These facts suggest that the weak radiative processes are favourable ones to search for the effect of the virtual charged Higgs scalar.

\section*{§ 3. Inclusive weak radiative decay and the parameter region in the two-Higgs-doublet model}

Taking into account the correction by QCD as discussed in § 2, we obtain the branching ratio $B(B \rightarrow X_s + \gamma)$ shown in Figs. 1(a) and (b) by using

\[
B(b \rightarrow ce\nu_e) = B(B \rightarrow ev_{\text{hadrons}}) = 10.7\%
\]

as the experimental value\textsuperscript{17) to estimate the inclusive weak radiative branching ratio. In Fig. 1(a), the $m_t$-dependence of the branching ratio is shown with $m_H = 200$ GeV and $\cot\beta = 0$ and 2 to be compared with SM. Though the predicted branching ratio in SM has rather small $m_t$-dependence within the region between 100 GeV and 180 GeV, the dependency on $m_t$ is rather strong in THDM. Figure 1(b) shows the dependence on $m_H$ at the typical $t$-quark mass $m_t = 140$ GeV.

Without QCD correction, the branching ratio for $b \rightarrow s + \gamma$ in SM is

\[
B(b \rightarrow s + \gamma) = 8.3 \times 10^{-5}. \quad (m_t = 140 \text{ GeV})
\]
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Fig. 1.(a) The inclusive branching ratio $B(B \rightarrow X_s + \gamma)$ versus $m_t$. The dashed line shows the SM prediction. The solid line shows the case $m_H=200$ GeV and $\cot\beta=0$. The dash-dotted line shows the case $m_H=200$ GeV and $\cot\beta=2$. The shaded area under the solid line shows the allowed region by the experimental upper limit whose value is $8.4 \times 10^{-4}$.

From Figs. 1(a) and (b) the branching ratio in SM with QCD correction is

$$B(b \rightarrow s + \gamma) = 3.3 \times 10^{-4}.$$  \hspace{1cm} (3·3)

So the QCD correction enhances the magnitude of the branching ratio up to nearly four times larger than the one without QCD correction.

It is to be remarked from Figs. 1(a) and (b) that the sizable enhancement exists even in the minimum case of $\cot\beta = 0$ for the range of $m_H$ less than 300 GeV. The range of $m_H$ less than 200 GeV with $\cot\beta \geq 2.5$ is already excluded by CLEO experiments which give the upper limit of $b \rightarrow s + \gamma$ branching fraction as

$$B(B \rightarrow X_s + \gamma) < 8.4 \times 10^{-4}.$$  \hspace{1cm} (3·4)

In Fig. 2 we show the allowed region of $m_H$ and $\cot\beta$ obtained from this experimental limit as well as the results of the other analyses which will be discussed at the next step.

There have been given several bounds on $m_H$ and $\cot\beta$ based on some theoretical speculations. One is to require that the width of the charged Higgs scalar should be less than half of its mass if $m_H \geq m_t + m_b$, which leads to $\cot\beta \leq 4$, while the case $m_t \geq m_H + m_b$ leads to $\cot\beta \leq 10$ from the same condition for the width of the $t$-quark.
The other is to ask for the perturbative Yukawa coupling for $\bar{t}bH^-$ and this leads to $\cot\beta \leq 5$.

Now we want to remark that the presence of the charged Higgs scalar affects also the other phenomena. In particular, it contributes significantly to the $B_d^0-\bar{B}_d^0$ mixing and the $\epsilon$-parameter in the neutral $K$ meson system.

In the following, we study these subjects by use of the recent improved experimental data. Let us start to consider the particle-antiparticle mixing. The short distance dispersive part of the neutral $P-\bar{P}$ mass matrix $M_{ij}$, where $P$ denotes the neutral $K$ or $B$ mesons, can be obtained by evaluating the box diagrams with $W$ bosons as well as charged Higgs scalars. The off diagonal component $M_{12}$ is

$$M_{12} = \frac{G_F^2}{12\pi^2} f_P^2 m_P B_P m_W^2 [ (\bar{M}_{12})_{WW} + (\bar{M}_{12})_{HH} + (\bar{M}_{12})_{HW} ], \quad (3.5)$$

where the definition of variables is given in Appendix B. The $B_d^0-\bar{B}_d^0$ mixing parameter $\Delta m_{Ba}$ is

$$\Delta m_{Ba} = 2 |M_{12}|. \quad (3.6)$$

The $CP$ violating parameter $\epsilon$ in the $K$ meson system is given by

$$\epsilon = \frac{e^{i(\pi/4)}}{\sqrt{2}\Delta m_K} [ \text{Im} M_{12} + 2\xi \text{Re} M_{12} ], \quad (3.7)$$

where $\xi = \text{Im} A_0/\text{Re} A_0$ with $A_0$ denoting $\Delta I = 1/2$ amplitude in $K \rightarrow \pi\pi$ decays. Both of $\epsilon$ and $\Delta m_{Ba}$ involve the KM matrix element $V_{td}$, which is determined from the observed $|V_{td}|$ value and the $CP$ violating phase $\phi(= -\arg V_{td})$. According to $\phi \rightarrow 0$, the value of $|V_{td}|$ decreases and reaches the constant value $|V_{td}| = |V_{ub}|$. So, the contribution of SM becomes minimum at $\phi = 0$. In the following calculation, we use $|V_{cb}| = 0.045$ and $|V_{ub}|/|V_{cb}| = 0.1$.

Then the allowed region of the Higgs parameters is derived as follows. First we note that the maximal estimate of the charged Higgs scalar contribution to the observed quantity is given when the allowed contribution of SM is estimated to be minimum. Therefore, we should estimate the contribution of SM to be as small as possible. The difference between the calculated values in SM and the experimental ones, for which we take the largest ones

$$|\epsilon| = 2.26 \times 10^{-3}, \quad \Delta m_{Ba} = 4.3 \times 10^{-13} \text{ GeV}, \quad (3.8)$$

within the experimental errors, could be attributed to the contribution of the charged Higgs scalar. The values of the theoretically ambiguous parameters $B_K$ and $f_{Ba}/B_{Ba}$ are also chosen as typical ones 0.7 and 0.15 GeV, respectively. Finally, we find that in the case $15^\circ \leq \phi \leq 55^\circ$ the most rigorous constraint for the Higgs parameters is given by $\epsilon$, while in the case $0^\circ < \phi < 15^\circ$, the $B_d^0-\bar{B}_d^0$ mixing dominantly limits the parameter region. There is no room for the contribution of the charged Higgs scalar for $\phi > 55^\circ$, because SM can saturate the whole contribution.

From the above arguments, the bounds are shown in Fig. 2 on the $m_H$-$\cot\beta$ plane.
both at $\phi=0^\circ$ where the constraint is from the $B_d^0 - \bar{B}_d^0$ mixing and at $\phi=45^\circ$ and $15^\circ$ where the constraint is from the $\epsilon$ parameter. These bounds are rather tight than that from the experimental upper limit of $B(B \to X_s + \gamma)$ as seen in Fig. 2. Note that these bounds depend on the values of $B_K$ and $f_{B_d}/B_{B_d}$. For example, the 30% decrease of $f_{B_d}$ loosens its bound about 30~40% in magnitude. On the other hand, the 30% decrease of $B_K$ loosens the bound up to about the factor two at $\phi\approx45^\circ$. Improvement of the theoretical estimate of these parameters is awaited to obtain more reliable bounds.

Here, we want to mention the constraint from the $\epsilon'/\epsilon$ ratio in the $K$ meson system. The constraint from the present data of this ratio is too loose to bound the Higgs parameters because of the large experimental error. In future, if the experimental value of $\epsilon'/\epsilon$ will be obtained more precisely, it can determine the phase $\phi$ and a tight bound for the Higgs parameters will be given.

Consequently, in the present situation, we may say that, throughout the whole region of $\cot\beta$, the decay $B \to X_s + \gamma$ is one of the most favourable processes to detect the virtual effects of the charged Higgs scalar if its mass is below 500 GeV.

§ 4. Exclusive decays into higher $K$-resonances

In the previous sections, the inclusive weak radiative $B$ meson decay is studied by being based on the free quark decay $b \to s + \gamma$. The existing experimental data on semileptonic $D$- and $B$-decay suggest that the final hadronic states of the radiative $B$ meson decay are dominated by the resonances with the $s\bar{q}(q= u$ or $d)$ flavor quantum number. The saturation by the resonance modes is also supported by the $1/N_c$ expansion approach to the hadronic matrix elements. The lowest $s\bar{q}$ resonance that contributes to the weak radiative decays is the $K^*(890)$ meson, whose case is analysed by several authors. However, this channel seems to be strongly suppressed by a hadronic form factor as discussed later. For higher-mass resonances, the rate may possibly be not suppressed by hadronic form factors. The decay rate is dominated by $s\bar{q}$ resonances with masses of order

$$\sqrt{m_b\Lambda}$$

in the limit of the large $b$-quark mass, where $\Lambda$ is the QCD energy scale. Therefore, it is significant to study the exclusive decays into higher $K$-resonances to search for new physics. Also the analyses of the exclusive decay are helpful for the future experimental study at $B$-factory. We shall intensively focus on the weak radiative two body decays of the $B$ meson.

In the $s\bar{q}(q= u$ or $d)$ system, a rich spectrum of states has been observed. The resonance state $K^{res}$ is specified by $n, L, s$ and $J,$ which denote the radial excitation quantum number, the orbital angular momentum, the sum of each spin of the two quarks and the total spin of the meson, respectively. We investigate the following states with the notation $n^{2s+1}L_J; 1^3S_0, 1^3P_0, 1^3P_1, 1^1P_1$ and $2^3S_1$. The radiative decays into $J=0$ states ($1^3S_0, 1^3P_0, 2^1S_0$) are forbidden by the law of angular momentum conservation. Allowed states are:
The spin 1 mesons $K_1(1270)$ and $K_1(1400)$ are nearly $45^\circ$ mixed states with $1^3P_1$ and $1^1P_1$.

Our formulation to calculate the decay matrix elements is as follows. For the decay $B \to K^{\text{res}} + \gamma$, only the operator $O_7$ given in Appendix A is relevant as in the inclusive case:

$$<K^{\text{res}}\gamma|H_{\text{eff}}|B> = -\frac{4G_F}{\sqrt{2}} V_{tb} V^*_{ts} C_7(m_b)<K^{\text{res}}\gamma|O_7(m_b)|B>.$$  

(4·3)

Using the Gordon identity, the $O_7$ operator is reduced to

$$O_7(m_b) = C((m_b + m_s) \bar{s} \gamma_\mu b - (m_b - m_s) \bar{s} \gamma_\mu (1 + \gamma_5) b) \gamma_\mu,$$  

(4·4)

where $C = -em_b/16\pi^2$ and $\gamma_\mu$ is the polarization vector of the photon. Then, the hadronic matrix elements are given in terms of several form factors at zero value of the squared transfer momentum as follows:

$$|<K^*(892)\gamma|O_7|B>|^2 = 2C^2[(m_b - m_s)^2 f(0)^2 + 4(m_b + m_s)^2 m_b^2 g(0)^2 p^2]$$  

(4·5)

for the $1^3S_1$ state,

$$|<K^*(1430)\gamma|O_7|B>|^2 = 2C^2[(m_b + m_s)^2 k(0)^2 + 4(m_b + m_s)^2 m_b^2 h(0)^2 p^2]$$  

(4·6)

for the $1^3P_1$ state,

$$|<K_1(1410)\gamma|O_7|B>|^2 = 2C^2[(m_b + m_s)^2 r(0)^2 + 4(m_b - m_s)^2 m_b^2 v(0)^2 p^2]$$  

(4·7)

for the $3^P_1$ state and

$$|<K^*(1410)\gamma|O_7|B>|^2 = 2C^2[(m_b - m_s)^2 f'(0)^2 + 4(m_b + m_s)^2 m_b^2 g'(0)^2 p^2]$$  

(4·8)

for the $2^3S_1$ state, where $p$ is the three momentum of the $K^{\text{res}}$ meson.

The form factors $f(0)$, $g(0)$, $k(0)$, $l(0)$, $q(0)$, $r(0)$, $v(0)$, $f'(0)$ and $g'(0)$ for each final state generally depend on the quark potential. Here we use the form factors given by Isgur, Scora, Grinstein and Wise (ISGW), which have successfully been applied to the electron energy spectra of semileptonic $D$ and $B$ meson decays. They have used the wave functions derived from the Coulomb plus linear potential. This simple model gives quite reasonable spin-averaged spectra of $c\bar{d}$ and $b\bar{d}$ mesons up to $L=2$. Those form factors, which are given in Appendix C, include the relativistic compensation factor, although their model is the nonrelativistic one. Since the radiative decays have the largest recoil point $q^2 = 0$, this compensation factor is important in our analyses. In the naive quark model calculations, the decay into the $1^1P_1$ state is forbidden, because the $O_7$ operator is a spin-flip one. So, the $K_1(1270)$ and $K_1(1400)$ mesons can decay only through the component of the $1^3P_1$ state.
The decay width of possible modes is generally
\[
\Gamma(B \to K_{\text{res}}^* \gamma) = \frac{a G_F^2}{64 \pi^2} |V_{tb} V_{ts}^\ast|^2 |C_7(m_b)|^2 \left( \frac{m_b}{m_B} \right)^2 |\langle K_{\text{res}}^* \gamma | O_7 | B \rangle|^2. \quad (4.10)
\]

Since the inclusive decay width is
\[
\Gamma(B \to X_{\gamma}) = \frac{a G_F^2}{4 \pi^2} |V_{tb} V_{ts}^\ast|^2 p_{\text{incl}}^2 |C_7(m_b)|^2 m_b^2 \quad (4.11)
\]

with \(p_{\text{incl}} = (m_b^2 - m_s^2)/2m_b\), the ratio
\[
R(K_{\text{res}}^*) = \frac{\Gamma(B \to K_{\text{res}}^* \gamma)}{\Gamma(B \to X_{\gamma})} = \frac{1}{16 p_{\text{incl}}^2 m_B^2} |\langle K_{\text{res}}^* \gamma | O_7 | B \rangle|^2, \quad (4.12)
\]
is independent of the value of \(C_7(m_b)\). Hence, this ratio is free from the QCD correction as well as from the charged Higgs contribution. Our predicted fractions are
\[
R(K^*(892)) = 7.0\%, \quad R(K^*_s(1430)) = 3.6\%,
\]
\[
R(K_l(1270)) = 4.2\%, \quad R(K_l(1400)) = 5.9\%, \quad R(K^*(1410)) = 17.4\%. \quad (4.13)
\]

Our value 7.0% for \(B \to K^*(892)\gamma\) is in agreement with the prediction in Ref. 23, in which the corrections due to the relativistic effects were also included. Recently Altomari\(^{24}\) and Ali, Ohl and Mannel\(^{25}\) predicted the branching fractions of \(B\)-decays into higher resonances without relativistic corrections. The results by the former show that there exists no enhanced channel and that the fractions of all the two body decay modes lie in the region of 3.5%–7.3%. In the results by the latter, where the

### Table I

Predicted branching ratios of radiative decays into higher resonances of the \(K\) meson. Predicted values are shown for each set of values of the parameters \((m_H, \cot \beta)\) at the typical \(t\)-quark mass \(m_t = 140\text{ GeV}\).

<table>
<thead>
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<th>Decays</th>
<th>(\cot \beta)</th>
<th>(m_H = 200\text{ GeV})</th>
<th>(m_H = 400\text{ GeV})</th>
<th>(m_H = 600\text{ GeV})</th>
<th>SM</th>
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<td>(B \to K^*(892)\gamma)</td>
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<td>4.0 \times 10^{-5}</td>
<td>3.5 \times 10^{-5}</td>
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<td></td>
<td>2</td>
<td>6.9 \times 10^{-5}</td>
<td>4.5 \times 10^{-5}</td>
<td>3.8 \times 10^{-5}</td>
<td>4.2 \times 10^{-5}</td>
</tr>
<tr>
<td>(B \to K_l(1270)\gamma)</td>
<td>0</td>
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<td>2.4 \times 10^{-4}</td>
<td>2.1 \times 10^{-4}</td>
<td>1.8 \times 10^{-4}</td>
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<td>2.7 \times 10^{-4}</td>
<td>2.3 \times 10^{-4}</td>
<td>2.5 \times 10^{-4}</td>
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<tr>
<td>(B \to K_l(1400)\gamma)</td>
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<td>4.4 \times 10^{-4}</td>
<td>3.4 \times 10^{-4}</td>
<td>3.0 \times 10^{-4}</td>
<td>3.0 \times 10^{-4}</td>
</tr>
<tr>
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<td>3.5 \times 10^{-4}</td>
<td>3.0 \times 10^{-4}</td>
<td>3.2 \times 10^{-4}</td>
</tr>
<tr>
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<td>2</td>
<td>5.8 \times 10^{-4}</td>
<td>3.8 \times 10^{-4}</td>
<td>3.4 \times 10^{-4}</td>
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<tr>
<td>(B \to K^*_s(1430)\gamma)</td>
<td>0</td>
<td>2.7 \times 10^{-4}</td>
<td>2.0 \times 10^{-4}</td>
<td>1.8 \times 10^{-4}</td>
<td>1.5 \times 10^{-4}</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>2.9 \times 10^{-4}</td>
<td>2.1 \times 10^{-4}</td>
<td>1.8 \times 10^{-4}</td>
<td>1.5 \times 10^{-4}</td>
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<tr>
<td></td>
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<td>3.5 \times 10^{-4}</td>
<td>2.3 \times 10^{-4}</td>
<td>1.9 \times 10^{-4}</td>
<td>1.9 \times 10^{-4}</td>
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<tr>
<td>(B \to K^*(1400)\gamma)</td>
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<td>8.8 \times 10^{-5}</td>
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heavy quark effective theory\cite{26} was applied to the higher resonances of the $K$ meson, the $B\to K^\ast(1430)\gamma$ decay is the main mode around $17\% \sim 37\%$ fraction. However, we believe that the spin structures of the higher resonances of the $K$ meson are too simplified in the heavy quark effective theory. The $1/m_s$ corrections are expected to play a significant role in these decays.\cite{27}

On the contrary, it is to be remarked from Eq. (4·13) that, in our result, the decay into the radial excitation state $K^\ast(1410)$ is strongly enhanced. In comparison with Ref. 25) we conclude that the observation of the main decay mode in future experiment at $B$-factory is very important to test the models.

In Table I, we show the branching ratios for the decays to the $K$ meson resonances, which depend on the QCD corrections and the charged Higgs contribution. As seen in Table I, the charged Higgs effect increases the branching ratios by about factor 2 compared with SM at the typical values as $m_H=200$ GeV and $\cot\beta=1$. If the Higgs mass is larger than 500 GeV it is very difficult to find the effects of the Higgs contribution through the weak radiative $B$-decays.

§ 5. Summary and discussion

The radiative rare decay of $B$-meson is the very important process to obtain the information on new physics beyond SM. Especially the charged Higgs effect is sizable for the appropriate parameter region for $m_H$ and $\cot\beta$ as discussed in this paper. The experimental upper limit is just close to the prediction given by the effect of charged Higgs scalar for the typical values of the parameters $m_H \approx 200$ GeV and $\cot\beta \approx 2$. So the experiments on the rate decay modes at the $B$-factory seem to be very important to get the signal of new physics, especially the charged Higgs scalar based on THDM in the low energy region.

The analyses of the exclusive $B$-meson decays suggest that the decay channel $B\to K^\ast(1410) + \gamma$ has the largest fraction in the exclusive weak radiative decays. This result is remarkably different from the recent predictions given in Ref. 25). The calculation of the exclusive decay fraction depends on the structure of form factors. So, it is necessary to check those form factors in other processes such as the non-leptonic decays.

Appendix A

For the weak radiative $B$-meson decay, the following four-quark operators and the magnetic-transition-type operators are relevant under the Hamiltonian

\begin{align}
H_{\text{eff}} &= \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts} \sum C_i(m_b) O_i(m_b), \\
O_1 &= (\bar{c}_{Ld}\gamma^\mu b_{Lb})(\bar{s}_{Lb}\gamma^\mu c_{Lb}), \\
O_2 &= (\bar{c}_{Ld}\gamma^\mu b_{Lb})(\bar{s}_{Lb}\gamma^\mu c_{Lb}), \\
O_3 &= (\bar{s}_{Lb}\gamma^\mu b_{Lb})(\sum_{\text{quarks}} \bar{q}_{Lb}\gamma^\mu q_{Lb}).
\end{align}
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\[ O_4 = (s_L \gamma^\mu b_{L\mu} \left( \sum_{\text{squarks}} q_{L\mu} q_{L\mu} \right), \]
\[ O_5 = (s_L \gamma^\mu b_{L\mu} \left( \sum_{\text{quarks}} q_{R\mu} \gamma^\mu q_{R\mu} \right), \]
\[ O_6 = (s_L \gamma^\mu b_{L\mu} \left( \sum_{\text{quarks}} \bar{q}_{R\mu} \gamma^\mu q_{R\mu} \right), \]
\[ O_7 = \frac{\alpha}{8\pi^2} m_b s_L \sigma^{\mu\nu} b_{R\mu} q_{L\nu}, \]
\[ O_8 = \frac{g_c}{8\pi^2} m_b s_L \sigma^{\mu\nu} T^a_{L\mu} b_{R\nu} q_{L\mu} \epsilon^a. \]

By using the renormalization group equations, the coefficients \( C_i(m_b) \) evolve according to

\[ \mu \frac{dC_i(\mu)}{d\mu} = -\sum_{i=1}^{a} (\gamma^\top)_{ij} C_j(\mu) = 0 \]  

(A.3)

with

\[ \mu \frac{d\gamma_c}{d\mu} = \beta(\gamma_c) = -\left(11 - \frac{2}{3} f\right) \frac{\gamma_c^3}{16\pi^2}. \]

(A.4)

Then we obtain the formula

\[ \frac{dC_i(\alpha_c)}{d\alpha_c} = -\frac{6}{23} \gamma_c \sum (\gamma^\top)_{ij} C_j(\gamma_c), \]

(A.5)

where \( \gamma = (g_c^2/8\pi^2)\gamma' \) and the matrix \( \gamma' \) is

\[
\begin{pmatrix}
-\frac{3}{N} & 3 & 0 & 0 & 0 & 0 & 0 & 3 T_F \\
3 & -\frac{3}{N} & -\frac{1}{3N} & 1 & 1 & \frac{1}{3} & \gamma^2 & \gamma^8 \\
0 & 0 & -\frac{11}{3N} & \frac{11}{3} & \frac{2}{3N} & \gamma^2 & \gamma^8 \\
0 & 0 & 3 & -\frac{f}{3N} & f & \gamma^2 & \gamma^8 \\
0 & 0 & 0 & 0 & \frac{3}{N} & -3 & -6 Q_d C_F & \gamma^8 \\
0 & 0 & -\frac{f}{3N} & \frac{f}{3} & -\frac{f}{3N} & -6 Q_d C_F & \gamma^8 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{16}{3} \\
0 & 0 & 0 & 0 & 0 & 0 & \gamma^2 & \gamma^8 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{16}{9} & \frac{14}{3}
\end{pmatrix}
\]

(A.6)

with
\[ \gamma_{27} = \left( 3Q_u - \frac{4}{9}Q_d \right) c_F, \]
\[ \gamma_{29} = \frac{23}{9} c_F - \frac{2}{3}, \]
\[ \gamma_{38} = 3fT_F + \frac{46}{9} c_F - \frac{4N}{3}, \]
\[ \gamma_{47} = \left( 6Q_u + 9Q_d - \frac{4}{9} fQ_d \right) c_F, \]
\[ \gamma_{48} = 6T_F + \frac{23}{9} f c_F - \frac{2}{3} fN, \]
\[ \gamma_{58} = -3fT_F - 6c_F + 6N, \]
\[ c_{67} = \left( 10NQ_d - 9Q_d - \frac{4}{9} fQ_d - 6Q_u \right) c_F, \]
\[ \gamma_{58} = -20T_F - \frac{31}{9} f c_F + \frac{5}{6} fN. \]

(A·7)

Here \( N = 3, c_F = 4/3 \) and \( T_F = 1/2 \) are the SU(3) colour factors and \( f = 5 \) is the flavour number.

The coefficients \( C_1 \) and \( C_2 \) are

\[
C_1(m_b) = \frac{1}{2} \left( \eta^{-(6/23)} - \eta^{(12/23)} \right) = -0.229, \\
C_2(m_b) = \frac{1}{2} \left( \eta^{-(6/23)} + \eta^{(12/23)} \right) = 1.097 ,
\]

(A·8)

where \( \eta = a_s(m_b)/a_s(m_w) = 1.718 \). \( C_i(i=3, 4, 5, 6, 7, 8) \) are numerically given as

\[
\begin{pmatrix}
C_3(m_b) \\
C_4(m_b) \\
C_5(m_b) \\
C_6(m_b)
\end{pmatrix} =
\begin{pmatrix}
-0.416 & -0.768 & -0.170 & -0.242 \\
-0.804 & 0.665 & -0.083 & 0.337 \\
0.324 & 0.064 & -0.956 & -0.053 \\
-0.274 & 0.131 & 0.351 & -0.994
\end{pmatrix}
\begin{pmatrix}
a \\
b \\
c \\
d
\end{pmatrix} =
\begin{pmatrix}
0.010 \\
-0.024 \\
0.007 \\
-0.029
\end{pmatrix},
\]

(A·9)

where

\[
a = -0.122(\eta^{-(3/23) \times (3,133)} - C_2(m_b)) + 0.089C_1(m_b) = 0.016, \\
b = 0.183(\eta^{-(3/23) \times (3,343)} - C_2(m_b)) + 0.244C_1(m_b) = -0.027, \\
c = -0.032(\eta^{-(3/23) \times (1,117)} - C_2(m_b)) + 0.045C_1(m_b) = -0.005, \\
d = 0.046(\eta^{-(3/23) \times (6,865)} - C_2(m_b)) + 0.024C_1(m_b) = 0.019
\]

(A·10)

and, finally

\[
C_7(m_b) = \eta^{-(3/23) \times 16/3}(C_7(m_w) - 2.667C_6(m_w) - 0.687) \\
+ 2.67 \eta^{-(3/23) \times 14/3}(C_8(m_w) + 0.582)
\]
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\[ -0.571 C_1(m_b) - 0.865 C_2(m_b) - 1.797 C_3(m_b) - 2.252 C_4(m_b) \]
\[ -2.302 C_5(m_b) + 1.282 C_6(m_b) \]
\[ = 0.686 C_7(m_w) + 0.088 C_8(m_w) - 0.191, \]
\[ C_8(m_b) = \eta^{-3(3/2)/14} (C_8(m_w) + 0.582) - 0.573 C_1(m_b) - 0.582 C_2(m_b) \]
\[ - 4.028 C_3(m_b) - 4.055 C_4(m_b) - 1.948 C_5(m_b) + 1.548 C_6(m_b) \]
\[ = 0.719 C_7(m_w) - 0.091. \]  

In the above numerical estimation, we used

\[ a_\epsilon(m_b) = \frac{12\pi}{(33-2f) \ln \frac{\mu^2}{\Lambda^2}} \left[ 1 - \frac{6(153 - 19f)}{(33-2f)^2} \ln \frac{\ln \frac{\mu^2}{\Lambda^2}}{\ln \frac{\mu^2}{\Lambda^2}} \right], \]

\[ \Lambda = 175 \text{ MeV}, \]
\[ m_w = 80.2 \text{ GeV}, \]
\[ m_b = 5.0 \text{ GeV}. \]  

Appendix B

The definition of variables in the off diagonal component \( M_{12} \) in Eq. (3·5):

\[ M_{12} = \frac{G_F^2}{12\pi^2} f_p^2 m_B m_w^2 [ (\bar{M}_{12})_{ww} + (\bar{M}_{12})_{HH} + (\bar{M}_{12})_{Hw} ] \]  

is as follows:

\[ (\bar{M}_{12})_{ww} = \lambda_c^2 \eta_1 S_{cc} + \lambda_t^2 \eta_2 S_{tt} + 2\lambda_c \lambda_t \eta_3 S_{ct}, \]
\[ (\bar{M}_{12})_{HH} = \lambda_c^2 T_{cc} + \lambda_t^2 T_{tt} + 2\lambda_c \lambda_t T_{ct}, \]
\[ (\bar{M}_{12})_{Hw} = \lambda_c^2 U_{cc}(z) + \lambda_t^2 U_{tt}(z) + 2\lambda_c \lambda_t U_{ct}(z) \]  

with

\[ \lambda_i = \begin{cases} V_{id} \sqrt{V_{is}} & \text{for } K^0, \\ V_{id} \sqrt{V_{ib}} & \text{for } B_d^0. \end{cases} \]  

Here,

\[ S_{ij} = x_i x_j \times \left[ \frac{1}{4} + \frac{3}{2(1-x_j)} - \frac{3}{4(1-x_j)^2} \ln x_j \right. \]
\[ \left. + (x_j \leftrightarrow x_i) - \frac{3}{4(1-x_j)(1-x_i)} \right], \]
\[ T_{ij} = \frac{z}{4} \cot^2 \beta L_2(y_i, y_j, 1), \]
\[ U_{ij}(z) = 2\cot^2 \beta \left( \frac{1}{4} L_2(x_i, x_j, z) - L_1(x_i, x_j, z) \right) \]  

The functions $L_1$ and $L_2$ are given by

$$L_1(x_i, x_j, z) = x_i x_j [F(x_i; x_j, z) + F(x_j; z, x_i) + F(z; x_i, x_j)],$$

$$L_2(x_i, x_j, z) = x_i x_j [x_i F(x_i; x_j, z) + x_j F(x_j; z, x_i) + z F(z; x_i, x_j)]$$

with

$$F(x_i; x_j, z) = \frac{x_i \ln x_i}{(x_i - 1)(x_i - x_j)(x_i - z)}.$$  

The $B_F$ parameter in Eq. (B·1) is defined by\(^\text{20}\)

$$\langle P^0 [(\bar{d}q)_\nu A(\bar{d}q)_{\nu A}] P^0 \rangle = B_F \frac{8}{3} f^2 m^2,$$

where $q = s$ and $b$ for the $K$ meson and the $B$ meson, respectively, and the decay constant $f_K$ is taken to be 161 MeV. The factors $\eta_i$ in Eq. (B·2) denote the QCD corrections to the box diagram. We take $\eta_1 = 0.85$, $\eta_2 = 0.61$ and $\eta_3 = 0.37$ for the $K$ meson system, and $\eta_2 = 0.8$ for the $B$ meson system. It is noted that $\eta_1$ and $\eta_3$ are not needed for the $B$ meson system because $\lambda^2 S_{cc}$ and $\lambda^2 S_{ct}$ are negligible compared to $\lambda^2 S_{ht}$.\(^\text{28}\)

### Appendix C

We show the form factors of the $B(b \bar{d}) \rightarrow X(s \bar{d})$ transition in the ISGW model.\(^\text{21}\)

Throughout the following, we employ the definitions:

$$F_n = \left[ \frac{\bar{m}_X}{\bar{m}_B} \right]^{1/2} \left[ \frac{\beta_B \beta_X}{\beta_{B_X}} \right] \exp \left[ -\frac{m^2}{4 \bar{m}_B \bar{m}_X} \right] \exp \left[ -\frac{t_m - t}{\kappa^2 \beta_{B_X}} \right],$$

where

$$t = (p_B - p_X)^2, \quad t_m = (m_B - m_X)^2,$$

and

$$\mu = \left[ \frac{1}{m_s} + \frac{1}{m_b} \right]^{-1},$$

$$\beta_{B_X}^2 = \frac{1}{2} (\beta_B^2 + \beta_X^2),$$

$$\bar{m}_B = m_b + m_d, \quad \bar{m}_X = m_s + m_d,$$

$\kappa = 0.7$ being the relativistic compensation factor.

The form factors are given by
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\[ f = 2m_b F_3, \quad g = \frac{1}{2} F_3 \left[ \frac{1}{m_s} - \frac{1}{2\mu-} \frac{m_d}{m_x} \frac{\beta_B^2}{\beta_{bx}} \right] \] (C.6)

for the \(^{1S}_1\) state,

\[ h = F_3 \frac{m_d}{2\sqrt{2}m_b \beta_B} \left[ \frac{1}{m_s} - \frac{m_d}{2m_x \mu-} \frac{\beta_B^2}{\beta_{bx}} \right], \quad k = \sqrt{2} F_3 \frac{m_d}{\beta_B} \] (C.7)

for the \(^{1P}_2\) state,

\[ l = -F_3 \frac{m_d}{m_b \beta_B} \left[ \frac{1}{\mu-} + \frac{m_d}{2m_b} \frac{t_m - t}{\kappa^2 \beta_B^2} \left[ \frac{1}{m_s} - \frac{m_d}{2m_x \mu-} \frac{\beta_B^2}{\beta_{bx}} \right] \right] \] (C.8)

\[ q = \frac{1}{2} F_3 \frac{m_d}{m_x \beta_B} \] (C.9)

for the \(^{1P}_1\) state,

\[ r = F_3 \frac{m \beta_B}{\sqrt{2} \mu+}, \quad v = F_3 \frac{m \beta_B}{4\sqrt{2} m_b m_s m_X} \] (C.10)

for the \(^{1P}_0\) state, and

\[ f' = \sqrt{6} F_3 m_B \left[ \frac{\beta_B^2 - \beta_x^2}{\beta_B^2 + \beta_x^2} + \frac{m_d^2}{6m_X m_B} \frac{\beta_x^2}{\beta_{bx}^2} \frac{t_m - t}{\beta_{bx}} \right] \] (C.11)

\[ g' = \sqrt{\frac{3}{8}} F_3 \left[ \left[ \frac{\beta_B^2 - \beta_x^2}{\beta_B^2 + \beta_x^2} + \frac{m_d^2}{6m_X m_B} \frac{\beta_x^2}{\beta_{bx}^2} \frac{t_m - t}{\beta_{bx}^2} \right] \right. \]

\[ \times \left[ \frac{1}{m_s} - \frac{m_d}{2m_X \mu-} \frac{\beta_B^2}{\beta_{bx}^2} \right] + m_d + \frac{m_d}{3\mu-} \frac{m_X}{m B} \frac{\beta_B^2}{\beta_{bx}^2} \] (C.12)

for the \(^{2S}_1\) state, respectively. The parameters which appeared in these form factors are fixed as \(m_s = 0.33\) GeV, \(m_b = 0.55\) GeV, \(m_B = 5.12\) GeV, \(\beta_B = 0.41\) GeV and \(\beta_x = 0.34\) for the \(S\)-state and 0.30 for the \(P\)-state.

References

1) For example, see the lectures given by G. Altarelli at the LVth Les Houches Summer School, France 30 June-26 July, 1991 (CERN-TH. 6305/91).
5) For the recent review on \(B\)-factory see Progress Report on Physics and Reactor at KEK Asymmetric \(B\) Factory, ed. B Physics Task Force (April, 1992, KEK).