Dynamics of a Scalar Field along a Flat Direction in de Sitter Space

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We compute the one-loop effective action of a scalar field $\phi$ along a flat direction in de Sitter space using the derivative expansion method. It is found that this scalar field rolls down the potential away from the origin if the initial value of a scalar field $\phi$ is much larger than the Hubble constant. We also see that this dynamics supports the Affleck-Dine mechanism for baryogenesis.

§ 1. Introduction

It is known that a scalar potential of a global supersymmetric model often has a special direction called the flat direction, along which the value of the scalar potential remains unchanged. The potential energy of a supersymmetric model is given by a linear combination of $F_i^*F_i$ and $D^aD^a$, where $F_i$ and $D^a$ are auxiliary fields in scalar multiplets and vector multiplets, respectively. Therefore, the flat direction is characterized by conditions $F_i(\phi)=0$ and $D^a(\phi)=0$. If these conditions have non-trivial solutions in terms of the scalar fields $\phi$, there exists a flat direction. An example in supersymmetric $SU(5)$ grand unified theory (GUT) can easily be found. The condition of the flat direction imply the supersymmetry is not spontaneously broken, while the presence of the vacuum expectation value along the direction in general breaks the gauge and global symmetries. It is also possible to show that, in the supersymmetric models, the flat direction does not receive any radiative correction as a result of the non-renormalization theorem. The unbroken supersymmetry and the absence of the radiative correction are interesting characteristics of the flat direction. With a supersymmetry breaking mass $m$, on the other hand, the direction receives a logarithmic radiative correction proportional to the breaking mass squared $m^2$.

In this paper we evaluate the one-loop effective action along the flat direction in the de Sitter space using the derivative expansion method. Our original interest is the effective potential along the flat direction in the inflationary expanding universe. Our analysis is relevant for the baryogenesis scenario by the Affleck-Dine mechanism, in which it is supposed that scalar fields along the flat direction initially have a large expectation value of the order of the GUT scale. Then after inflation and when the Hubble parameter becomes of order of supersymmetric breaking mass scale $m$ (that is of order of the weak scale $m_w$), the scalar fields begin to fall down along the flat direction. The motion of scalar fields (say the squark fields) generates a condensation of the baryon number density and decays into light particles.

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* We call these conditions $F$-term flat and $D$-term flat, respectively.
** More precisely we will evaluate the effective action on Euclidean section of the de Sitter space, namely on the four-sphere $S^4$. 

In flat space, as mentioned above, the flat direction receives rather small radiative corrections. However, it is not clear if this is held even in curved spaces. In particular the typical mass (or curvature) scale during inflation, namely the Hubble parameter $H$, may be of order of the GUT scale or the intermediate scale $\sim \sqrt{m_{\text{soft}} \cdot m_{\text{Pl}}}$.

It may then be natural to expect the flat direction receives a correction of order of $g^2 H^2 \phi^2$ where $g$ is typical coupling constant. If this is the case, the scalar fields along the flat direction may be damped away during inflation and cannot have a sufficiently large initial value for a successful Affleck-Dine scenario.

To analyze the above problem, however, we have to specify how the global supersymmetric model is coupled to gravity. Though the supergravity model might be a natural framework, this theory is not renormalizable. Since we do not know any renormalizable supergravity model, we will consider renormalizable models that reduce to global supersymmetric models (with soft supersymmetric breaking masses) in the flat space limit. Here, the gravitational interactions break the supersymmetry explicitly, but it is still consistent as long as we do not include the radiative corrections due to the graviton.

Note that the renormalizability in curved space allows a curvature coupling term $\xi R \phi^2$ for the flat direction $\phi$ if it does not contradict with the gauge and the global symmetries. Since the scalar curvature $R$ is given by $R=12H^2$ in de Sitter space, if $\xi$ is of order of unity, the direction is not flat even at the tree level. In this case, the Affleck-Dine mechanism does not work.

In the following section, we will review that one-loop radiative corrections to the potential induce a term $\sim -3g^2 H^2 \phi^3 \ln \phi^2/(16\pi^2)$ for a large value of $\phi$. If the (renormalized) curvature coupling is nearly zero, $\xi \approx 0$, and the soft breaking mass is sufficiently small as $m^2 < g^2 H^2/\pi^2$, this correction may drastically modify the tree potential and may even give an unbounded potential. We will argue that this asymptotic behavior is unavoidable irrespective of the matter contents and the details of the model if the flat direction couples to a scalar multiplet. A realistic supersymmetric model should satisfy this requirement because a scalar field along the flat direction, say the squark field, must have a Yukawa coupling in the superpotential. So, hereafter we refer to flat direction at tree level as fiat direction. Before we conclude that scalar fields $\phi$ along the fiat direction keep rolling down for a large value of $\phi$, we should take into account the derivative term of the scalar field along fiat direction to understand the dynamics of this field. We will calculate the quantum correction to the kinetic term assuming the slow rolling of the scalar field along fiat direction. But we show this correction is small compared to the tree contribution (and the one-loop correction to the effective potential) contrary to the case of potential, so we conclude the scalar field along fiat direction rolls down along the unbounded potential in the region where the loop expansion is reliable.

In §2, we shall give an explicit evaluation of one-loop effective potential in Wess-Zumino model with the superpotential $W=g\phi_1 \phi_2 \phi_3/2$. This model has an $F$-term flat direction and gives an example for radiative corrections due to a scalar multiplet. We will see that the radiative correction induces the potential term $\sim -3g^2 H^2 \phi^2/(16\pi^2)$ as mentioned above. The general case including $D$-term flat direction is also examined.
In §3, we shall give an explicit evaluation of one-loop correction to the kinetic term of the scalar field along flat direction in de Sitter space.

In conclusion and discussion, we argue the time evolution of the scalar field along flat direction in de Sitter space and suggest that the Affleck-Dine mechanism likely takes place.

In the remaining of this section, we summarize our framework for the evaluation of the one-loop effective potential in Euclidean de Sitter space \((S^4)\).

As is well known the one-loop effective potential is given by a path integral

\[
V_{\text{eff}}(\phi) = V(\phi) - \frac{1}{\Omega} \ln \left\{ \int \! d[B] \exp(-S_2[B]) \right\}
\]

\[
= V(\phi) - \frac{1}{\Omega} \ln(\text{Det} \, a^2 \mathcal{A})^{-1/2},
\]

where \(B\) represents a fluctuation around the vacuum expectation value \(\phi\) and \(S_2[B]\) is the Euclidean action of the fluctuation up to the quadratic order; the wave operator of \(B\) is denoted as \(\mathcal{A}\). We have defined the path integral measure so that it gives a dimensionless combination \(a^2 \mathcal{A}\) in the determinant where \(a\) is the radius of the Euclidean de Sitter space \((S^4)\). (The Hubble parameter \(H\) is given by \(H = 1/a\).) The power of determinant \(-1/2\) in Eq. (1) is for one bosonic degree of freedom and should be read as \(+1/2\) for one fermionic degree of freedom. \(\Omega\) is the four-volume of \(S^4\) and is given by \(\Omega = 8\pi^2 a^4/3\).

We regularize the determinant factor in Eq. (1) by using the proper time cutoff

\[
\ln \text{Det}(a^2 \mathcal{A}) = \text{Tr} \ln(a^2 \mathcal{A}) = \sum_{n=0}^{\infty} g_n \ln \lambda_n = -\lim_{\epsilon \to 0} \int_{\epsilon}^{\infty} t^{-1} Y(t) \, dt + \text{const},
\]

where \(g_n\) and \(\lambda_n\) are the multiplicity and the eigenvalue of the dimensionless wave operator \(a^2 \mathcal{A}\) on \(S^4\). The function \(Y(t)\) is defined by

\[
Y(t) = \sum_{n=0}^{\infty} g_n e^{-\lambda_n t}.
\]

The eigenvalue \(\lambda_n\) and the multiplicity \(g_n\) of wave operator \(a^2 \mathcal{A}\) are exactly known\(^5\) for typical tensorial fields due to the large symmetry of \(S^4\). One may thus directly evaluate the definition (2). Here, we instead utilize a relation

\[
-\lim_{\epsilon \to 0} \int_{\epsilon}^{\infty} t^{-1} Y(t) \, dt
\]

\[
= -\frac{1}{2} \text{Res} [\zeta(2)](aA)^4 - \text{Res} [\zeta(1)](aA)^2 + \zeta(0)[\gamma - \ln(aA)^2] - \zeta'(0),
\]

where the dimensionless cutoff \(\epsilon\) is defined as \(\epsilon = (aA)^{-2}\) (\(A\) corresponds to the momentum cutoff in the flat space). In the above expression, \(\gamma\) is the Euler constant and \(\zeta(s)\) is the generalized zeta function\(^6\) that is defined by

\[
\zeta(s) = \sum_{n=0}^{\infty} g_n \lambda_n^{-s}, \quad (\text{Re}(s) > 2)
\]

and the analytic continuation. Equation (4) is formally the same as the relation
Table I. Some values of the generalized zeta function.

<table>
<thead>
<tr>
<th>Real scalar $\phi$</th>
<th>$\text{Res}[\zeta(2)]$</th>
<th>$\text{Res}[\zeta(1)]$</th>
<th>$\zeta(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{6}$</td>
<td>$-\frac{1}{6}m^2a^2+\frac{1}{3}$</td>
<td>$\frac{1}{12}m^4a^4-\frac{1}{3}m^2a^2+\frac{29}{90}$</td>
<td></td>
</tr>
<tr>
<td>Majorana spinor $\phi$</td>
<td>$\frac{1}{3}$</td>
<td>$-\frac{1}{3}m^2a^2-\frac{1}{3}$</td>
<td>$\frac{1}{6}m^4a^4+\frac{1}{3}m^2a^2+\frac{11}{180}$</td>
</tr>
<tr>
<td>Transverse massive vector $A^\mu$</td>
<td>$\frac{1}{2}$</td>
<td>$-\frac{1}{2}m^2a^2+\frac{1}{2}$</td>
<td>$\frac{1}{4}m^4a^4-\frac{1}{2}m^2a^2-\frac{7}{60}$</td>
</tr>
<tr>
<td>Rarita-Schwinger $\phi^\mu$</td>
<td>$\frac{2}{3}$</td>
<td>$-\frac{3}{2}m^2a^2-\frac{3}{3}$</td>
<td>$\frac{1}{3}m^4a^4+\frac{8}{3}m^2a^2+\frac{401}{90}$</td>
</tr>
<tr>
<td>Symmetric transverse traceless tensor $\eta^\mu$</td>
<td>$\frac{5}{6}$</td>
<td>$-\frac{5}{6}m^2a^2-\frac{5}{3}$</td>
<td>$\frac{5}{12}m^4a^4+\frac{5}{3}m^2a^2-\frac{1}{18}$</td>
</tr>
</tbody>
</table>

which is introduced by Schwarz. The one-loop effective potential is then given by

$$V_{\text{eff}}(\phi) = V(\phi) + \frac{3}{16\pi^2} \left\{ -\frac{1}{2} \text{Res}[\zeta(2)] A^4 - \text{Res}[\zeta(1)] a^{-2} A^2 \right\}$$

$$+ \zeta(0) \left[ \gamma - \ln(aA)^2 \right] a^{-4} - \zeta'(0) a^{-4} \right\}.$$  \hspace{1cm} (6)

We will use the generalized zeta function on $S^4$ evaluated by Allen. It is convenient to translate his results in terms of the effective mass in the wave operator $\Delta = -\nabla_a \nabla^a + m^2$. For spinorial fields $\Delta = (\gamma_\mu \nabla^\mu - m)^4 (\gamma_\nu \nabla^\nu - m)$. Some relevant values are summarized in Table I. On the other hand, the derivative of zeta function at $s=0$, $\zeta'(0)$, is given by

$$\zeta'(0) = -\frac{1}{3} (2L+1) \int_0^{\psi(L)} \left[ x^3 + \left( L + \frac{1}{2} \right)^2 x \right] \left[ \phi \left( L + \frac{1}{2} + i\gamma \right) + \phi \left( L + \frac{1}{2} - i\gamma \right) \right] dx$$

$$+ c(L) + \frac{2}{3} (2L+1) \left[ \zeta(\phi(-3, L + \frac{3}{2}) - \left( L + \frac{1}{2} \right)^2 \zeta(1, L + \frac{3}{2}) \right],$$  \hspace{1cm} (7)

where constants $\gamma(L)$ and $c(L)$ are given in Table II. In the above expression, $\phi(s)$ is the digamma function and $\zeta(s, a)$ is the extended Riemann's zeta function. The integral in Eq. (7) cannot be done analytically but we can evaluate the asymptotic behavior at $m^2a^2 \to \infty$ by using the asymptotic form of the digamma function. We may also evaluate the integral Eq. (7) numerically. In Table III, the asymptotic form of $\zeta'(0)$ at $m^2a^2 \to \infty$ are presented.

| Table II. Some constants appearing in Eq. (7). |
|---|---|---|
| $L$ | $\gamma(L)$ | $c(L)$ |
| $\phi$ | $0 \sqrt{m^2a^2-\frac{9}{4}}$ | $\frac{1}{12}m^4a^4-\frac{7}{18}m^2a^2+\frac{29}{64}$ |
| $\phi$ | $\frac{1}{2} |ma|$ | $\frac{1}{6}m^4a^4+\frac{1}{18}m^2a^2$ |
| $A^\mu$ | $1 \sqrt{m^2a^2-\frac{13}{4}}$ | $\frac{1}{4}m^4a^4-\frac{7}{6}m^2a^2+\frac{221}{192}$ |
| $\phi^\mu$ | $\frac{3}{2} |ma|$ | $\frac{1}{3}m^4a^4+\frac{13}{9}m^2a^2$ |
| $\eta^\mu$ | $2 \sqrt{m^2a^2-\frac{17}{4}}$ | $\frac{5}{12}m^4a^4-\frac{5}{18}m^2a^2-\frac{3655}{576}$ |

§ 2. Wess-Zumino model

The simplest non-trivial example that has a $F$-term flat direction is given by the
Table III. The asymptotic forms of a derivative of the generalized zeta function at $s=0$.

<table>
<thead>
<tr>
<th></th>
<th>$\xi'(0)$ at $m^2a^2 \to \infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>$-\frac{1}{12} m^4 a^3 \left( \ln m^2 a^2 - \frac{3}{2} \right) + \frac{1}{3} m^2 a^2 \ln m^2 a^2 - 1 - \frac{29}{90} \ln m^2 a^2 + \cdots$</td>
</tr>
<tr>
<td>$\psi$</td>
<td>$-\frac{1}{6} m^4 a^3 \left( \ln m^2 a^2 - \frac{3}{2} \right) - \frac{1}{3} m^2 a^2 \ln m^2 a^2 - 1 - \frac{11}{180} \ln m^2 a^2 + \cdots$</td>
</tr>
<tr>
<td>$A^\mu$</td>
<td>$-\frac{1}{4} m^4 a^3 \left( \ln m^2 a^2 - \frac{3}{2} \right) + \frac{1}{2} m^2 a^2 \ln m^2 a^2 - 1 + \frac{7}{60} \ln m^2 a^2 + \cdots$</td>
</tr>
<tr>
<td>$\phi^\mu$</td>
<td>$-\frac{1}{3} m^4 a^3 \left( \ln m^2 a^2 - \frac{3}{2} \right) - \frac{8}{3} m^2 a^2 \ln m^2 a^2 - 1 - \frac{401}{90} \ln m^2 a^2 + \cdots$</td>
</tr>
<tr>
<td>$h^\mu$</td>
<td>$-\frac{5}{12} m^4 a^3 \left( \ln m^2 a^2 - \frac{3}{2} \right) - \frac{5}{3} m^2 a^2 \ln m^2 a^2 - 1 + \frac{1}{18} \ln m^2 a^2 + \cdots$</td>
</tr>
</tbody>
</table>

Wess-Zumino model with the superpotential $W=g \Phi_1 \Phi_2 \Phi_3/2$. We define the corresponding Euclidean action using the four component spinors $\bar{\phi}$

$$S=\int \left[ \left( \partial_{\mu} \phi_1 \right)^* \left( \partial^\mu \phi_1 \right) + V(\phi_2, \phi_2) + \frac{1}{2} \left( \bar{\phi}_1, \bar{\phi}_2 \right) \left[ \gamma_5 P^\mu - M(\phi_1, \phi_2) \right] \left( \phi_1 \right) \left( \phi_2 \right) \right] \sqrt{g} d^4 x, \quad (8)$$

where $P^\mu$ is covariant derivative and the potential term $V$ is defined as

$$V(\phi_1, \phi_2) = (m_1^2 + \xi_1 R) \phi_1^* \phi_1 + \frac{1}{4} g^2 (\phi_2^* \phi_2)^2$$

$$+ (m_2^2 + \xi_2 R) \phi_2^* \phi_2 + g^2 (\phi_1^* \phi_1)(\phi_2^* \phi_2), \quad (9)$$

and the mass matrix $M(\phi_1, \phi_2)$ is given by

$$M(\phi_1, \phi_2) = \begin{pmatrix} 0 & \text{Re}(g \phi_2) - \text{Im}(g \phi_2) \gamma^5 \\ \text{Re}(g \phi_2) - \text{Im}(g \phi_2) \gamma^5 & \text{Re}(g \phi_1) - \text{Im}(g \phi_1) \gamma^5 \end{pmatrix}. \quad (10)$$

$R$ is the scalar curvature and $R=12a^{-2}$ on $S^4$. In the scalar potential (8), we have introduced the supersymmetry soft breaking mass terms $m_i^2 \phi_i^* \phi_i$ and the non-minimal curvature couplings $\xi_i R \phi_i^* \phi_i$, in addition to the minimal extension of the Wess-Zumino model in curved space. Since the non-minimal coupling receives the renormalization as will be seen below, the bare term $\xi_i R \phi_i^* \phi_i$ is necessary for this model to be renormalizable. On the other hand, in de Sitter space it is well known that a minimal ($\xi=0$) massless ($m=0$) scalar field is pathological in the sense that there exists no de Sitter invariant Fock vacuum. It may therefore be necessary to keep either of the soft breaking mass or the (renormalized) non-minimal coupling to be non-zero.

From the tree potential (9), we see $\phi_2=0$ is actually a flat direction in this model; namely the potential energy remains flat for any values of $\phi_1$ except for the supersymmetric breaking mass term and the non-minimal coupling term.

Now let us consider a real vacuum expectation value along the flat direction and the one-loop effective potential. Since $\phi_2=0$ is the flat direction, we decompose the scalar fields as

*) The present Euclidean convention is the same as in Ref. 3).
\[ \phi_1 = \phi + \varphi_1 + i \varphi_2, \quad \phi_2 = \varphi_3 + i \varphi_4, \]  

(11)

where all the fields are real and \( \phi \) is the classical field along the flat direction. Then the quadratic action of the fluctuations around the classical field \( \phi \) reads

\[
S_2 = \int \left( \sum_{i=1}^{2} \varphi_i (-\Box + m_i^2 + \xi_i R) \varphi_i + \sum_{j=3}^{4} \varphi_i (-\Box + m_j^2 + |g|^2 \phi^2 + \xi_j R) \varphi_i \right) + \frac{1}{2} \bar{\varphi}_1 \gamma_{\mu} \nabla^\mu \varphi_1 + \frac{1}{2} \bar{\varphi}_2 (\gamma_{\mu} \nabla^\mu - [\text{Re}(g\phi) - \text{Im}(g\phi) \gamma^5]) \varphi_2 \sqrt{g} \, d^4 x, \tag{12}
\]

and we can read off the effective mass of the fluctuations that depends on the expectation value of \( \phi \). The result is summarized in Table IV. For the spinorial fields \( \varphi_1 \) and \( \varphi_2 \), the corresponding second order wave operator is given by \( \Delta = (\gamma_{\mu} \nabla^\mu - m)^2 (\gamma_{\mu} \nabla^\mu - m) \). From the mass generating pattern in Table IV and general rule in Tables I and III, we can obtain the one-loop effective potential \( V_{\text{eff}}(\phi) \) in Eq. (4) in the flat space limit \( a \to \infty \),

\[
V_{\text{eff}}(\phi) = (m_1^2 + 12 \xi_1 a^{-2}) \phi^2 + \frac{3}{16} \left[ \gamma - \ln(a A)^2 \right] \left[ \frac{1}{3} m_2^2 + (4 \xi_2 - 1) a^{-2} \right] g^4 \phi^2
\]

\[
+ \frac{1}{6} (g^2 \phi^4 + m_3^2) \left( \ln [(g^2 \phi^2 + m_3^2) a^2] - \frac{3}{2} \right) - \frac{1}{6} g^4 \phi^2 \left[ \ln (g^2 \phi^2 a^2) - \frac{3}{2} \right]
\]

\[
- \frac{2}{3} (1 - 6 \xi_3) a^{-2} (g^2 \phi^2 + m_2^2) \left( \ln [(g^2 \phi^2 + m_2^2) a^2] - 1 \right)
\]

\[
- \frac{1}{3} a^{-2} g^2 \phi^3 \left[ \ln (g^2 \phi^2 a^2) - 1 \right] + O(a^{-4} \ln a) \right). \tag{13}
\]

(We have omitted \( \phi \)-independent terms.) We may rewrite the expression in terms of the renormalized parameters. Note that the tree potential along the flat direction only has the mass and the non-minimal coupling terms. Therefore, we may set the renormalization condition as*)

\[ \left. \frac{\partial^2 V_{\text{eff}}}{\partial \phi^2} \right|_{\phi = M^2} \to 2 m_1^2 + 24 \xi_1 a^{-2}, \tag{14} \]

when \( a \to \infty \), where \( M \) is the renormalization point (we cannot take \( M = 0 \) because of

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* More appropriately, we should specify the renormalized parameters as \( \frac{\partial^2 V_{\text{eff}}}{\partial \phi^2} \to 2 m_1^2 + 24 \xi_1 a^{-1} \) by using the renormalized field, \( \phi_s^2 = Z^{-1} \phi^2 \). The barred parameters and the unbarred ones in the text are related as \( m_{1s} = Z m_1 \) and \( \xi_{1s} = Z \xi_1 \). Nevertheless, in the one-loop approximation, the effective potential in terms of the barred parameters and the renormalized field \( \phi_s \), has precisely the same form as that in terms of the unbarred renormalized parameters defined by (14) and the bare field \( \phi \); namely Eqs. (17) and (18). Therefore we can regard the effective potential in Eqs. (17) and (18) are written in terms of the physically observable quantities.
the infrared divergence in the fermionic sector). Then the relations between the renormalized parameters and the bare ones become

$$m_{ir}^2 = m_i^2 + \frac{3g^2}{16\pi^2} \left( \frac{1}{3} [\gamma - \ln(aA)] m_2^2 \right)$$

$$+ \left( g^2 M^2 + \frac{1}{3} m_2^2 \right) \left[ \ln \left( (g^2 M^2 + m_2^2) a^2 \right) - 1 \right] - g^2 M^2 \left[ \ln \left( g^2 M^2 a^2 \right) - 1 \right]$$

(15)

and

$$\xi_{ir} = \xi_1 + \frac{g^2}{64\pi^2} \left[ \left[ \gamma - \ln(aA) \right] (4\xi_2 - 1) \right] - \frac{2}{3} \left( 1 - 6\xi_2 \right) \left[ \ln \left( (g^2 M^2 + m_2^2) a^2 \right) + 2 \right]$$

$$- \frac{1}{3} \left[ \ln \left( g^2 M^2 a^2 \right) + 2 \right] + \frac{4}{3} \left( 1 - 6\xi_2 \right) \frac{m_2^2}{g^2 M^2 + m_2^2}.$$  

(16)

Note that the right-hand side of the above expressions is independent of $a$. Then the one-loop effective potential in terms of the renormalized parameters reads (we have omitted the subscript $R$)

$$V_{\text{eff}}(\phi) = (m_i^2 + 12\xi_1 a^{-2}) \phi^2 - \frac{3}{16\pi^2} \zeta'(0) a^{-4} + \bar{V}_{\text{counter}}(\phi),$$

(17)

where a finite part of the counter term $\bar{V}_{\text{counter}}(\phi)$ is given by

$$\bar{V}_{\text{counter}}(\phi) = (m_i^2 - m_{ir}^2) \phi^2 + 12(\xi_1 - \xi_{ir}) a^{-2} \phi^2 + \frac{3}{16\pi^2} \zeta(0) \left[ \gamma - \ln(aA) \right] a^{-4}$$

$$- g^2 M^2 \phi^2 \left[ \ln \left( g^2 M^2 a^2 \right) - 1 \right] - \frac{2}{3} \left( 1 - 6\xi_2 \right) a^{-2} \phi^2 \left[ \ln \left( (g^2 M^2 + m_2^2) a^2 \right) + 2 \right]$$

$$- \frac{1}{3} a^{-2} \phi^2 \left[ \ln \left( g^2 M^2 a^2 \right) + 2 \right] + \frac{4}{3} \left( 1 - 6\xi_2 \right) a^{-2} \frac{m_2^2}{g^2 M^2 + m_2^2} \phi^2.$$  

(18)

The flat space limit $a \to \infty$ of Eq. (17) gives

$$V_{\text{eff}}(\phi) = (m_i^2 + 12\xi_1 a^{-2}) \phi^2 + \frac{3}{16\pi^2} \left( \frac{1}{6} (g^2 \phi^2 + m_2^2) \right) \left[ \ln \left( \frac{g^2 \phi^2 + m_2^2}{g^2 M^2 + m_2^2} \right) - \frac{1}{2} \right]$$

$$- \frac{1}{6} g^4 \phi^4 \left[ \ln \left( \frac{g^2 \phi^2 + m_2^2}{g^2 M^2 + m_2^2} \right) - \frac{1}{2} \right] + g^4 M^2 \phi^2 \ln \left( \frac{g^2 M^2}{g^2 M^2 + m_2^2} \right)$$

$$- \frac{2}{3} \left( 1 - 6\xi_2 \right) a^{-2} \left( g^2 \phi^2 + m_2^2 \right) \ln \left( \frac{g^2 \phi^2 + m_2^2}{g^2 M^2 + m_2^2} \right) - \frac{1}{3} a^{-2} g^2 \phi^2 \ln \left( \frac{\phi^2}{M^2} \right)$$

$$+ 3(1 - 4\xi_2) a^{-2} g^2 \phi^2 - \frac{4}{3} \left( 1 - 6\xi_2 \right) a^{-2} \frac{g^2 m_2^2}{g^2 M^2 + m_2^2} \phi^2 + O(a^{-3}).$$

(19)

All the parameters in the above expression are the renormalized ones, $m_{ir}$ and $\xi_{ir}$. It is easy to see that all the corrections up to $O(a^{-2} \phi^2)$ vanishes for $m_{2R} = 0$ and $\xi_{2R} = 1/4$.

Let us now consider the behavior of the effective potential at a large vacuum
expectation value. For a large value of $\phi$, $\phi^2 \gg M^2 + m_2^2/\phi^2$, the effective potential in Eq. (19) reduces to

$$V_{\text{eff}}(\phi) = (m_1^2 + 12\xi_1 a^{-2})\phi^2 + \frac{g^2}{16\pi^2}(m_2^2 + 12\xi_2 a^{-2} - 3a^{-2})\phi^3 \ln\left(\frac{\phi^2}{M^2}\right) + \cdots,$$

where we have only retained the tree potential and the leading contribution of the one-loop corrections. For general cases, we cannot draw any conclusion from the asymptotic form Eq. (19). However if we assume that all the soft breaking masses and all the non-minimal couplings are of the same order of magnitude, respectively, $m_1 \sim m_2$ and $\xi_1 \sim \xi_2$, the expression is further simplified. First we note the coupling constant $g$ should be small $g \ll 1$ for the perturbation to be reliable. Moreover the one-loop approximation is meaningful only for a region that a logarithmic combination $g^2 \ln(\phi^2/M^2)/\pi^2$ is sufficiently small. This is because higher power of the combination is expected to appear in higher orders of the loop expansion. Therefore in the reliable region of our calculation, Eq. (20) is simplified as

$$V_{\text{eff}}(\phi) \sim (m_1^2 + 12\xi_1 a^{-2})\phi^2 - \frac{3g^2}{16\pi^2}a^{-2}\phi^3 \ln\left(\frac{\phi^2}{M^2}\right),$$

because we should have $g^2(m_1^2 + 12\xi_2 a^{-2})\ln(\phi^2/M^2)/\pi^2 \ll m_1^2 + 12\xi_2 a^{-2}$ under the above assumption, $m_1 \sim m_2$ and $\xi_1 \sim \xi_2$. From Eq. (21), we realize the following.

(1) If $\xi_1$ is of order unity or larger, say $\xi_1 = 1/6$, an inequality $g^2 \ln(\phi^2/M^2)/\pi^2 \ll O(1)$ should be satisfied in the reliable region of the one-loop approximation. Therefore the effective potential Eq. (21) is essentially given by the tree potential. The one-loop correction in Eq. (21) as it stands can exceed the tree potential for a quite large $\phi$ and it even gives an unbounded potential. This is an artifact in our approximation because such an unboundedness implies a large logarithmic factor, $g^2 \ln (\phi^2/M^2)/\pi^2 \gg 1$, and is outside the expected validity of the one-loop approximation. Thus we cannot obtain any predictions for such a quite large $\phi$ from the one-loop calculation Eq. (21).

(2) If $\xi_1 = 0$ or much smaller than unity as $\xi_1 \ll m_1^2 a^2$, the effective potential is further simplified as

$$V_{\text{eff}}(\phi) \approx m_1^2\phi^2 - \frac{3g^2}{16\pi^2}a^{-2}\phi^3 \ln\left(\frac{\phi^2}{M^2}\right),$$

because we should have $g^2(m_1^2 + 12\xi_2 a^{-2})\ln(\phi^2/M^2)/\pi^2 \ll m_1^2 + 12\xi_2 a^{-2}$ under the above assumption, $m_1 \sim m_2$ and $\xi_1 \sim \xi_2$. From Eq. (21), we realize the following.

Therefore if

$$m_1^2 < g^2 a^{-2}/\pi^2,$$

the tree potential and one-loop correction in (22) can be comparable. Note that this does not imply the breaking of the one-loop approximation; it is possible to satisfy the condition Eq. (23) while the coupling constant $g$ is kept small $g \ll 1$. Moreover under the condition, it is also possible that the tree level potential and the one-loop correction in Eq. (22) become comparable even if $g^2 \ln(\phi^2/M^2)/\pi^2 \ll 1$. The situation is quite similar to the famous massless scalar QED in Ref. 9. There, the existence of two coupling constants $\lambda$ and $e$ and a condition $\lambda \approx O(e^4)$ are essential to conclude that the one-loop correction qualitatively modify the tree potential.
Dynamics of a Scalar Field along a Flat Direction in de Sitter Space

The effective potential in Eq. (17) combined with a numerical integration of Eq. (7) is depicted in Figs. 1~3. We have fixed the coupling constant $g=0.1$. Figure 1 corresponds to case (1) above ($\xi_i=1/4$ and $m_i^2a^2=0$) and we cannot see any deviation from the tree potential. Note that in this case, the finite part of the counter term Eq. (18) vanishes and the effective potential is independent on the renormalization point $M$. On the other hand, the case in Figs. 2 and 3 corresponds to (2) above ($\xi_i=0$ and $m_i^2a^2=0.001<g^2/\pi^2$). As is expected, we see significant modifications of the tree potential by the radiative correction. We again stress this does not imply the breaking of the one-loop approximation. In fact for the range of the figures, $ga\phi \leq 10$, we have $g^2\ln(\phi^2/M^2)/\pi^2 < 0.02$.

In summary, if $\xi_i=0$ and $m_i^2/\pi^2<g^2a^{-2}$, we can expect a qualitative modification of the tree potential within our one-loop calculation. The one-loop correction, however, gives an unbounded potential for a large value of $\phi$. In §4 of Ref. 3, it was argued that if all the soft breaking masses and all the non-minimal couplings are in the same orders, the one-loop effective potential along the flat directions is basically given by Eq. (21) irrespective of the details of the model. Therefore the discussion above can essentially be applied for any models. The only way to get stable flat
direction is to set \( m_i = 0, \xi_i = 0 \) and \( \xi_2 = 1/4 \). But this combination is somewhat artificial. So we will not pursue this possibility.

§ 3. Derivative expansion of the effective action

In the previous section we saw that flat direction receives large radiative correction and is modified qualitatively. So in this section, to know the dynamics of scalar field along flat direction we compute the one-loop effective action to the quadratic order of derivatives of the field. The effective action is inherently nonlocal. But we are interested only in slow rolling region of scalar field \( \dot{\phi} \ll H \phi = \phi/a \), the direction which the scalar field starts to roll to, so it is adequate for us to compute the effective action to the quadratic order in covariant derivatives of the field. We utilize the relation (4) again and get the one-loop contribution to the effective action

\[
\Gamma^{(1)}[\phi] = \frac{1}{2} \left\{ -\frac{1}{2} \text{Res}[\xi(2)] a^4 \lambda^4 - \text{Res}[\xi(1)] a^2 \lambda^2 + \xi(0)(\gamma - \ln a^2 \lambda^2) - \xi'(0) \right\} \quad (24)
\]

In this case generalized zeta functions are expressed as follows,

\[
\text{Res}[\xi(2)] = \text{tr} \frac{1}{6}, \quad (25)
\]

\[
\text{Res}[\xi(1)] = \frac{3}{8 \pi^2 a^4} \text{tr} \int \sqrt{g} d^4 x \frac{a^2}{6} (f_1 - \tilde{m}^2), \quad (26)
\]

\[
\xi(0) = \frac{3}{8 \pi^2 a^4} \text{tr} \int \sqrt{g} d^4 x \frac{a^4}{12} (\tilde{m}^4 - 2 \tilde{m}^2 f_1 + 2 f_2), \quad (27)
\]

\[
\xi'(0) = \frac{3}{8 \pi^2 a^4} \text{tr} \int \sqrt{g} d^4 x \left\{ \left( \gamma + \frac{3}{2} \right) \frac{a^4}{12} (\tilde{m}^4 - 2 \tilde{m}^2 f_1 + 2 f_2) + \frac{1}{12} \int_0^\infty dz \ln z \frac{\partial^3}{(\partial z)^3} (e^{-\pi a z} F(x, x'; a^2 z)) \right\}, \quad (28)
\]

where \( F(x, x'; a^2 z) = \sum_{i=0}^\infty (a^2 z)^i f_i(x, x') \) and \( f_i(x, x') \) are well-known Seeley-deWitt coefficients. The integral in Eq. (28) can be done in dimensional regularization sense and we get the one-loop contribution to the effective lagrangian,

\[
L^{(1)} = \frac{3}{8 \pi^2 a^4} \left[ -\text{tr} \frac{1}{24} \Lambda^4 a^4 - \text{tr} \frac{1}{12} \Lambda^2 a^4 (f_1 - \tilde{m}^2) + \frac{1}{24} (\gamma - \ln a^2 \lambda^2) a^4 (\tilde{m}^4 - 2 f_1 \tilde{m}^2 + 2 f_2) + \frac{1}{24} a^2 \left\{ \tilde{m}^4 \left[ \ln \tilde{m}^2 a^2 - \frac{3}{2} \right] - 2 f_1 \tilde{m}^2 (\ln \tilde{m}^2 a^2 - 1) + 2 f_2 \ln \tilde{m}^2 a^2 \right\} - \text{tr} \frac{1}{12} a^4 \sum_{i=0}^\infty f_i \tilde{m}^{4-2i} \Gamma(i - 2) \right]. \quad (29)
\]

In our Wess-Zumino model

\[
\tilde{m}^2 = m^2 + g^2 \phi^2 \quad \text{for spin 0} \quad (30)
\]
\[ m^2 = g^2 \phi^2 + g \gamma \cdot \partial \phi \quad \text{and} \quad \xi = \frac{1}{4} \quad \text{for spin} \quad \frac{1}{2}, \]

where \( g \gamma \cdot \partial \phi \) comes out when we square up the Dirac operator. In the limit \( g a \phi \gg 1 \) we can evaluate the one-loop contribution to kinetic term knowing only first three Seeley-deWitt coefficients which depend also on space-time derivatives of \( \phi \). They are already computed, see for example, Guven in Ref 10).

\[ f_1(x, x) = \frac{2-12\xi}{a^2}, \quad (32) \]

\[ f_2(x, x) = -\frac{1}{6} \Box m^2 - \frac{1}{15 a^4} + \frac{1}{2 a^4} (2-12\xi)^2. \quad (33) \]

For \( f_3(x, x) \) we present only leading contribution to kinetic term in the above limit,

\[ f_3(x, x) = \frac{1}{12} \nabla_\mu m^2 \nabla^\mu m^2 + \cdots. \quad (34) \]

Contrary to the effective potential case we can evaluate the effective action only in this limit. But our interest is in large \( \phi \) region where the one-loop effective potential becomes unbounded below. So it is adequate for us to know the one-loop correction to the kinetic term in the above limit. Now we get the one-loop contributions to coefficient of kinetic term, \( Z^{(1)}(\phi) \). We get

\[ Z^{(1)}(\phi) \big|_{boson} = \frac{g^2}{48 \pi^2} \]

for the boson contribution and

\[ Z^{(1)}(\phi) \big|_{fermion} = -\frac{g^2}{32 \pi^2} \left( \ln \frac{g^2 \phi^2}{\Lambda^2} + \gamma + \frac{2}{3} \right) \]

for fermion contribution. There is only finite contribution to \( Z^{(1)}(\phi) \) in boson case as is well known. Both contributions are the same as those calculated in flat space.\(^{11}\)

Collecting both contributions up to \( O((g^2 a^2 \phi^3)^0) \) we get

\[ Z^{(1)}(\phi) = \frac{g^2}{32 \pi^2} \left[ \ln \frac{A^2}{g^2 \phi^2} - \gamma \right]. \quad (37) \]

We set the renormalization condition at the renormalization point \( M \) as

\[ 1 + Z^{(1)}(\phi) \big|_{\phi = M^2} = 1 \quad (38) \]

in the flat limit \( a \to \infty \) or large \( \phi \) limit. Then the relation between the renormalized wave function and the bare one becomes

\[ \partial_\mu \phi_R \cdot \partial^\mu \phi_R = \left( 1 + \frac{g^2}{32 \pi^2} \left[ \ln \frac{A^2}{g^2 M^2} - \gamma \right] \right) \partial_\mu \phi \cdot \partial^\mu \phi. \quad (39) \]

Finally we can get the effective kinetic term at large \( \phi_R \) limit

\[ L_{\text{kin}} = (\partial_\mu \phi \cdot \partial^\mu \phi - \partial_\mu \phi_R \cdot \partial^\mu \phi_R) + \partial_\mu \phi_R \cdot \partial^\mu \phi_R + (1 \text{ loop correction to } \partial_\mu \phi_R \cdot \partial^\mu \phi_R) \]
At the last expression we have omitted the subscript $R$. Now we can discuss the space-time dependence of the scalar field along flat direction.

§ 4. Conclusion and discussion

It has been reviewed that in de Sitter space flat direction is in general made unstable by radiative corrections, if the curvature coupling $\xi$ is renormalized as smaller-than unity and the soft breaking masses are sufficiently small as in Eq. (23). The asymptotic form of the effective potential then reads

$$V_{\text{eff}}(\phi) \rightarrow -\frac{3g^2}{16\pi^2} H^2 \phi^2 \ln\left(\frac{\phi^2}{M^2}\right).$$

(41)

Since this problem is in a large logarithm region, the renormalization group improvement is useful. The renormalization group analysis in appendix of Ref. 3) suggests that the unbounded potential is reliable as long as $\phi^2 \ll M^2 \exp(1/g\, R^2)$. Keeping this fact in mind, in this paper we calculated the one-loop contribution to kinetic term of scalar field along flat direction in de Sitter space. That is because whether the scalar field along flat direction rolls down the unbounded potential or not depends also on the kinetic term. But as is read in Eq. (36), in the region that loop expansion is reliable tree contribution is dominant. In addition, in the slow roll region ($\dot{\phi} \ll H\phi$) the magnitude of one-loop correction to the kinetic term is much smaller than that of $V_{\text{eff}}(\phi)$ in (41). So, as far as slow rolling and large $\phi$ condition

$$\dot{\phi} \ll H\phi \ll g\phi^2$$

(42)

holds, the scalar field along flat direction rolls down the unbounded potential in the large $\phi$ region. But if the value of $\phi$ exceeds Planck scale we expect that non-renormalizable interaction effect may make the potential bounded from below. So the scalar field along flat direction rolls down to large value in the inflationary era. And after the cosmological constant becomes small the scalar field begins to fall down slowly the tree potential with soft breaking mass to the origin. This is because as Hubble constant becomes smaller, the loop correction Eq. (41) also does. Our result implies that the initial condition for the Affleck-Dine mechanism is likely realized if we take into account the quantum correction in de Sitter space.

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* We note this behavior is also true for general spacetime whose typical curvature is smaller than the scale of $\phi$, because our result for the $a \to \infty$ coincides with adiabatic (or the small curvature) expansion. 10
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