Direction dependence and non-Gaussianity in the high-redshift supernova data

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ABSTRACT
The most detailed constraints on the accelerating expansion of the universe and the nature of dark energy are derived from the high-redshift supernova data, assuming that the luminosity distance versus redshift relation is isotropic and the errors in the measurements are Gaussian. There is a possibility that there is a systematic direction dependence in the data, either due to uncorrected, known physical processes or because there are tiny departures from the cosmological principle, making the universe slightly anisotropic. To investigate this possibility, we introduce a statistic based on extreme value theory and apply it to the gold data sets from Riess et al. Our analysis indicate a systematic, directional dependence in the supernova data in both sets, which using the bootstrap distribution comes to about 80 per cent level of confidence for Riess et al. and 90 per cent for Riess et al. Equally importantly, we show that while the 2007 data fit the cold dark matter (ΛCDM) model better than the 2004 data, the level of non-Gaussianity, quantified by departures of our statistic from the Gaussian predictions has become worse. In fact, we find that Riess et al. data lie totally outside the distribution obtained by assuming the noise to be Gaussian.

Key words: supernovae: general – cosmological parameters – large-scale structure of Universe.

1 INTRODUCTION
During the last decade the possibility that the expansion of our Universe is accelerating has been put on a firm footing. The combined analysis of high-redshift supernova data (Riess et al. 1998, 2002; Perlmutter et al. 1999; Riess et al. 2004, hereafter GD04; Riess et al. 2007, hereafter GD07), along with observations of cosmic microwave background (Benoit et al. 2003; Page et al. 2007) or large-scale structure (Percival et al. 2002; Tegmark et al. 2004) indicate a spatially flat universe with low-matter density (around one-third of the critical density), the rest of the closure density is believed to be in an unknown form, generically termed as the dark energy. It is this component that drives the late time acceleration of the expansion of the universe.

The simplest possibility, which fits the data well, is that the acceleration is caused by the presence of a cosmological constant term, called the Λ term. When combined with the usual matter term, the resultant model is referred to as the Λ cold dark matter (ΛCDM) model. In this model, the Hubble parameter asymptotically approaches a constant at late times, thus causing the universe to accelerate. There are compelling theoretical reasons to believe that the dark-energy density may not be a strict constant. There are several physical models where dark energy is generated dynamically from an evolving scalar field (called quintessence) or even from alternate theories of gravity (see Sahni & Starobinsky 2006, for a recent review of models and methods of reconstruction of cosmic history). Due to its simplicity, it has become popular to phenomenologically model dark energy as an ideal fluid with an equation of state given by $p = w\rho$, where $w$ is allowed to be negative. In this model, $w = -1$ gives the usual cosmological constant. Models where $w$ is a constant other than $-1$ or a simple function of redshift have also been considered.

Cosmological data are rapidly approaching a quality where we can start discriminating competing models of dark energy. The effect of small departures from a strict cosmological constant on cosmological observations is sufficiently small to render such analysis unreliable if the nature of statistical noise in data is not well understood. Since the most detailed constraints on dark energy are derived from the luminosity distance to distant supernovae, we would like to be certain that their statistics is well understood. The central limit theorem (Kendall & Stuart 1977) ensures that, to a very good approximation, statistical noise due to a large number of random influences can be treated as Gaussian. In practice this limit may not be realized, perhaps due to non-random systematic influences, such as uncorrected extinction due to a statistically isotropic dust.
Apart from non-Gaussianity in an otherwise isotropic data, the supernovae distances could suffer from direction dependent deviations in the distance-redshift relation. There are several possible sources of deviations from an isotropic data; according to Kolatt & Lahav (2001) these could be (1) statistical scatter due to location in the host galaxy and galaxy type (2) scatter due to dust absorption in the host galaxy, inter galactic medium or in our own Galaxy or (3) due to lensing along the line-of-sight. Some of these processes are corrected for in the data reduction. It is also possible that the observed anisotropy is a result of collation of data of disparate quality, perhaps due to differences in seeing condition or in the data reduction process (Nessris & Perivolotapoulos 2004, 2005, 2007; Jain & Ralston 2006; Corasaniti 2006; Jain, Modgil & Ralston 2007).

Modern cosmology is based on the Cosmological Principle (CP) (Peebles 1993), which states that on the large scales the universe is statistically homogenous and isotropic. Even if supernovae were perfect indicators of distance, and if statistical noise in the supernova distances were Gaussian, an anisotropic universe could produce a systematic, direction dependent modulation in the data. Another possibility is that our galaxy could contain anisotropic, gray dust patches, and since the dust correction depends on reddening in the spectrum, this sort of dust could remain uncorrected. Above arguments suggest that there is a strong case for investigating direction dependence in the supernova data.

Kolatt & Lahav (2001) have investigated the possibility of detecting cosmic anisotropy with 79 high-redshift supernovae obtained from Riess et al. (1998) and Perlmutter et al. (1999). In this paper, we use a different statistic than used by them. We use the gold data set (a) containing 157 supernovae (GD04) (b) and GD07 containing 182 supernovae, for our analysis. Our results on anisotropy can be interpreted as a systematic directional dependence in the data due to any of the above-mentioned possibilities.

The outline of this paper is as follows. In Section 2, we describe our methodology in detail. We present our results in Section 3 and our conclusions in Section 4.

2 METHODOLOGY: EXTREME VALUE STATISTIC

Throughout our analysis we have assumed a flat Friedmann–Robertson–Walker universe. We analyse each data set by the following scheme: We find that the $\Lambda$CDM model fits the data reasonably well, therefore, we first obtain the best-fitting models to the complete gold data sets and calculate the dispersion normalized residuals $\chi_i = [\mu_i - \mu_{\Lambda CDM}(z_i; H_0, \Omega_m)]/\sigma_m(z_i)$ for each supernova, where the distance modulus $\mu = 5 \log (d_L/\text{Mpc}) + 25$, the observed values being $\mu_i$ for a supernova at redshift $z_i$, and $\sigma_m(z_i)$ is the observed standard error. We shall consider subsets of the two data sets to construct our statistic. We define the reduced $\chi^2$ in terms of $\chi_i$ as follows:

$$\chi^2 = \frac{1}{N_{\text{subset}}} \sum_i \chi_i^2,$$

where it should be noted that by ‘reduced’ we do not mean ‘per degree of freedom’, since we do not fit the model separately to the subsets of the data. Here, $\chi^2$ is an indicator of the statistical scatter of the subset from the best-fitting $\Lambda$CDM model.

If the CP holds then the apparent magnitude of a supernova should be drawn from some unknown, direction dependent probability distribution. We note that since the same supernova appears in several subsets, the maximization is not done over statistically independent measures of our statistic. Another noteworthy fact is that if the direction dependence has a forward-backward symmetry then this statistic will not be able to detect it. However, due to its ease of construction and use we first consider only this simplest of possible statistics.

To interpret our results we need to know the range of $\Delta$ that we can expect if there were no directional dependence on the luminosity distance redshift relation, and the noise in the measurements were Gaussian. The distribution of supernovae is not uniform on the sky, therefore, the number of supernovae in the two hemispheres, for a given direction, varies with the direction $\hat{n}$ in a complicated manner. Therefore one might expect the probability distribution function $P(\Delta)$ to be extremely complicated, however, extreme value theory (Kendall & Stuart 1977) shows that the distribution is, in fact, a simple, two parameter Gumbel distribution, characteristic of extreme value distribution type I:

$$P(\Delta) = \frac{1}{\delta} \exp \left[ - \frac{\Delta - \mu}{\delta} \right] \exp \left[ - \exp \left( - \frac{\Delta - \mu}{\delta} \right) \right],$$

where the position parameter $\mu$ and the scale parameter $\delta$ completely determine the distribution.

To quantify the significance of any departures from isotropy, we need to know the theoretical distribution $P_{\text{theory}}(\Delta)$ in the isotropic case with Gaussian noise. It is difficult to obtain the parameters $\mu$ and $\mu$ analytically, therefore, we have calculated this distribution numerically by simulating several sets of Gaussian distributed $\chi_i$ on the two gold sets supernovae positions and obtaining $\Delta$ from each realization. We only provide a graphical comparison with the Gumbel distribution and quote the various probabilities that have been obtained numerically. We plot this distribution for GD04 in Fig. 1 and for GD07 in Fig. 2 as the broken curve. We find that, as expected, the distribution closely approximates the Gumbel distribution.

If the noise in the data were Gaussian in nature then the above distribution would adequately quantify the directional dependence of the normalized residuals. However, if the data contain non-Gaussian noise then the theoretical distribution, represented by the broken curve, cannot be used to quantify the level of significance of our possible discovery of anisotropy.

We have constructed an independent test for directional dependence by obtaining the bootstrap distribution $P_{\text{BS}}(\Delta)$, which is constructed in the following manner. The observed $\chi_i$s are assumed to be drawn from some unknown, direction dependent probability distribution. We shuffle the data values $z$, $m(z)$ and $\sigma_m(z)$ over the supernovae positions, thus destroying any directional alignment they might have had due to anisotropy. Thus, we are able to generate several realizations of data and estimate the distribution $P_{\text{BS}}(\Delta)$. We show in the next section that this distribution allows us to quantify the anisotropy in the data.

3 RESULTS

Before listing our main results we would like to discuss a problem with the bootstrap distribution. We expect it to be shifted slightly for the two hemispheres separately to obtain $\Delta \chi^2 = \chi^2 - \chi^2_{\text{south}}$, where we have defined ‘north’ as that hemisphere towards which the direction vector $\hat{n}$ points. We are only interested in the magnitude of this difference, therefore, we take the absolute value of $\Delta \chi^2$, and then vary the direction $\hat{n}$ across the sky to obtain the maximum absolute difference

$$\Delta = \max \left\{ |\Delta \chi^2| \right\}.$$

We note that since the same supernova appears in several subsets, the maximization is not done over statistically independent measures of our statistic. Another noteworthy fact is that if the direction dependence has a forward-backward symmetry then this statistic will not be able to detect it. However, due to its ease of construction and use we first consider only this simplest of possible statistics.
to the left of the theoretical distribution. This is due to the fact that theoretical distribution is obtained by assuming $\chi_i$s to be Gaussian random variates with a zero mean and unit variance. Therefore, theoretical $\chi_i$s are unbounded. However, the bootstrap distribution is obtained by shuffling through a specific realization of $\chi_i$, and they have a maximum value such that $|\chi_i| < \chi_0$. Since the bootstrap realizations have bounded $\chi_i$, they should produce, on the average, slightly smaller values of $\Delta$ as compared to what one expects from an unbounded Gaussian distributed $\chi_i$. For a large number of supernovae, this bias is expected to vanish.

Since the data sets contain only 157 and 182 supernovae, respectively, a valid concern might be that the difference between the broken and solid curves is due to the small sample size, and not (as we will discuss below) due to any non-Gaussianity present in the noise. To test this we have simulated mock data sets with the same number of supernovae as in the GD04 set, with randomly chosen positions on the sky, and with Gaussian noise added to an isotropic luminosity distance redshift relation. We then process the simulated data in exactly the same way as we do in the case of the actual data – that is we compare the ‘theoretical’ distribution of $\Delta$ (obtained from a large number of Monte Carlo realizations) with the distribution obtained from bootstrap resembling of a single realization. A typical result is shown in Fig. 3. There are a few things to be noted. The simulated data give a $\Delta$ distribution that is consistent with the theoretical distribution, indicating that a large data set (i.e. much larger than GD04) is not a strict requirement for the application of the bootstrap method. However, this shift is found to be of the order of 10 per cent for a data set that has size similar to GD04. This test has been done assuming Gaussian noise, although a more appropriate test would be if we had assumed a direction dependent noise. However, since this requires a specific choice of anisotropy, and since our conclusions would depend on this choice, we have decided to skip this test for this paper.

3.1 GD04

Our main result for the GD04 data set is plotted in Fig. 1. We find that the theoretical distribution $P_{\text{theory}}(\Delta)$ (broken line) generated assuming Gaussian noise, shows that our universe is consistent with the theoretical distribution at about one sigma level and has a smaller anisotropy than the peak of the distribution. However, as mentioned in the last section, if the residuals $\mu_i$ are non-Gaussian then a more appropriate estimate of departure from directional anisotropy would be the bootstrap distribution $P_{\text{BS}}(\Delta)$. We have plotted $P_{\text{BS}}(\Delta)$ (obtained by randomly shuffling $\chi_i$ on the given supernovae positions) in the same figure. We find that the observed value of $\Delta$ is away by about 80 per cent from the peak of the bootstrap distribution and 69 per cent away from the peak of the theoretical distribution. If one takes into account the general shifting of the bootstrap distribution leftwards due to reasons mentioned earlier, the true level of significance of the result is closer to about one sigma. Kolatt & Lahav (2001) also find a direction dependent systematic at about the same significance level.

Fig. 1 shows that the bootstrap distribution has a mean that is about 40 per cent shifted from the theoretical distribution, and has a different shape, indicating non-Gaussianity. As noted above, the expected shift based on Gaussian isotropic noise is on the order of 40 per cent. The excess leftward shift cannot be reconciled with the theoretical distribution by a simple scaling of the error bars.
To produce a rightward shift we would need to increase $\Delta$, which can be done by decreasing the error bars on supernovae by a constant scale factor. However, this would also produce a larger $\chi^2$ for the best-fitting $\Lambda$CDM model. The data actually give $\chi^2 = 1.14$, and scaling would make it larger, thus making the primary fit worse.

We find that the gold data set GD04 is maximally asymmetric in the direction ($l = 100^\circ$, $b = 45^\circ$). We designate the two hemispheres as ‘hot’ or ‘cold’ according to largeness or smallness of their reduced $\chi^2$ (as given in equation 1) with respect to the best-fitting $\Lambda$CDM model. The parameter estimation for these subsets of supernovae is tabulated in Table 1, where we find that the best-fitting $\Lambda$CDM model for hot supernovae give $\Omega_m = 0.30$ and the cold ones give $\Omega_m = 0.31$, so the difference is only a few per cent. However, the situation is not the same for a model with a constant equation of state $p = w\rho$. We find that the model parameters for the hot supernovae in this model are $\Omega_m = 0.51$ and $w = -4.53$, and for the cold supernovae $\Omega_m = 0.32$ and $w = -1.03$. The value of the Hubble constant is relatively quite robust, showing that most of the direction dependence is due to the high-redshift supernovae. The large difference in the values for the constant $w$ model shows that the level of non-Gaussianity indicates that constraints on a more complicated dark-energy model are not very reliable. Perhaps this explains the intriguing result in Alam et al. (2004), that the data seem to fit a $\Lambda$CDM model as well as a model with a strongly evolving dark energy.

3.2 GD07

The results of our analysis for GD07 are plotted in Fig. 2. Here, too we find evidence for direction dependence and non-Gaussianity. The most striking thing about this figure is that GD07 set sits completely outside the theoretical Gumbel distribution $P_{\text{theor}}(\Delta)$ (broken line) generated assuming Gaussian noise. This appears, therefore, to indicate strong evidence that the noise in GD07 is non-Gaussian. Another notable difference is that the observed value of $\Delta$ is shifted from the peak of the bootstrap distribution at a 90 per cent confidence level, which is marginally more significant evidence in support of anisotropy than Kolatt & Lahav (2001).

The gold set GD07 is maximally asymmetric in the direction ($l = 343^\circ$, $b = 22^\circ$). The parameter estimation for the hot and cold subsets of supernovae for this data set is tabulated in Table 2. In the $\Lambda$CDM cosmology, the cold supernovae sit close to the global fit but now we find that the hot set does not give the global fit but instead gives $\Omega_m$ about 43 per cent shifted from the best-fitting value. This is in stark contrast to the GD04 data, where the difference is only about 3 per cent.

4 CONCLUSIONS

We have constructed and used a statistic based on the extreme value theory on the gold data sets from GD04 and GD07. We have found non-Gaussianity and direction dependence in both data sets. We have found that the direction dependence, based on the bootstrap distribution, is significant at about 80 per cent significance level for the GD04 data and 90 per cent for the GD07 data. In either case, the significance of the result is not very strong. We also find that the direction of maximal asymmetry changes from one set to the other, which perhaps indicates that anisotropy and non-Gaussianity in the two data sets arise due to some observational cause (that is different in the two sets) rather than due to a failure of the CP. The amount of non-Gaussianity in GD07 is at a level that $\Delta$ for GD07 lies completely outside the theoretical curve based on the assumption of Gaussianity, strongly indicating the presence of non-Gaussian noise. Although the data of GD07 fit the $\Lambda$CDM model more precisely than the older GD04 data, the non-Gaussian features have seemingly increased. We also find that our results cannot be trivially understood by a simple scaling of error bars. We have discussed that care should be taken while interpreting dark energy beyond the

![Figure 3](https://academic.oup.com/mnras/article-abstract/388/1/242/1011739/Direction-dependence-and-non-Gaussianity-in-the-supernova-data/243)

**Figure 3.** Here we plot a typical result of our simulation with 157 supernovae to test for the efficacy of our technique with a small number of data points. The supernovae positions were generated randomly and populated with Gaussian noise. Similar to Figs 1 and 2 of this plot shows the Gaussian versus the bootstrap distribution. The simulated universe is seen to be consistent with the theoretical distribution, thus indicating that our statistic does not require a dense sky coverage.

**Table 1.** The model parameters for the hot and cold set for GD04 are tabulated here. WCDM refers to a model where the dark energy has a constant equation of state.

<table>
<thead>
<tr>
<th>Model</th>
<th>Subset</th>
<th>$\Omega_m$</th>
<th>$w$</th>
<th>$H_0$</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda$CDM</td>
<td>Hot</td>
<td>0.30</td>
<td>$-1$</td>
<td>64.80</td>
<td>1.55</td>
</tr>
<tr>
<td>$\Lambda$CDM</td>
<td>Cold</td>
<td>0.31</td>
<td>$-1$</td>
<td>63.88</td>
<td>0.70</td>
</tr>
<tr>
<td>WCDM</td>
<td>Hot</td>
<td>0.51</td>
<td>$-4.53$</td>
<td>68.46</td>
<td>1.49</td>
</tr>
<tr>
<td>WCDM</td>
<td>Cold</td>
<td>0.32</td>
<td>$-1.03$</td>
<td>63.90</td>
<td>0.71</td>
</tr>
</tbody>
</table>

**Table 2.** The model parameters for the hot and cold set for GD07 are tabulated here. WCDM refers to a model where the dark energy has a constant equation of state.

<table>
<thead>
<tr>
<th>Model</th>
<th>Subset</th>
<th>$\Omega_m$</th>
<th>$w$</th>
<th>$H_0$</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda$CDM</td>
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<td>0.43</td>
<td>$-1$</td>
<td>60.18</td>
<td>1.12</td>
</tr>
<tr>
<td>$\Lambda$CDM</td>
<td>Cold</td>
<td>0.29</td>
<td>$-1$</td>
<td>64.54</td>
<td>0.58</td>
</tr>
<tr>
<td>WCDM</td>
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<td>0.53</td>
<td>$-2.01$</td>
<td>61.76</td>
<td>1.12</td>
</tr>
<tr>
<td>WCDM</td>
<td>Cold</td>
<td>0.40</td>
<td>$-1.4$</td>
<td>65.14</td>
<td>0.58</td>
</tr>
</tbody>
</table>
cosmological constant model since it is possible that systematic noise may masquerade as evolving dark energy. Although we have found evidence for non-Gaussianity and directional dependence, we have not been able to single out the primary cause for this. Further work is required to fully understand our results and the statistic that we have introduced for our analysis, and will be discussed in greater detail in a future work.

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REFERENCES

Nesseris S., Perivolaropoulos L., 2005, Phys. Rev. D, 72, 123519

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