The effects of twisted magnetic field on coronal loop oscillations and dissipation

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ABSTRACT
The standing magnetohydrodynamic (MHD) modes in a cylindrical incompressible magnetic flux tube modelled as a straight core surrounded by a magnetically twisted annulus, both embedded in a straight ambient external field, are considered. The dispersion relation for the MHD waves is derived and solved numerically to obtain the frequencies of both the kink (m = 1) and fluting (m = 2, 3) waves. Damping rates due to both viscous and resistive dissipations in the presence of the twisted magnetic field are derived and solved numerically for both the kink and fluting waves.

Key words: Sun: corona – Sun: magnetic fields – Sun: oscillations.

1 INTRODUCTION
The solar corona is highly structured by magnetic flux tubes in the form of coronal loops. Transverse oscillations of coronal loops were first identified by Aschwanden et al. (1999) and Nakariakov et al. (1999) using the observations of TRACE (the Transition Region And Coronal Explorer). Edwin & Roberts (1983) elaborated on the dispersion relation for a magnetic cylinder embedded in a magnetic environment typical of that of the solar photosphere and corona. They found that the existence of inhomogeneities in the form of structuring of the magnetic field enables loops to act as wave guides for a variety of different modes. Karami, Nasiri & Sobouti (2002) used the model of Edwin & Roberts (1983), and solved numerically the dispersion relation for each mode in its full generality. They obtained that in the presence of weak viscous and ohmic dissipations, the damping rate is inversely proportional to the Reynolds and Lundquist numbers, \( R \) and \( \xi \), respectively.

An additional feature of the flux tube is that of twist. Bennett, Roberts & Narain (1999) examined the influence of magnetic twist on the modes of oscillations of a magnetic flux tube. They found that twist introduces an infinite band of body modes. Klimchuk, Antiochos & Norton (2000) introduced twist to resolve the internal structure on an individual loop embedded within a much larger dipole configuration. Mikhailyaev & Solov’ev (2005) investigated the magnetohydrodynamic (MHD) waves in a double magnetic flux tube embedded in a uniform external magnetic field. The tube consists of a dense hot cylindrical cord surrounded by a co-axial shell. They found that two slow and two fast magnetosonic modes can exist in the thin double tube.

Verwichte et al. (2004), using the observations of TRACE, detected the multimode oscillations for the first time. They found that two loops are oscillating in both the fundamental and the first-overtone standing kink modes. According to the theory of MHD waves, for uniform loops, the period ratio \( P_1/2P_2 \) of the fundamental mode and its first overtone are exactly 1. But the ratios found by Verwichte et al. (2004) are 0.91 ± 0.04 and 0.79 ± 0.03, and thus clearly differ from 1. This may be caused by different factors such as the effects of curvature (see e.g. Van Doorsselaere et al. 2004), leakage (see De Pontieu, Martens & Hudson 2001), density stratification in the loops (see e.g. Andries et al. 2005; Erdélyi & Verth 2007 and Karami & Asvar 2007) and magnetic twist (see Erdélyi & Fedun 2006 and Erdélyi & Carter 2006).

McEwan et al. (2006) studied the departure of the period ratio \( P_1/2P_2 \) from unity in the presence of the transverse, longitudinal structuring and gravitational stratification. Structure along the loop was found to be the dominant effect. They concluded that this deviation can be used as a seismological tool in the corona.

Erdélyi & Fedun (2006) studied the wave propagation in a twisted cylindrical magnetic flux tube embedded in an incompressible but also magnetically twisted plasma. They found that magnetic twist will increase, in general, the periods of waves approximately by a few per cent when compare to their untwisted counterparts. Erdélyi & Carter (2006) used the model of Mikhailyaev & Solov’ev (2005) but for a fully magnetically twisted configuration consisting of a core, annulus and external region. They investigated their analysis by considering magnetic twist just in the annulus, the internal and external regions having straight magnetic field. Two modes of oscillations occurred in this configurations: surface and hybrid modes. They found that when the magnetic twist is increase, the hybrid modes cover a wide range of phase speeds, centred around the annulus, longitudinal Alfvén speed for the sausage modes.

Carter & Erdélyi (2007) investigated the oscillations of a magnetic flux tube configuration consisting of a core, annulus and external region, each with a straight distinct magnetic field in an

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incompressible medium. They found that there are two surface modes arising for both the sausage and kink modes for the annulus-core model where the monolithic tube has solely one surface mode for the incompressible case. They also showed that the existence and width of an annulus layer has an effect on the phase speeds and periods. Carter & Erdélyi (2008) used the model introduced by Erdélyi & Carter (2006) to include the kink modes. They found for the set of kink body modes, the twist increases the phase speeds of the modes. Also, they showed that there are two surface modes for the twisted shell configuration, one due to each surface, where one mode is trapped by the inner tube, and the other by the annulus itself.

In this work, our aim is to investigate the effects of the twisted magnetic field on oscillations and damping of standing MHD waves in the coronal loops observed by Verwichte et al. (2004) deduced from the TRACE data. This paper is organized as follows. In Section 2, we use the model introduced by Erdélyi & Carter (2006) to derive the equations of motion, to introduce the relevant boundary conditions and to obtain the dispersion relation. In Section 3, we discuss resistive and viscous dissipations to calculate the contributions of the different modes to the heating of the coronal loops. In Section 4, we give numerical results. Section 5 is devoted to conclusions.

2 EQUATIONS OF MOTION

The linearized MHD equations for an incompressible plasma are:

\[ \frac{\partial \delta v}{\partial t} = \nabla \cdot (\delta \mathbf{B}) + \frac{1}{\rho} \nabla \cdot \mathbf{B} + \frac{\nabla \times \delta \mathbf{B} \times (\nabla \times \mathbf{B})}{\rho} \delta v + \frac{\nabla \times \nabla^2 \delta v}{\rho}, \]

(1)

\[ \frac{\partial \delta \mathbf{B}}{\partial t} = \nabla \times (\delta \mathbf{v} \times \mathbf{B}) + \frac{\nabla^2 \delta \mathbf{B}}{4\pi \rho}, \]

(2)

\[ \nabla \cdot \delta \mathbf{v} = 0, \]

(3)

where \( \delta \mathbf{v}, \delta \mathbf{B} \) and \( \delta p \) are the Eulerian perturbations in the velocity, magnetic field and thermal pressure, respectively; \( \rho, \sigma, \nu, v \) and \( c \) are the mass density, the electrical conductivity, the viscosity and the speed of light, respectively. Note that equation (3) satisfies the incompressibility condition.

The simplifying assumptions are as follows.

(i) From Erdélyi & Carter (2006), background magnetic field is assumed to be

\[ \mathbf{B} = \begin{cases} (0, 0, B_r), & r < a, \\ (0, A_0, B_0), & a < r < R, \\ (0, 0, B_r), & r > R, \end{cases} \]

where \( A_0, B_0, B_0 \) are constant and \( a \) and \( R \) are radii of the core and tube, respectively.

(ii) \( \rho \) is constant over the loop but different in the interior, annulus and exterior regions and denoted by \( \rho_c, \rho_0 \) and \( \rho_e \), respectively.

(iii) The equilibrium condition, i.e. radial force balance equation \( \frac{d^2}{dr^2} [p(r) + \frac{\rho(r)}{\rho_0}] \) gives the equilibrium plasma pressure \( p(r) \) as

\[ p(r) = \begin{cases} p_i, & r < a, \\ p_0(a) + \frac{A_0^2}{4\pi} (a^2 - r^2), & a < r < R, \\ p_e, & r > R, \end{cases} \]

where \( p_i, p_e \) are the uniform plasma pressures in the untwisted internal and external regions and \( p_0(a) \) is the plasma pressure at the inner boundary of the twisted annulus layer.

(iv) Tube geometry is a circular with cylindrical coordinates, \( (r, \phi, z) \).

(v) There is no initial steady flow over the tube.

(vi) Viscous and resistive coefficients, \( \nu \) and \( \sigma \), respectively, are constants.

(vii) \( r, \phi \) and \( z \)-dependence for any of the components \( \delta \mathbf{v} \) and \( \delta \mathbf{B} \) is \( \exp \{i (m \phi + k_z z - \omega t)\} \), where \( k_z = \ell \pi / L \), \( L \) is the length of the tube, and \( \ell = (1, 2, \ldots) \) and \( m = (0, 1, 2, \ldots) \) are the longitudinal and azimuthal mode numbers, respectively.

We will further assume that the dissipative terms in equations (1) and (2) are much smaller. We will first solve the problem without these terms and reintroduce them later as small corrections in calculating the contributions of the different modes to the heating of the corona. Taking time derivative of equation (2) and substituting for \( \delta \mathbf{v} / \partial t \) from equation (1), the resulting equation yields to Bessel’s equation for the Eulerian perturbation in the total pressure \( \delta p_t \) as

\[ \left[ \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \left( \frac{m^2}{r^2} + \frac{\beta_0^2}{L^2} \right) \right] \delta p_t = 0, \]

(4)

where

\[ \delta p_t = \delta p + \frac{B \cdot \delta B}{4\pi}, \]

(5)

and

\[ m_0^2 = k_0^2 \left[ 1 - \frac{A_0^2 \omega_0^2}{\pi \rho_0 (\omega^2 - \omega_0^2)} \right], \]

(6)

\[ \omega_{\alpha_0} = \frac{1}{\sqrt{4\pi \rho_0}} (mA_n + k_z B_0). \]

(7)

Equation (4) is same as the result exactly derived by Bennett et al. (1999) and Erdélyi & Carter (2006). Note that subscripts 0 (which correspond to annulus) are replaced by \( i \) and \( e \) corresponding to the internal and external regions, respectively. Since for the internal and external regions \( A_i = A_e = 0 \), hence \( m_i^2 = m_e^2 = k_i^2 > 0 \) and \( \omega_{\alpha_i} = \frac{k_i m_i}{\rho_0}, \omega_{\alpha_e} = \frac{k_i m_e}{\rho_0} \).

Solutions of equation (4) are

\[ \delta p_t = \epsilon I_m(k_i r) \]

(8)

for the interior region \( (r < a) \).

\[ \delta p_t = \left\{ \begin{array}{ll} \beta I_n(m_0 r) + \gamma K_n(m_0 r), & m_0^2 > 0, \\
\beta J_n(n_0 r) + \gamma Y_n(n_0 r), & n_0^2 = -m_0^2 > 0 \end{array} \right. \]

(9)

for the annulus region \( (a < r < R) \) and

\[ \delta p_t = \epsilon K_m(k_e r) \]

(10)

for the exterior region \( (r > R) \). Where \( (J_n, \ Y_n) \) and \( (I_n, \ K_n) \) are the Bessel and modified Bessel functions of the first and second kind, respectively. The coefficients \( \epsilon, \beta, \gamma \) and \( \sigma \) are determined by the boundary conditions. From both Karami et al. (2002) and Erdélyi & Carter (2006), the necessary boundary conditions are that: at the boundaries \( r = a \) and \( r = R \), both the total Lagrangian pressure and \( \delta \mathbf{v} \), should be continuous. These conditions yield to the dispersion relations for surface, \( m_i^2 > 0 \), and hybrid, \( m_e^2 < 0 \), modes which are same as the results obtained by Erdélyi & Carter (2006) in equations (28a) and (28b), respectively. Note that numerical solution of the dispersion relation yields to eigenfrequencies, which are characterized by a trio of wavenumbers \( (n, m \) and \( \ell) \) that actually count the number of nodes or antinodes along \( r, \phi \) and \( z \)-directions, respectively.
3 DISSIPATIVE PROCESSES

The recent observations by TRACE indicate that coronal oscillations are damped (Aschwanden et al. 1999; Nakariakov et al. 1999). By damping we mean that the oscillation decays in time; this may indicate a local physical damping with heating resulting or it may indicate a transfer of energy from the visible oscillations to some other agency (with no heating resulting). Both effects are quite possibly occurring (Roberts 2000). Hence, the observed decay may be due to several effects. Ruderman & Roberts (2002), for example, showed how resonant absorption may be the cause with no significant heating involved. They elaborated that the damping is due to resonant absorption, acting in the inhomogeneous regions of the tube, which leads to a transfer of energy from the kink mode to Alfvén (azimuthal) oscillations within the inhomogeneous layer. They found that the loop oscillations decay occurs principally because of inhomogeneities in the loop. Laing & Edwin (1994) studied the dissipation of fast magnetoacoustic waves by viscosity, heat conduction and radiation. They found that these waves are not readily dissipated in the solar corona.

Following Karami & Asvar (2007), the finite conductivity and viscosity of plasma cause an exponential time decay of disturbances. Hence for weak dissipations, one may assume

\[ \delta B_{\text{dissipative}} = \delta B(r) e^{-\omega t+i\omega t}, \]

\[ \delta v_{\text{dissipative}} = \delta v(r) e^{-\omega t+i\omega t}, \]  

where \( \omega, \delta B \) and \( \delta v \) on the right-hand side are the solutions of the equations (1) and (2) in the absence of dissipations. Substituting equation (11) in equations (1) and (2), cancelling out the non-dissipative terms and keeping only the first-order terms in \( \alpha, c^2/4\pi\sigma \) and \( v \) gives

\[ 2i\alpha \omega \delta B = \frac{c^2}{4\pi\sigma} \left\{ \nabla^2[(B \cdot \nabla)\delta v] - \nabla^2[(\delta v \cdot \nabla)B] \right\}, \]

\[ + \left( \frac{\omega}{\rho} \right) \left( B \cdot \nabla \right) \nabla \delta v - \left( \nabla^2 \delta v \cdot \nabla \right) B, \]  

(12)

where

\[ \delta v = \frac{-i\omega}{4\pi\rho (\omega^2 - \omega_\alpha^2)} \left\{ 4\pi\nabla \delta P_t - 2(\delta B \cdot \nabla)B \right\} \]

\[ = \frac{\omega}{\omega_\alpha \sqrt{4\pi\rho}} \delta B. \]  

(13)

Rewriting equation (12) for either the transverse or the \( z \)-component and substituting for all quantities in terms of \( \delta P_t \) gives

\[ \alpha = \frac{k^2 - m^2}{2} \left( \frac{c^2}{4\pi\sigma} + \frac{\omega}{\rho_0} \right), \]

\[ = \frac{\omega_\alpha k A_0}{\rho_0} \left[ \frac{\omega_\alpha^2}{2\pi(\omega^2 - \omega_\alpha^2)} \right]^2 \left( \frac{1}{S} + \frac{1}{R} \right), \]  

(14)

where \( \omega_\alpha = \frac{\omega}{\omega_\alpha} \), the Lundquist number \( S = (4\pi\rho R^2)/(2\pi R_\alpha^2) \), is the ratio of the resistive time-scale to the Alfvén crossing time and the Reynolds number \( R = (R^2)/(2\pi R_\alpha^2) \) is the ratio of the viscous time-scale to the Alfvén crossing time. Equation (14) shows that when the twist is absent, \( A_0 = 0 \), the damping rate is vanished. Whereas for a compressible plasma, it is not zero. See Karami et al. (2002).

4 NUMERICAL RESULTS

As typical parameters for a coronal loop, we assume \( L = 109 \times 10^3 \) km, \( R/L = 0.01, \rho_0/\rho_1 = 0.1, \rho_0/\rho_1 = 0.5, \rho_1 = 2 \times 10^{-14} \) gr cm\(^{-3} \),

\[ B_0/\rho_1 = 1, B_0/\rho_1 = 1 = 1 = 100 \text{ G}. \]

(100, the damping becomes stronger and the ratio \( \omega/\alpha \) decreases two order of magnitude compared with \( l = 1, 2 \). See again Figs 1 and 2.)
Here in our calculations, the sausage modes \((m = 0)\) are absent. Because following Edwin & Roberts (1983) and Karami et al. (2002), the sausage modes have a lower longitudinal cut-off and they are only expected in fat and dense loops. For instance, according to Aschwanden (2005) for a typical active region, loops which have a density contrast of the order of \(\rho_e/\rho_i \approx 0.1–0.5\) would be required to have width-to-length ratios of \(L/(2R) \approx 1–2\).

The period ratio \(P_1/2P_2\) of the fundamental and first overtone, \(l = 1, 2\) modes of both the kink \((m = 1)\), and fluting \((m = 2, 3)\) surface waves versus the twist parameter plotted in Figs 8–10, respectively. Figs 8–10 show that: (i) for a given relative core width, the period ratio \(P_1/2P_2\) decreases when the twist parameter increases. For \(a/R = 0.65\), for instance, \(P_1/2P_2\) decreases from 1 (for untwisted loop) and approaches below 0.95, 0.88 and 0.82 for \(m = (1, 2 \text{ and } 3)\), respectively, with increasing the twist parameter. Note that when...
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Figure 6. Same as Fig. 1, for the first-overtone fluting \((m = 3)\) surface modes.

Figure 7. Same as Fig. 1, for the kink \((m = 1)\) surface modes with \(l = 100\).

Figure 8. The period ratio \(P_1/2P_2\) of the fundamental and its first-overtone kink \((m = 1)\) surface modes versus the twist parameter for different relative core width \(a/R\) = 0.65 (solid line), 0.8 (dashed line) and 0.9 (dash–dotted line). Auxiliary parameters as in Fig. 1.

Figure 9. Same as Fig. 8, for fluting \((m = 2)\) surface modes.

Figure 10. Same as Fig. 8, for fluting \((m = 3)\) surface modes.

The twist is zero, the diagrams of \(P_1/2P_2\) do not start exactly from unity. This may be caused by the radial structuring \((\rho_0 \neq \rho_i, \rho_e \neq \rho_i)\). But for the selected thin tube with \(R/L = 0.01\), this departure is very small, \(O(10^{-4})\), and does not show itself in the diagrams (see McEwan et al. 2006). (i) For a given twist parameter, the period ratio \(P_1/2P_2\) increases when the relative core width increases. Fig. 8 clears that for kink modes \((m = 1)\) with \(B_\phi/B_z = 0.0065\) and \(a/R = 0.65\), the ratio \(P_1/2P_2\) is 0.941. This is in good agreement with the period ratio observed by Verwichte et al. (2004), \(0.91 \pm 0.04\) deduced from the observations of TRACE. See also McEwan, Díaz & Roberts (2008).

Fig. 11 displays the frequency bandwidth, \(\Delta \omega_1\), including infinite set of the fundamental kink \((m = 1)\) hybrid modes versus the twist parameter and for different relative core width. Fig. 11 presents that: (i) for a given twist parameter, \(\Delta \omega_1\) increases when the relative core width decreases. (ii) For a given relative core width, \(\Delta \omega_1\) increases when the twist parameter increases. This is in good agreement with the result obtained by Carter & Erdélyi (2008).
5 CONCLUSIONS

Oscillations and damping of standing MHD surface and hybrid waves in coronal loops in the presence of the twisted magnetic field are studied. To do this, a typical coronal loop is considered as a straight cylindrical incompressible flux tube with a magnetic twist just in the annulus and a straight magnetic field in the internal and external regions. The linearized MHD equations, when the dissipation is absent, are reduced to a Bessel’s equation for the total Eulerian pressure. The dispersion relation is obtained and solved numerically for obtaining the frequencies of both the kink and fluting modes. The damping rates of oscillations due to the resistive and viscous dissipation in the presence of the magnetic twist are obtained and solved numerically. Our numerical results show that

(i) for a given relative core width, frequencies and damping rates of both the kink \((m = 1)\) and fluting \((m = 2, 3)\) surface waves increase when the twist parameter increases;

(ii) the period ratio \(P_1/P_2\) for both the kink \((m = 1)\) and fluting \((m = 2, 3)\) surface modes is lower than 1 (for an untwisted loop) in the presence of the twisted magnetic field. The result of \(P_1/P_2\) for kink modes is in accordance with the TRACE observations;

(iii) the frequency bandwidth of the fundamental kink \((m = 1)\) hybrid modes increases when the twist parameter increases.

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